F–1911

Sub. Code
7MMA1C1

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

ALGEBRA — I

(CBCS – 2017 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part A　　　　(10 × 2 = 20)

Answer **all** questions.

1. Suppose H and K are subgroups of a group G of order 10 and 21 respectively. Find $O(H\ K)$.

2. Define automorphism of a group.

3. Define normalizer of an element in G.

4. Show that the group of order 21 is not simple.

5. Define commutative ring with an example.

6. Define a ring homomorphism.

7. If D is an integral domain with finite characteristic prove that the characteristic of D is a prime.

8. If I is an ideal of a ring R containing the unit element, show that $I = R$.

9. Let R be an Euclidear domain. Suppose that $a, b, c \in R$, a/bc but $(a, b) = 1$. Then prove that a/c.

10. Define content of the polynomial.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that if H and K are finite subgroups of G of order $O(H)$ and $O(k)$ respectively, then $O(HK) = \frac{O(H).\, O(k)}{O(H \cap K)}$.

Or

(b) Let $\theta : G \to H$ be a onto group homomorphism with kernel K. Prove that G/K is isomorphic to H.

12. (a) Suppose that G is a finite direct sum of $N_1, N_2, .. N_r$. Prove that for $i \neq J$, $N_i \cap N_j = \{e\}$ and if $a \in N$;, $b \in N$; then $ab = ba$.

Or

(b) Show that any two P-sylow subgroups of a group G are conjugate.

13. (a) Prove that a ring homomorphism $\phi : R \to R'$ is one to one if and only if the Kernel of ϕ is zero submodule.

Or

(b) Prove that a finite integral domain is a field.

14. (a) Let R, R' be rings and ϕ a homomorphism of R onto R' with Kernel U. Then prove that R' is isomorphic of R/U.

Or

(b) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

15. (a) If R is an integral domain, then prove that $R[x_1, x_2,..x_n]$ is also an integral domain.

Or

(b) If R is an Euclidean domain prove that any two elements a and b in R have greatest common divisor.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Cayley's theorem.

17. State and prove third part of sylow's theorem.

18. If ϕ is a homomorphism if R into R' with Kernel $I(\phi)$ then prove that

(a) $I(\phi)$ is a subgroup of R under addition.

(b) If $a \in I(\phi)$ and $r \in R$ then both ar and ra ae in $I(\phi)$.

19. (a) Prove that every integral domain can be imbedded in a field.

(b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. The prove that R is a field.

20. (a) State and prove the division algorithm

(b) State and prove the Gauss lemma.

F–1912

Sub. Code
7MMA1C2

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

ANALYSIS — I

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. For $x \in R'$ and $y \in R'$, define $d_1(x, y) = (x - y)^2$, determine, whether it is a metric or not.

2. Define the cantor set. Give an example.

3. If $\overline{E}$ is the closure of a set E in a metric space X, then prove that $diam\overline{E} = diam E$.

4. State the ratio test.

5. Find the radius of convergence of the power series $\sum \frac{2^n}{n^2} z^n$.

6. If Σa_n converges absolutely, then prove that Σa_n converges.

7. Define a continuous function with an example.

8. Define the following terms :

   (a) Monotonically increasing function

   (b) Monotonically decreasing function.

9. Let f be defined on $[a, b]$. If f is differentiable at a point $x \in [a, b]$, then prove that f is continuous at x.

10. State the L'Hospital's rule.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that every k-cell is compact.

Or

   (b) Let P be a nonempty perfect set in R^k. Prove that P is uncountable.

12. (a) Prove the following :

   (i) If $p > 0$, then $\lim_{n \to \infty} \frac{1}{n^p} = 0$

   (ii) If $p > 0$, then $\lim_{n \to \infty} \sqrt[n]{p} = 1$.

Or

   (b) Define e. Prove that e is irrational.

13. (a) Derive partial summation formula.

Or

   (b) State and prove Mertens theorem

14. (a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Prove that $f(X)$ is compact.

Or

(b) If f is a continuous mapping of a metric space X into a metric space Y, and if E is a connected subset of X, then prove that $f(E)$ is connected.

15. (a) State and prove Cauchy mean value theorem.

Or

(b) Suppose f is a continuous mapping of $[a, b]$ into R^k and f is differentiable in (a, b). Prove that there exists $x \in (a, b)$ such that $|f(b) - f(a)| \le (b - a)|f'(x)|$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Suppose $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

17. (a) Define a cauchy sequence. In any metric space X, prove that every convergent sequence is a cauchy sequence.

(b) Suppose $\{s_n\}$ is monotonic. Prove that $\{s_n\}$ converges if and only if it is bounded.

18. Let Σa_n be a series of real numbers which converges, but not absolutely. Suppose $-\infty \le \alpha \le \beta \le \infty$. Prove that there exists a rearrangement $\Sigma a'_n$ with partial sums s'_n such that $\liminf_{n \to \infty} s'_n = \alpha$, $\limsup_{n \to \infty} s'_n = \beta$.

19. Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

20. State and prove Taylor's theorem.

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F–1913

Sub. Code
7MMA1C3

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the binormal line at a point on a curve.

2. What is meant by radius of curvature?

3. Define a right helicoid.

4. When will you say that the components are called direction coefficients?

5. State the Whitehead theorem.

6. Write short notes on Geodesic polar.

7. Define the normal curvature.

8. State the Minding's theorem.

9. Define asymptotic directions.

10. What is meant by the polar developable?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Calculate the curvature and torsion of the cubic curve given by $\vec{r} = (u, u^2, u^3)$.

Or

(b) If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma \rho') + \frac{\rho}{\sigma} = 0$.

12. (a) Calculate the fundamental coefficients E, F, G and H for the paraboloid $\vec{r} = (u, v, u^2 - v^2)$.

Or

(b) Find a surface revolution which is isometric with a region of the right helicoid.

13. (a) Prove that on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$, for all values of u.

Or

(b) Discuss the geodesic parallels.

14. (a) State and prove Gauss-Bonnet theorem.

Or

(b) Find the area of geodesic triangle ABC on a sphere of radius a. Also find the total curvature for the whole sphere.

15. (a) Derive the second fundamental form.

Or

(b) State and prove the Euler's theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that the intrinsic equations of the curve given by $x = ae^u \cos u,\ y = ae^u \sin u,\ z = be^u$ are

$$\kappa = \frac{\sqrt{2}\,a}{(2a^2+b^2)^{\frac{1}{2}}}\cdot\frac{1}{s},\ \tau = \frac{b}{(2a^2+b^2)^{\frac{1}{2}}}\cdot\frac{1}{s}.$$

17. A helicoid is generated by the screw motion of a straight line which meets the axis at an angle α. Find the orthogonal trajectories of the generators. Find also the metric of the surface referred to the generators and their orthogonal trajectories as parametric curves.

18. Derive the differential equations for a geodesic using the normal property.

19. If (λ, μ) is the geodesic curvature vector then prove that $Kg = \dfrac{-H\lambda}{Fu' + Gv'} = \dfrac{H\mu}{Eu' + fv'}$.

20. (a) Define the following terms :

(i) Mean curvature

(ii) Gaussian curvature.

(b) Derive Rodrigue's formula.

F–1914

Sub. Code
7MMA1C4

M.Sc. DEGREE EXAMINATION, APRIL 2019.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. State uniqueness theorem for linear equation with constant coefficients $L(y) = 0$.

2. Compute the Wronskian of $\phi_1(x) = x^2, \phi_2(x) = 5x^2$.

3. Write down the Chebyshev equation.

4. Prove that $P_n(-1) = (-1)^n$.

5. Define indicial polynomial.

6. Find the singular points of the equation $x^2 y'' + (x + x^2)y' - y = 0$ and determine whether they are regular singular points or not.

7. Show that the solution ϕ of $y' = y^2$ which passes through the point (x_0, y_0) is given by $\phi(x) = \dfrac{y_0}{1 - y_0 (x - y_0)}$.

8. Find an integrating factor for the equation
$\cos x \cos y dx - 2\sin x \sin y \, dy = 0$.

9. Let $f(x,y) = \frac{\cos y}{1-x^2} \left(|x|<1\right)$, show that every initial value problem $y' = f(x,y)$, $y(0) = y_0, \left(|y_0| < \infty\right)$ has a solution which exists for $|x|<1$.

10. Consider the initial value problem
$y_1' = y_2^2 + 1$
$y_2' = y_1^2$.
$y_1(0) = 0, y_2(0) = 0$
Compute the first three successive approximations ϕ_0, ϕ_1, ϕ_2.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) The functions ϕ_1, ϕ_2 defined below exist for $-\infty < x < \infty$. Determine whether they are linearly dependent or independent these.

$\phi_1(x) = x, \phi_2(x) = e^{rx}$, r is a complex constant.

Or

(b) Find all solutions of the equation $y'' + 9y = \sin 3x$.

12. (a) Show that $\int_{-1}^{1} P_n^2(x)\, dx = \frac{2}{2n+1}$.

Or

(b) One solutions of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

13. (a) Show that –1 and +1 are regular singular points for the Legendre equation

$$(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0 .$$

Or

(b) Show that $x^{1/2} J_{1/2}(x) = \frac{\sqrt{2}}{\Gamma(1/2)} \sin x$.

14. (a) Find all real valued solutions of the equation $y' = \frac{x + x^2}{y - y^2}$.

Or

(b) Compute the first four successive approximation $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y' = 1 + xy,\ y(0) = 1$.

15. (a) Let $f(x,y) = \frac{\cos y}{1-x^2}, (|x| < 1)$. Show that f satisfies a Lipschitz condition on every strip $S_a : |x| \leq a$, where $0 < a < 1$.

Or

(b) Consider the system :

$$y'_1 = 3y_1 + xy_3$$
$$y'_2 + y_2 + x^2 y_3$$
$$y'_3 = 2xy_1 - y_2 + e^x y_3$$

Show that every initialo value problem for this system has a unique solution which exists for all real x.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let ϕ be any solution of $L(y) = y^{(n)} + a, y^{(n-1)} + \ldots + a_n y = 0$ on an interval I containing a point x_0. Prove that for all x in I $\|\phi(x_0)\| e^{-k|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k|x-x_0|}$. Whose $k = 1 + |a_1| + \ldots + |a_n|$.

17. Solve $(1-x^2)y'' - 2x + y' + \alpha(\alpha+1)\, y = 0$.

18. Derive Bessel function of order α of the first kind $J_\alpha(x)$.

19. Let M, N be two real – valued functions which have continuous first partial derivatives on some rectangle

    $R : |x - x_0| \leqq a, |y - y_0| \leqq b$. Prove that the equation $M(x,y) + N(x,y)y' = 0$ is exact in R if and only if, $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ in R.

20. Let f be a real – valued continuous function on the strip $S : |x - x_0| \leqq a, \quad |y| < \infty, \quad (a > 0)$ and suppose that f satisfies on S a Liphchitz condition with constant $k > 0$. Prove that the successive approximation $\{\phi_k\}$ for the problem $y' = f(x,y), y(x_0) = y_0$ exist on the entire interval $|x - x_0| \leqq a$ and converge these to a solution ϕ of $y' = f(x,y), y(x_0) = y_0$.

F–1915

Sub. Code
7MMA1E1

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

Elective – NUMBER THEORY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. State Euclid's lemma.

2. Define the following terms with an example for each:

   (a) Prime number;

   (b) Composite number.

3. Prove that $\varphi(mn) = \varphi(m)\,\varphi(n)$ if $(m, n) = 1$.

4. Define Liouville's function $\lambda(n)$.

5. Define Euler's constant.

6. Prove that $[-x] = \begin{cases} -[x] & \text{if } x = [x], \\ -[x]-1 & \text{if } x \neq [x] \end{cases}$.

7. If $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$ where $(m, n) = 1$, then prove that $a \equiv b \pmod{mn}$.

8. State the Wilson's theorem.

9. What are the quadratic residues and non-residues mod 13?

10. Define the Jacobi symbol.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that if $2^n + 1$ is prime, then n is a prove of 2.

Or

(b) State and prove the Eulidean algorithm.

12. (a) For $n \geq 1$, prove that $n \prod_{p/n} \left(1 - \frac{1}{p}\right)$.

Or

(b) Prove that $d(n)$ is odd if and only if n is a square.

13. (a) If $x \geq 1$, prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + 0\left(\frac{1}{x}\right)$.

Or

(b) Prove that $\sum_{k=1}^{n} \left[\frac{k}{2}\right] = \left[\frac{n^2}{4}\right]$. Also deduce that $\sum_{k=1}^{n} \left[\frac{k}{3}\right] = \left[\frac{n(n-1)}{6}\right]$.

14. (a) Prove that $5n^3 + 7n^2 \equiv 0 \pmod{12}$ for all integers n.

Or

(b) State and prove the chinese remainder theorem.

15. (a) Show that Legendre's symbol (n/p) is a completely multiplicate function of n.

Or

(b) Determine whether 219 is a quadratic residue or non residue mod 383.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the fundamental theorem of arithmetic.

(b) Given x and y, let $m = ax + by$, $n = cx + dy$, where $ad - bc = \pm 1$. Prove that $(m, n) = (x, y)$.

17. Let f be multiplicative. Prove that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

18. Derive Dirichlet's asymptotic formula.

19. (a) Solve the congruence $5x \equiv 3 \pmod{24}$.

(b) Prove: For any prime p all the coefficients of the polynomial $f(x) = (x-1)(x-2) \ldots (x-p+1) - x^{p-1} + 1$ are divisible by p.

20. State and prove the quadratic reciprocity law.

F–1916		Sub. Code
		7MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

ALGEBRA — II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say a vector space is homomorphism?

2. Define the following terms:

   (a) Linearly independent

   (b) Linear span.

3. What is meant by the annihilator of W?

4. Define an orthonormal set.

5. Define an algebraic number.

6. Write short notes on the splitting field over F.

7. Define the fixed field of G.

8. What is meant by the Galois group of $f(x)$?

9. Define a characteristic vector of T.

10. Define the following terms:

(a) Self-adjoint

(b) Skew-Hermitian.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that the intersection of two subspaces of V is a subspace of V.

Or

(b) If $v_1, v_2, ...v_n$ are in V then prove that either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1, v_2, ...v_{k-1}$.

12. (a) If V is finite-dimensional and $v \neq 0 \in V$, then prove that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

Or

(b) State and prove the Bessel inequality.

13. (a) State and prove the Remainder theorem.

Or

(b) For any $f(x), g(x) \in F[x]$ and any $\alpha \in F$, prove the following :

(i) $(f(x) + g(x))' = f'(x) + g'(x)$;

(ii) $(\alpha f(x))' = \alpha f'(x)$;

(iii) $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.

14. (a) If k is a field and if $\sigma_1, \sigma_2, ..., \sigma_n$ are distinct automorphisms of k, then prove that it is impossible to find elements $a_1, a_2, ..., a_n$, not all 0, in k such that $a_1\sigma_1(u) + a_2\sigma_2(u) + ...a_n\sigma_n(u) = 0$ for all $u \in k$.

Or

(b) Let k be a normal extension of F and let H be a subgroup of $G(k, F)$; let $k_H = \{x \in k | \sigma(x) = x$ for all $\sigma \in H\}$ be the fixed field of H. Prove that

(i) $[k : k_H] = 0(H)$;

(ii) $H = G(k, k_H)$.

15. (a) If V is finite-dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

Or

(b) Let $v = F^{(3)}$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of $T \in A(V)$ in the basis $v_1 = (1, 0, 0)$, $v_2 = (0, 1, 0)$, $v_3 = (0, 0, 1)$. Find the matrix of T in the basis $u_1 = (1,1,1)$, $u_2 = (0,1,1)$. Find the matrix of T in the basis $u_1 = (1,1,1)$, $u_2 = (0,1,1)$, $u_3 = (0, 0, 1)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If S, T are subsets of V, then prove the following

(a) SCT implies $L(S) \subset L(T)$;

(b) $L(S \cup T) = L(S) + L(T)$;

(c) $L(L(S)) = L(S)$.

17. Let V be a finite-dimensional inner product space, then prove that V has an orthonormal set as a basis.

18. Prove that the element $a \in k$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

19. State and prove the fundamental theorem of Galois theory.

20. (a) Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

(b) If N is normal and $AN = NA$, then prove that $AN^* = N^*A$.

F–1917

Sub. Code
7MMA2C2

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

ANALYSIS — II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define the upper and lower Riemann integrals of f over $[a, b]$.

2. Define a refinement of a partition.

3. Define an equicontinuous function on a set.

4. Define an algebra of a family $\mathcal{A}$ of Complex functions defined on a set E.

5. Prove that $\lim_{h=0} \frac{E(z+h)-E(z)}{h} = E(z)$.

6. Prove that the function E is periodic, with period $2\pi i$.

7. Define a measurable set.

8. Let c be a constant and f be a measurable real-valued function defined on the domain. Then prove that the function $f + c$ is also measurable.

9. Define an integrable function over the measurable set.

10. Let f be a non negative measurable function. Show that $\int f = 0$ implies $f = 0$ a.e.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$ if and only if for every $\Sigma > 0$ there exists a partition p such that $U(p, f, \alpha) - L(p, f, \alpha) < \Sigma$.

Or

(b) Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha' \in \mathscr{R}$.

12. (a) Show by an example that the limit of the integral need not be equal to the integral of the limit, even if both are finite.

Or

(b) Prove that a sequence $\{f_n\}$ converges to f with respect to the metric of $\mathscr{C}(X)$ if an only if $f_n \to f$ uniformly on X.

13. (a) Let e^x be defined on R' by $E(x) = e^x$ and $E(x) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$. Then prove that

(i) e^x is continuous and differentiable for all x.

(ii) $(e^x)' = e^x$

(iii) $e^{x+y} = e^x e^y$.

Or

(b) State and prove parseval's theorem.

14. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Then prove that $m^*(\bigcup A_n) \leq \sum m^* A_n$.

Or

(b) Let $< E_n >$ be an infinite decreasing sequence of measurable sets. Let mE_1 be finite then prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} mE_n$.

15. (a) Let ϕ and ψ be simple functions which vanish outside a set of finite measure. Then prove that $\int (a\phi + b\psi) = a\int \phi + b\int \psi$ and if $\phi \geq \psi$ a.e., then prove that $\int \phi \geq \int \psi$.

Or

(b) Show that a necessary and sufficient condition for the Riemann integrability of a bounded function.

Part C $(3 \times 10 = 30)$

Answer any **three** of the following.

16. If r' is continuous on $[a, b]$, then prove that r is rectifiable and $\wedge(r) = \int_a^b |r'(t)| dt$.

17. Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then prove that $\{f_n\}$ converges uniformly on $[a, b]$, to a function f, and $f'(x) = \lim_{n\to\infty} fn'(x)$ $(a \leq x \leq b)$.

18. State and prove Taylor's theorem.

19. Prove that the outer measure of an interval is its length.

20. State and prove Monotone Convergence theorem.

F–1918

Sub. Code
7MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Show that the set of parametric equations $x = a\sin u\cos v$, $y = a\sin u\sin v$, $z = a\cos u$ and the set $x = a\dfrac{1-v^2}{1+v^2}\cos u$, $y = a\dfrac{1-v^2}{1+u^2}\sin u$, $z = \dfrac{2av}{1+v^2}$ both yield the surface $x^2 + y^2 + z^2 = a^2$.

2. What is meant by orthogonal trajectories?

3. Eliminate the arbitrary function f from the equation $z = f(x^2 + y^2)$.

4. Define the general integral of $F(x, y, z, p, q) = 0$.

5. When we say that a first-order partial differential is separable?

6. Write down the fundamental idea of Jacobi's method.

7. Eliminate the arbitrary functions f and g from the equation $z = f(x+ay) + g(x-ay)$.

8. Give an example for $F(D,D')$ is reducible.

9. Write down the exterior Dirichlet boundary value problem for Laplace's equation.

10. Write down the interior Neumann boundary value problem.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Find the integral curves of the equation $\dfrac{dx}{y(x+y)+az} = \dfrac{dy}{x(x+y)-az} = \dfrac{dz}{z(x+y)}$.

Or

(b) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections, with the family of planes parallel to $z = 0$.

12. (a) Find the general integrals of the linear partial differential equation $z(xp - yq) = y^2 - x^2$.

Or

(b) Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.

13. (a) Show that the equations $xp - yq = x$, $x^2 p + q = xz$ are compatible and find their solution.

Or

(b) Solve the equation $p^2 x + q^2 y = z$ using Jacobi's method.

14. (a) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.

Or

(b) Solve the equation $r + s - 2t = e^{x+y}$.

15. (a) Establish a necessary condition for the existence of the solution of the interior Neumann problem.

Or

(b) Derive d'Alembert's solution of the one-dimensional wave equation.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that a necessary and sufficient condition that the pfaffian differential equation $X.dr = 0$ should be integrable is that $X.Curl\,X = 0$.

17. Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis.

18. Find a complete integral of the partial differential equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$.

19. By separating the variables, find the solution of the one-dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{K}\frac{\partial z}{\partial t}$.

20. Determine the temperature $\theta(p, t)$ in the infinite cylinder $0 \le p \le a$ when the initial temperature is $\theta(p, o) = f(p)$ and the surface $p = a$ is maintained at zero temperature.

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F–1919

Sub. Code
7MMA2C4

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

MECHANICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. State d'. Alembert's principle.

2. State the principle of work and kinetic energy.

3. Write down the standard form of Lagrange's equation for non holonomic system.

4. Explain double pendulum.

5. State multipliers rule.

6. Define geodesic problem.

7. Explain first p and off system.

8. State stackel's theorem.

9. Define homogeneous canonical transformation.

10. Define momentum transformation.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain generalised co-ordinates.

Or

(b) Explain non holonomic constraints.

12. (a) Explain natural systems.

Or

(b) Explain Routhian functions.

13. (a) Explain Legendre transformation.

Or

(b) Derive Jacobi's form of principle of least action.

14. (a) Explain Liouville's system.

Or

(b) Derive Hamilton – Jacobi equation.

15. (a) Explain the principle of point transformation.

Or

(b) Consider the transformation $Q = q - tp + \frac{1}{2}gt^2, P = p - gt.$ Find K-H and the generating functions.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Explain virtual work.

17. Find the differential equations of motion for a spherical pendulum of length l.

18. Derive Euler-Lagrange's equation.

19. State and prove Jacobi's theorem.

20. Explain poisson covariant.

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F–1920

Sub. Code
7MMA2E1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

Elective — GRAPH THEORY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a complete bipartite graph with an example.

2. Define a forest. Give an example.

3. What is meant by the k-connected graph? Give an example for a 3-connected graph.

4. Write short notes on Konigsberg problem.

5. Define a perfect matching. Give an example.

6. Find the edge chromatic number of a Petersen graph.

7. Determine the value of $r(2, l)$ and $r(k, 2)$.

8. When will you say a critical graph is block?

9. What is meant by a Jordan curve?

10. Give an example for dual graphs.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If k-regular bipartite graph with $k > 0$ has bipartition (X, Y), then prove that $|X| = |Y|$.

Or

(b) Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

12. (a) Define a block of a graph with an example. If G is a block with $\nu \geq 3$, then prove that any two edges of G lie on a common cycle.

Or

(b) Show that $c(G)$ is well defined.

13. (a) State and prove the Berge theorem.

Or

(b) Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

14. (a) Prove that a set $S \subseteq V$ is an independent set of G if and only if $V - S$ is a covering of G. Also deduce that $\alpha + \beta = \nu$.

Or

(b) State and prove Dirac theorem.

15. (a) Enumerate the dual graphs. Also prove that $\sum_{f \in F} d(f) = 2E$ if G is a plane graph.

Or

(b) (i) If G is a simple planar graph with $\nu \geq 3$, then prove that $\varepsilon \leq 3\nu - 6$.

(ii) Show that $K_{3,3}$ is non planar.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Prove that a graph is bipartite if and only if it contains no odd cycle.

(b) If e is a link of G, then prove that

$\tau(G) = \tau(G - e) + \tau(G.e)$.

17. (a) State and prove the Chvatal theorem.

(b) If G is a simple graph with $\nu \geq 3$ and $\varepsilon > \binom{\nu - 1}{2} + 1$, then prove that G is Hamiltonian.

18. If G is simple graph, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

19. State and prove the Brook's theorem.

20. Show that every planar graph is 5-vertex-colourable.

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F–1921

Sub. Code
7MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say a function is said to be harmonic?

2. Distinguish between translation, rotation and inversion.

3. Define the winding number of γ with respect to a.

4. State the Cauchy's estimate theorem.

5. Define zero and pole. Give an example.

6. Show that the function $\sin z$ have essential singularity at ∞.

7. Find the poles and residue of the function $\frac{1}{\sin^2 z}$.

8. How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2?

9. Write down the formula for the series expansion of are $\tan z$.

10. State the Weierstrass's theorem on power series.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Derive the complex form of the Cauchy-Riemann equations.

Or

(b) Define a linear transformation. Also prove that a linear transformation carries circles into circles.

12. (a) Prove that the line integral $\int_{\gamma} pdx + 2dy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q$.

Or

(b) State and prove the Cauchy's representation formula.

13. (a) State and prove the Taylor's theorem.

Or

(b) State and prove the Schwarz lemma.

14. (a) State and prove the arguments principle.

Or

(b) Evaluate $\int_{0}^{\frac{\pi}{2}} \frac{dx}{a + \sin^2 x}, |a| > 1$.

15. (a) Define an entire function with an example. Also prove that every function which is meromorphic in the whole plane is the quotient of two entire functions.

Or

(b) Derive the Jensen's formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Show that a harmonic function satisfies the formula differential equation $\frac{\partial^2 u}{\partial z\, \partial \bar{z}} = 0$.

(b) Define cross ratio if z_1, z_2, z_3 and z_4 are distinct points in the extended plane and T any linear transformation, then prove that $(T_{z_1}, T_{z_2}, T_{z_3}, T_{z_4}) = (Z_1, Z_2, Z_3, Z_4)$.

17. State and prove Cauchy's theorem for a rectangle.

18. State and prove the local mapping theorem.

19. Show that $\int_0^{\pi} \log \sin \theta \, d\theta = \pi \log\left(\frac{1}{2}\right)$.

20. Obtain the Laurent expansion $\sum_{n=\infty}^{\infty} A_n (z-a)^n$ for the function $f(z)$ analytic in $R_1 < |z - a| < R_2$.

F–1922

Sub. Code
7MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by discrete topology? Give an example.

2. Define the subspace topology with an example.

3. Define the box topology. Give an example.

4. What do you mean by the uniform topology?

5. Define a linear continuum.

6. What is meant by locally path connected?

7. Is the interval $(0,1)$ compact? Justify your answer.

8. State the Lebesgue number lemma.

9. Define a first – countable.

10. What is meant by completely regular space?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11\. (a) If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then prove that the collection $\mathcal{D} = \{B \times C / B \in \mathcal{B}$ and $\mathcal{C} \in \mathcal{C}\}$ is a basis for the topology of $X \times y$.

Or

(b) Let Y be a subspace of X and let A be a subset of Y. Let $\overline{A}$ denote the closure of A in X. Prove that the closure of A in Y equals $\overline{A} \cap Y$.

12\. (a) State and prove the pasting lemma.

Or

(b) Let $f : A \to \overset{\pi}{\underset{\alpha \in J}{}} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let $\pi\, x_\alpha$ have the product topology. Prove that the function f is continuous if and only if each function f_α is continuous.

13\. (a) Show that the union of a collection of connected subspaces of x that have a point in common is connected.

Or

(b) Prove that a space X is locally connected if and only if for every open set U of X each component of U is open in X.

14. (a) Let Y be a subspace of X. Prove that Y is compact if and only if every conversing of Y by sets open in X contains a finite subcollection covering Y.

Or

(b) State and prove uniform continuity theorem.

15. (a) Show that every metrizable space with a countable dense subset has a countable basis.

Or

(b) Prove that a product of completely regular spaces is completely regular.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Prove that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another.

(b) What is meant by an order topology? Give an example.

(c) Write down the Hausdorff axiom.

17. Let $\bar{d}(a,b) = \min\{|a-b|, 1\}$ be the standard bounded metric on $\mathbb{R}$. If x and y are two points of $\mathbb{R}^w$, define $D(x,y) = \sup\left\{\frac{\bar{d}(x_i, y_i)}{i}\right\}$. Prove that D is a metric that induces the product topology on $\mathbb{R}^w$.

18. If L is a linear continuum in the order topology, then prove that L is connected, and so are intervals and rays in L.

19. Let X be a metrizable space. Prove the following are equivalent :

    (a) X is compact.

    (b) X is limit point compact.

    (c) X is sequentially compact.

20. State and prove the Urysohr lemma.

F–1923

Sub. Code
7MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. If the sample space is $\mathcal{C} = C_1 \cup C_2$ and if $P(C_1) = 0.8$ and $P(C_2) = 0.5$, then find $P(C_1 \cap C_2)$.

2. Find the mode of the distribution $f(x) = \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3, \cdots,$ zero elsewhere.

3. Define the distribution function of X and Y.

4. Prove that $E[(X - \mu_1)(Y - \mu_2)] = E(XY) - \mu_1\mu_2$.

5. Define a negative binomial distribution.

6. Define an exponential distribution.

7. Write down the m.g.f. of $Y = \sum_{i=1}^{n} X_i$

8. Write down the mean and variance of the beta distribution.

9. When we say that the sequence of random variables converges in distribution to a random variable with distribution function.

10. When we say that a sequence of random variables $X_1, X_2, \cdots$ converges in probability to a random variable X?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If C_1 and C_2 are subsets of $\mathcal{C}$, then prove that $P(C_1 \cup C_2) = P(C_1) + P(C_2) - P(C_1 \cap C_2)$.

Or

(b) Let the space of random variable X be $\mathcal{A} = \{x : 0 < x < 10\}$ and let $P(A_1) = \frac{3}{8}$, where $A_1 = \{x : 1 < x < 5\}$. Show that $P(A_2) \leq \frac{5}{8}$, where $A_2 = \{x : 5 \leq x < 10\}$.

12. (a) Prove that :

(i) $E[E(X_2/X_1)] = E(X_2)$ and

(ii) $\text{var}[E(X_2/X_1)] \leq \text{var}(X_2)$.

Or

(b) Find $\Pr\left(0 < X_1 < \frac{1}{3}, 0 < X_2 < \frac{1}{3}\right)$ if the random variables X_1 and X_2 have the joint p.d.f. $f(x_1, x_2) = 4x_1(1 - x_2)$, $0 < x_1 < 1$, $0 < x_2 < 1$, zero elsewhere.

13. (a) Compute the measures of skewness and kurtosis of the binomial distribution $b\,(n,\,p)$.

Or

(b) Derive the mean and the variance of a gamma distribution.

14. (a) Let X have the p.d.f. $f\,(x) = \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3, \cdots$ zero elsewhere. Find the p.d.f. of $Y = X^3$.

Or

(b) Let X and Y be random variables with $\mu_1 = 1$, $\mu_2 = 4$, $\sigma_1^2 = 4$, $\sigma_2^2 = 6$, $\rho = \frac{1}{2}$. Find the mean and variance of $Z = 3X - 2Y$.

15. (a) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N\left(\mu_1\ \sigma^2\right)$. Find the limiting distribution of $\overline{X}_n$.

Or

(b) Let Z_n be $\chi^2(n)$. Then prove that the random variable $Y_n = (Z_n - n)/\sqrt{2n}$ has a limiting standard normal distribution.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X have the p.d.f. $f(x) = (x+2)/18$, $-2 < x < 4$, zero elsewhere. Find $E\,(X)$, $E\left[(x+2)^3\right]$ and $E\left[6X - 2\,(X+2)^3\right]$.

17. Let the random variables X and Y have the joint p.d.f.

$$f\,(x,\,y) = \begin{cases} x + y, & 0 < x < 1, \quad 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Compute the correlation coefficient of X and Y.

18. Find the m.g.f. of a normal distribution and hence find mean and variance of a normal distribution.

19. Derive the p.d.f. of *F*-distribution.

20. State and prove the central limit theorem.

F–1924

Sub. Code
7MMA3E1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

Elective — DISCRETE MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Prove that an identity element for any binary operation, if it exists, is unique.

2. Define a monoid homomorphism.

3. Calculate F_4 of the Fibonacci numbers using recursion.

4. Write $p(x) = x^5 + 3x^4 - 15x^3 + x - 10$ in telescopic form.

5. Define a regular function with an example.

6. Define $f(x) = 1 + x$ and $g(x) = +\sqrt{x}$. Then find $g(f(x))$.

7. Let $X = \{1, 2, 3, 4, 6, 12\}$ and $\leq$ be the usual less than or equal to relation. Then draw the Hasse diagram of $(X, \leq)$.

8. Prove that every distributive lattice is modular.

9. Define a sum-of-products canonical form.

10. If a and b are two distinct atoms, then prove that $a \wedge b = 0$.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If a and b are the elements of a group $(G,*)$, then prove that $(a*b)' = b'*a'$.

Or

(b) If $(a*b)^2 = a^2 * b^2$, for all $a.\, b \in G$, then show that G is abelian.

12. (a) Prove that $\sqrt{2}$ is irrational.

Or

(b) Prove by Mathematical induction that $2^n > n$ for all $n \in N$.

13. (a) Find the generating function of the recurrence relation $S(K) = 2S(K-1), S(0) = 1$.

Or

(b) Show that $f(x) = \frac{x}{2}$ is partial recursive.

14. (a) State and prove Cancellation rule for distributive lattices.

Or

(b) If G is a group, then prove that the set of all normal subgroups of G forms a modular lattice.

15. (a) Write down the minterm normal form of $f(x_1, x_2) = \overline{x}_1 \vee \overline{x}_2$.

Or

(b) Let B be the Boolean algebra $\{0, 1\}$ with the usual $\wedge, \vee$ operations, and B be any given finite Boolean algebra. Let $p, q \in P_n$. If $p_B = q_B$, then prove that $p_B = q_B$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Show that $(Z_n, +n)$ is an abelian group.

17. Write the recurrence relation for Fibonacci numbers and solve it.

18. Solve the recurrence relation $S(n) = S(n-1) + 2(n-1)$ with $S(0) = 3, S(1) = 1$ by finding its generating function.

19. Let L be a complemented distributive lattice. For $a, b \in L$, prove that the following are equivalent.

(a) $a \leq b$

(b) $a \wedge b' = 0$

(c) $a' \wedge b = 1$

(d) $b' \leq a'$.

20. Describe the addition of two one-digit binary numbers.

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F–1925

Sub. Code
7MMA3E4

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

***Elective* — FUZZY MATHEMATICS**

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define L-Fuzzy sets.

2. When will you say that a fuzzy sets are called extension principle?

3. Define the Sugeno class.

4. Define the union of two fuzzy sets. Give an example.

5. What is meant by the height of a fuzzy relation?

6. Write down the algorithm for transitive closure $R_T(X, X)$.

7. Define a fuzzy measure with an example.

8. State the Bayesian belief measures.

9. Define a grouping requirement.

10. What is meant by measure of dissonance?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Narrate the following terms with suitable example for each :

(i) height of a fuzzy

(ii) normalized fuzzy set

(iii) α-cut of a fuzzy set.

Or

(b) Show that all α-cuts of any fuzzy set A defined on $\mathbb{R}^n$ $(n \geq 1)$ are convex if and only if

$\mu_A[\lambda r + (1-\lambda)s] \geq \min[\mu_A(r), \mu_A(s)]$ for all $r, s \in \mathbb{R}^n$ and all $\lambda \in [0, 1]$.

12. (a) If C is a continuous fuzzy complement, then prove that C has a unique equilibrium.

Or

(b) For all $a, b \in [0, 1]$, prove that $i(a, b) \geq i_{\min}(a, b)$.

13. (a) Let a binary fuzzy relation R be defined by the following membership matrix :

$$M_R = \begin{bmatrix} 0.7 & 0.4 & 0 \\ 0.9 & 1 & 0.4 \\ 0 & 0.7 & 1 \\ 0.7 & 0.9 & 0 \end{bmatrix}$$. Obtain its resolution form.

Or

(b) Describe the concept of fuzzy ordering relations in detail.

14. (a) Write down the axioms of fuzzy measures. Also explain Dempster's rule of combination.

Or

(b) Show that a belief measures that satisfies $\eta(A \cap B) = \min[\eta(A), \eta(B)]$ is based on nested focal elements.

15. (a) Consider two fuzzy sets, A and B, defined on the set of real numbers $X = [0, 4]$ by the membership grade functions $\mu_A(x) = \frac{1}{1+x}$ and $\mu_B(x) = \frac{1}{1+x^2}$.

Draw graphs for these functions and their standard classical complements.

Or

(b) Let m_X and m_Y be marginal basic assignments on sets X and Y, respectively, and let m be a joint basic assignment on $X \times Y$ such that $m(A \times B) = m_X(A) \cdot m_Y(B)$ for all $A \in \mathcal{P}(X)$ and $\mathcal{P}(Y)$. Prove that $C(m) = C(m_X) + C(m_Y)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let the membership grade functions of sets A, B and C defined on the set $X = \{0,1,..., 10\}$ by $\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = 2^{-x}$, $\mu_C(x) = \frac{1}{1+10(x-2)^2}$. Let $f(x) = x^2$, for all $x \in X$. Use the extension principle to derive $f(A)$, $f(B)$ and $f(C)$.

17. (a) Show that $\lim_{w \to \infty} \min[1, (a^w + b^w)^{\frac{1}{w}}] = \max(a, b)$.

(b) Prove that the following operations on fuzzy sets satisfy De Morgan's laws :

$u_{\max}, i_{\min}, C(a) = 1 - a$.

18. Solve the following fuzzy relation equation :

$$p \circ \begin{bmatrix} 0.9 & 0.6 & 1 \\ 0.8 & 0.8 & 0.5 \\ 0.6 & 0.4 & 0.6 \end{bmatrix} = [0.6 \quad 0.6 \quad 0.5]$$

19. Prove : Given a consonant body of evidence ($\mathfrak{F}$, m), the associated consonant belief and plausibility measures passes the following properties :

(a) $Bel(A \cap B) = \min[Bel(A), Bel(B)]$ for all $A, B, \in \mathcal{P}(X)$.

(b) $Pl(A \cup B) = \max[Pl(A), Pl(B)]$ for all $A, B, \in \mathcal{P}(X)$.

20. (a) Derive the Gibb's inequality.

(b) Narrate the Boltzmann entropy.

F–1926

Sub. Code
7MMA3E5

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

Elective – STOCHASTIC PROCESSES

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say that a stochastic process is said to be a second order process?

2. Define a Gaussian process.

3. Define a Markov chain with an example.

4. What is meant by the mean recurrence time for the state j?

5. Define stationary distribution.

6. Define an intree T_j to a specific point j in a directed graph G.

7. What do you mean by a point process?

8. Define transition density matrix of the process.

9. Define a renewal process.

10. State the Blackwell's theorem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the mean and variance of the following probability distribution.

$$P_r \{x(t) = n\} = \frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1, 2, ...$$

$$= \frac{at}{1+at}, n = 0.$$

Or

(b) Show that every stochastic process $\{X_t, t = 0, 1, 2, ...\}$ with independent increments is a Markov process. Is the converse true?

12. (a) An urn contains b black and r red balls. A ball is drawn at random and is replaced after the drawing, that is drawing is with replacement. Find the outcome at the n^{th} drawing is either a black ball or a red ball.

Or

(b) Classify the Markov chain whose transition matrix is

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{2} \end{pmatrix} \end{matrix}$$

13. (a) If state j is persistent, then prove that for every state k that can be reached from state j, $F_{kj} = 1$.

Or

(b) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that

$$P_r \{N(s) = k \mid N(t) = n\} = \binom{n}{k} (s/t)^k (1 - (s/t))^{n-k}.$$

14. (a) Enumerate Yule – Furry process.

Or

(b) Write down the differential – difference equations for the linear growth process with immigration having $\lambda_n = n\lambda + a$, $\mu_n = n\mu$. Show that $M(t) = E\{x(t)\}$ satisfies the differential equation $M'(t) = (\lambda - \mu) M(t) + a$.

15. (a) Discuss the renewal interval.

Or

(b) Show that the renewal function M satisfies the equation $M(t) = F(t) + \int_0^t M(t-x)\, dF(x)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) What is meant by a martingale process?

(b) Let $\{Z_i\}$, $i = 1, 2, \ldots.$ be a sequence of $i.i.d.$ random variables with mean 0 and let $X_n = \sum_{i=1}^{n} Z_i$. Prove that $\{X_n, n \geq 1\}$ is a martingale.

(c) Find the covariance function of $\{Y_n, n \geq 1\}$ given by $Y_n = a_0 X_n + a_1 X_{n-1} + \ldots. + a_k X_{n-k}$, $n = 1, 2, \ldots.,$ where a's are constants and X n's are uncorrelated random variables.

17. If state K is persistent null, then prove that for every j, $\lim_{n \to \infty} p_{jk}^{(n)} \to 0$. Also if state K is a periodic, persistent non-null then prove that $\lim_{n \to \infty} p_{jk}^{(n)} \to \frac{F_{jk}}{\mu_{kk}}$.

18. (a) Write any three postulates for Poisson process.

(b) With the usual notations, prove that the Poisson process $P_n(t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$, $n = 0, 1, 2, 3, \ldots..$

19. Derive the differential equation of the birth-death process.

20. (a) State and prove Wald's equation.

(b) With probability 1, prove that $\frac{N(t)}{t} \to \frac{1}{\mu}$ as $t \to \infty$, where $\mu = E(X_n) \leq \infty$.

F–1927

Sub. Code
7MMA3E6

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

Elective – COMBINATORIAL MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. What is meant by ordinary generating functions?
2. What is the coefficient of the term x^{23} in $(1 + x^5 + x^9)^{100}$?
3. Define the Fibonacci numbers and form the recurrence relation.
4. Give an example for a linear difference equation with constant coefficients.
5. Write the principle of inclusion and exclusion identity.
6. Define a derangement of integers with an example.
7. Prove that the inverse of any element in a group is unique.
8. Define a permutation group of a set.
9. What is a normal matrix? Give an example.
10. What is a block design? Give an example.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the number of r–digit quaternary sequences in which each of the digits $1, 2$ and 3 appears at least once.

Or

(b) In how many ways can three numbers be selected from the numbers $1, 2, ...300$ such that their sum is divisible by 3?

12. (a) Solve the difference equation $a_n + 2a_{n-1} = n + 3$, with the boundary condition $a_0 = 3$.

Or

(b) Find the number of ways to parenthesize the expression $w_1 + w_2 + ... + w_{n-1} + w_n$ so that only two terms will be added at one time.

13. (a) Find the number of integers between 1 and 250 that are not divisible by any of the integers 2,3,5 and 7.

Or

(b) Find the rook polynomial for the standard 8 × 8 chess board.

14. (a) Find the distinct ways of painting the eight vertices of a cube with two colours x and y.

Or

(b) Show that the binary relation induced by Q is an equivalence relation.

15. (a) Prove that there are no integers a, b, c such that $a^2 + b^2 = 6c^2$, apart from $a = b = c = 0$.

Or

(b) Prove that the Kronecker product of two Hadmard matrices is a Hadamard matrix.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Evaluate the sum $\sum_{i-0}^{r} \frac{r!}{(n-i+1)!(i+1)!}$.

17. State and solve the Taver of Hanoi Problem.

18. If d_n is number of derangement of n objects prove that $d_n - nd_{n-1} = (-1)^n$.

19. State and prove Burnside theorem.

20. In a symmetric balanced in complete block design, prove that every two blocks have exactly λ objects in common.

F–1928

Sub. Code
7MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define bounded linear map.

2. State Jensen's inequality.

3. Let X be a normed space over k, and f be a non zero linear functional on X. If E is an open subset of X, then prove that $f(E)$ is an open subset of k.

4. Let X be a linear space over C and u a real-linear functional on X. Define $f(x) = u(x) - iu(ix)$, $x \in X$. Prove that f is complex-linear functional on X.

5. Let X be a metric space and Y be a metric space. If $F : X \to Y$ is continuous and $G : X \to Y$ is closed, then prove that $F + G : X \to Y$ is closed.

6. Define continuous seminorm.

7. State Riesz representation theorem for L^p.

8. Define dual of a normed space. Give an example.

9. Let $x \in X$, X a linear space. Prove that $\langle x, y \rangle = 0$ for all $y \in X$ if and only if $x = 0$.

10. Define orthogonal projection.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let Y be a subspace of a normed space X. Prove that Y and its closure $\overline{Y}$ are normed spaces with the induced norm.

Or

(b) Let X be a normed space and Y be a subspace of X. If Y is finite dimensional, prove that Y is complete.

12. (a) State and prove Hahn-Banach extension theorem.

Or

(b) Prove that a Banach space cannot have a denumerable basis.

13. (a) Let X and Y be normed spaces and $F : X \to Y$ be linear. Prove that F is continuous if and only if $g \circ f$ is continuous for every $g \in Y'$.

Or

(b) Let X and Y be normed spaces and $F : X \to Y$ be linear. Prove that F is an open map if and only if there exists some $\gamma > 0$ such that for every $y \in Y$, there is some $x \in X$ with $F(x) = y$ and $\|x\| \leq x\|y\|$.

14. (a) Let X be a normed space. Proved that X' is separable, then so is X.

Or

(b) State and prove closed range theorem of Banach.

15. (a) Let X be an inner product space and $f \in X'$. Let $\{u_\alpha\}$ be an orthonormal set in X and $E_f = \{u_\alpha : f(u_\alpha) \neq 0\}$. Then E_f is a countable set say $\{u_1, u_2, ...\}$. If E_f is denumerable, then prove that $f(u_n) \to 0$ as $n \to \infty$.

Or

(b) Prove that $\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2)$ for all $x, y \in X$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X denote a subspace of $B(T)$ with the sup norm, $1 \in X$ and f be a linear functional on X. If f is continuous and $\|f\| = f(1)$, then prove that f is positive conversely, if $\operatorname{Re} x \in X$. Whenever $x \in X$ and if f is positive then prove that f is continuous and $\|f\| = f(1)$.

17. State and prove Hahn-Banach separation theorem.

18. State and prove open mapping theorem.

19. (a) Let X be a normed space and $A \in BL(X)$. Prove that $\sigma(A') \subset \sigma(A)$.

(b) If X is a banach space, then prove that $\sigma(A) = \sigma_a(A) \cup \sigma_e(A') = \sigma(A')$.

20. State and prove Riesz representation theorem on Hilbert space.

F–1929

Sub. Code
7MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write any two rules for constructing the network.

2. What is the difference between PERT and CPM?

3. Define lead time.

4. Write the formula for purchasing cost per unit time in EOQ with price breaks.

5. Define the lack of memory in queuing system.

6. Write a short notes on truncated Poisson distribution.

7. Draw the transition - rate diagram.

8. Find L_s of the model $(M/G/1):(GD/\infty/\infty)$.

9. Write a short notes on dichotomous search method.

10. Define the general constrained non linear programming problem.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Explain Three-Jug puzzle with an illustration.

Or

(b) Construct the network diagram comprising activities $B, C, \ldots.Q$ and N such that the following constraints are satisfied.

$B < E, F$; $C < G, L$; $E, G < H$; $L, H < I$; $L < M$; $H < N$, $H < J$; $I, J < P$; $P < Q$. The notation $X < Y$ means that the activity X must be finished before y can begin.

12. (a) An aircraft company uses rivets at an approximate customer rate of 2,500 kg per year. Each unit cost Rs. 30 per kg and the company personnel estimate that it costs Rs. 130 to place an order, and that the carrying cost of inventory is 10% per year. How frequently should orders for rivets be placed. Also determine the optimum size of each order.

Or

(b) Enumerate the no-setup model.

13. (a) What is a queue? Give an illustration. Also describe the basic elements of a queuing model.

Or

(b) Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following :

(i) The average number of births per year

(ii) The probability that no births will occur in any one day.

(iii) The probability of issuing 50 birth certificates in 3 hours given that 40 certificates were issued during the first 2 hours of the 3-hour period.

14. (a) First bank of Springdale operates a one-lane drive-in ATM machine. Cars arrive according to a Poisson distribution at the rate of 12 cars per hour. The time per car needed to complete the ATM transaction is exponential with mean 6 minutes. The lane can accommodate a total of 10 cars. Once the lane is full, other arriving cars seek service in another branch. Determine the following :

(i) The probability that an arriving car will not be able to use the ATM machine because the lane is full

(ii) The probability that a car will not be able to use the ATM machine immediately on arrival

(iii) The average number of cars in the lane.

Or

(b) Derive L_s, W_q, L_q, W_q for $(M/M/R):(GD/K/K)$, $R < K$ queuing model.

15. (a) Solve maximize $f(x)=\begin{cases} 3x, & 0 \le x \le 2 \\ \frac{1}{3}(-x+20), & 2 \le x \le 3 \end{cases}$ by using golden section method. Given the maximum value of $f(x)$ occurs at $x=2$ and $\Delta = 0.10$.

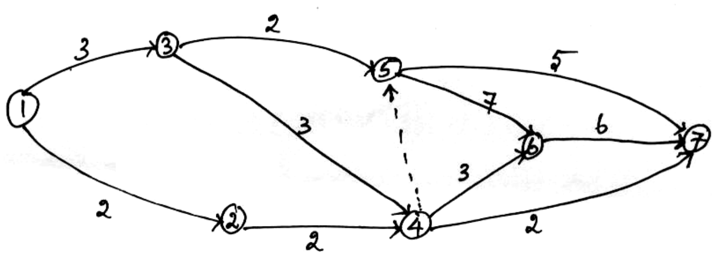
Or

(b) Discuss briefly about "Separable convex programming".

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find the critical path for the following project network.



17. (a) Find the optimal order quantity for a product for which the price breaks are as follows :

Quantity	Unit cost (Rs.)
$0 \le y < 500$	10
$500 \le y < 750$	9.25
$750 \le y$	8.75

The monthly demand for the product is 200 units, storage cost is 2% of the unit cost, and cost of ordering is Rs. 100.

(b) Narrate the multi item EOQ with shortage limitation.

18. The florist section in a grocery store stocks 18 dozen roses at the beginning of each week. On the average, the florist sells 3 dozens a day (one dozen at a time), but the actual demand follows a poisson distribution. Whenever the stock level reaches 5 dozens, a new order of 18 new dozens is placed for delivery at the beginning of the following week. Because of the nature of the item, all roses left at the end of the week per disposed of. Determine the following :

(a) The probability of placing an order in any one day of the week.

(b) The average number of dozen roses that will be discarded at the end of the week.

19. Visitors parking at Ozark college is limited to five spaces only. Cars making use of this space arrive according to a Poisson distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors who cannot find an empty space on arrival may temporarily wait inside the lot until a parked car leaves. The Temporary space can hold only three cars. Other cars that cannot park or find a temporary waiting space must go elsewhere. Determine the following :

(a) The probability, P_n of n cars in the system.

(b) The effective arrival rate for cars that actually use the lot.

(c) The average number of cars in the lot.

(d) The average time a car waits for a parking space inside the lot.

(e) The average utilization of the parking lot.

20. Solve the following problem using restricted basis method

Maximize $Z = x_1 + x_2^4$

Subject to

$3x_1 + 2x_2^2 \leq 9$

$x_1, x_2 \geq 0.$

F–1930

Sub. Code
7MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

TOPOLOGY – II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Is the space $\mathbb{R}^n$ locally compact? Justify your answer.

2. What is meant by subnet of (X_α)?

3. Define a completely regular space.

4. When will you say that two compactification is equivalent?

5. Define G_δ set in a space X.

6. What is meant by closed refinement?

7. Give an example of a Cauchy sequence in Q that is not convergent in it.

8. Define a Peano space.

9. When will you say that a space X is said to be compactly generated?

10. Is the space Q of rationals a Baire space? Justify your answer?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Define the one-point compactification of X. Also prove that $[0,1]^{\omega}$ is not locally compact in the uniform topology.

Or

(b) Let X be a space and let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Let $D \in \mathcal{D}$. Show that $A \in \mathcal{D}$ if $A \supset D$.

12. (a) Show that a subspace of a completely regular space is completely regular.

Or

(b) If X is completely regular and noncompact, then prove that $\beta(X)$ is not metrizable.

13. (a) Let $\mathcal{A}$ be the following collection of subsets of $\mathbb{R}$: $\mathcal{A} = \{(n, n+2) / n \in Z\}$. Which of the following collections refine $\mathcal{A}$?

$\mathcal{B} = \{(x, x+1) / x \in \mathbb{R}\}$, $\mathcal{C} = \{(n, n+3/2) / n \in z\}$, $\mathcal{D} = \{(x, x+3/2) / x \in \mathbb{R}\}$.

Or

(b) Let X be normal and let A be a closed G_δ set in X. Prove that there is a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) Define a Cauchy sequence in a metric space. Also prove that a metric space X is complete if every Cauchy sequence in X has a convergent subsequence.

Or

(b) Let X be a compactly generated space and let (Y, d) be a metric space. Prove that $\mathcal{C}(X,Y)$ is closed in Y^X in the topology of compact convergence.

15. (a) Let X be locally compact Hausdorff and let $\mathcal{C}(X,Y)$ have the compact open topology. Prove that the map $\mathcal{C} : X \times \mathcal{C}(X, Y) \to Y$, defined by the equation $\mathcal{C}(x, f) = f(x)$ is continuous.

Or

(b) Show that every locally compact Hausdorff space is a Baire space.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Tychonoff theorem.

17. Let X be a completely regular space. If Y_1 and Y_2 are two compactifications of X satisfying the extension property. Prove that Y_1 and Y_2 are equivalent.

18. Show that a space X is metrizable if and only if X is regular and has a basis that is countably locally finite.

19. Let $I = [0, 1]$. Prove that there exists a continuous map $f : I \to I^2$ whose image fills up the entire square I^2.

20. State and prove the Ascoli's theorem.

F–1931

Sub. Code
7MMA4E1

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics

Elective — ADVANCED STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the confidence interval for μ when σ is known is $N(\mu, \sigma^2)$.

2. Define two – sided test.

3. State the properties of sufficient statistic.

4. Distinguish between completeness and uniqueness.

5. What is meant by efficient estimator of θ.

6. Write a short notes on Bayes confidence interval.

7. Define a best critical region of size α.

8. What is meant by likelyhood ratio test?

9. What is analysis of variance?

10. Define the correlation coefficient of the random sample.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $x_1, x_2, ...x_n$ denote a random sample from a distribution having the following probability density function $f(x;\theta)=\left(\frac{1}{\theta}\right)e^{-x/\theta}, 0<x<\infty, 0<\theta<\infty$,

$= 0$ elsewhere

find the maximum likelihood estimator $\hat{\theta}$ of θ.

Or

(b) Let x have a p.d.f of the form $f(x\,;\theta)=\theta x^{\theta-1}$, $0<x<1$, zero elsewhere, where $\theta\in\{\theta:\theta=1,2\}$. To test the simple hypothesis $H_0:\theta=1$ against the alternative simple hypothesis $H_1:\theta=2$. Use a random sample x_1, x_2 of size $n=2$ and define the critical region to be $c=\left\{(x_1.\,x_2)=\frac{3}{4}\le x_1.\,x_2\right\}$, find the power function of the test.

12. (a) Let $x_1, x_2,...,x_n$ denote a random sample from a normal distribution with mean zero and variance $\theta, 0<\theta<\infty$. Show that $\sum_1^n \frac{x_i^2}{n}$ is an unbiased estimator of θ and has variance $\frac{2\theta^2}{n}$.

Or

(b) Let the random variables x and y have the joint p.d.f. $f(x,y)=\left(\frac{2}{\theta^2}\right)e^{-(x+y)/\theta}, \theta<x<y<\infty$, zero elsewhere. Find the mean and variance of y.

13. (a) Narrate the Bayesian estimation.

Or

(b) Define the fisher information. Let x have a gamma distribution with $\alpha = 4$ and $\beta = \theta > 0$. Find the Fisher Information I (θ).

14. (a) Discuss the uniformly most powerful test.

Or

(b) Let x be $N(\theta, 100)$. Find the sequential probability ratio test for testing $H_0 : \theta = 75$ against $H_1 : \theta = 78$.

15. (a) Define a non-central χ^2 variate and obtains its p.d.f.

Or

(b) State and prove Boole's inequality.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Derive the confidence intervals for difference of means.

(b) Enumerate the independence of attributes rising Chi-square test.

17. State and prove Rao-Blackwell theorem.

18. (a) Derive the Rao-Cramer inequality.

(b) Discuss the limiting distribution of maximum likelihood estimator.

19. State and prove Neyman – Pearson theorem.

20. With the usual notations, prove the following :

(a) $Q = Q_2 + Q_4 + Q_5$

(b) $$\sum_{i=1}^{n}\left[y_i - \alpha - \beta\left(x_i - \bar{x}\right)\right]^2 = n\left(\hat{\alpha} - \alpha\right)^2 + \left(\hat{\beta} - \beta\right)^2$$

$$\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 + \sum_{i=1}^{n}\left[y_i - \hat{\alpha} - \hat{\beta}\left(x_i - \bar{x}\right)\right]^2.$$

F–1933

Sub. Code
7MMA4E3

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

Elective — NUMERICAL METHODS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. State sturm's theorem.

2. Write the sufficient condition on $\phi(x)$ for the convergence in Iteration methods.

3. Prove that the infinite series converges $I + A + A^2 + \ldots$ converges to $(I - A)^{-1}$ if $\lim_{m \to \infty} A^m = 0$.

4. Define eigen value of a matrix.

5. Write the Hermite interpolating polynomial.

6. State Weierstrass approximation theorem.

7. Define order of a numerical differentiation method.

8. Define Lobatto integration methods.

9. Write down the order of the single step method.

10. Define periodic stability.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Obtain the complex roots of the equation $f(z) = z^3 + 1 = 0$ correct to eight decimal places.

Or

(b) Perform one iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$, take $p_0 = 0.5,\ q_0 = 0.5$.

12. (a) Prove that the iteration method of the form $x^{(k+1)} = H x^{(k)} + c,\ k = 0, 1, 2, ..$ for the solution of the system $A \times = 6$ converges to the exact solution for any initial vector, if $\|H\| < 1$.

Or

(b) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ using the iterative method, given that its approximate inverse is $B = \begin{pmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{pmatrix}$.

13. (a) Find the least squares approximation of second degree for the discrete data

x :	–2	–1	0	1	2
$f(x)$:	15	1	1	3	19

Or

(b) Obtain the cubic spline approximation for the function given in the tabular form

x:	0	1	2	3
$f(x)$:	1	2	33	244

and $M(0) = 0,\ M(3) = 0$.

14. (a) Find the Jacobian matrix for the system of equations $f_1(x, y) = x^2 + y^2 - x = 0$,

$f_2(x, y) = x^2 - y^2 - y0$ at the point $(1, 1)$ using the methods

$$\left(\frac{\partial f}{\partial x}\right)_{(x_i,\, y_j)} = \frac{f_{i+1},-f_{i-1,j}}{2h}, \qquad \left(\frac{\partial f}{\partial y}\right)_{(x_i,\, y_j)} = \frac{f_{i,\, j+1} - f_{i},\, _{j-1}}{2k}$$

with $h = k = 1$.

Or

(b) Evaluate the integral $I = \int_{-1}^{1} \left(1 - x^2\right)^{3/2} \cos x \, dx$ using Gauss – Legendre three point formula.

15. (a) Solve the initial value problem $u' = -2\, t u^2$, $u(0) = 1$ using mid-point method with $h = 0.2$ over the interval $(0,1)$.

Or

(b) Given the initial value problem $u' = t^2 + u^2$, $u(0) = 0$ determine the first three non-zero terms in the Taylor series for $u(t)$ and hence obtain the value for $u(6)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find all the roots of the polynomial $x^3 - 4x^2 + 5x - 2 = 0$ using the Graeffe's root squaring method.

17. Find all the eigenvalues of the matrix $A = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$ using the Rutishauser method.

18. Using the Gram-Schmidt orthogonalization process, compute the first three orthogonal polynomials $p_0(x)$, $p_1(x)$, $p_2(x)$ which are orthogonal on $[0, 1]$ with respect to the weight function $w(x) = 1$. Using these polynomials, obtain the lest square approximation of second degree for $f(x) = x^{1/2}$ on $[0, 1]$.

19. For the method

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h} + \frac{h^2}{3} f'''(\varepsilon),\ x_0 < \varepsilon < x_2$$

determine the optimal value of h, using the criteria

(a) $|RE| = |TE|$

(b) $|RE| + |TE| =$ minimum

Using this method and the value of h obtained from the criterion $|RE| = |TE|$, determine an approximate value of $f'(2.0)$ from the following tabulated values of $f(x) = \log x$

x:	2.0	2.01	2.02	2.06	2.12
$f(x)$:	0.69315	0.69813	0.70310	0.72271	0.75142

Given that the maximum round off error in function evaluation is 5×10^{-6}.

20. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 1]$. use the fourth order classical Runge-Kutta method.