E–0423

Sub. Code
1MMA1C4

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Verify that the function $\varphi_1(x) = x, x > 0$ satisfies the equation $x^2 y'' - xy' + y = 0$.

2. Define the Wronskian of n functions.

3. State the Euler equation.

4. Compute the indical polynomial and its roots of the equation $x^2 y'' + (x + x^2)\, y' - y = 0$.

5. Eliminate the constants a and b from the equation $Z = (x + a)\,(y + b)$.

6. Find the complete integral of the equation $pq = 1$.

7. Show that $F(D, D')e^{ax+by} = F(a, b)e^{ax+by}$.

8. Write down the one-dimensional diffusion equation.

9. What is meant by a boundary value problem for Laplace's equation?

10. Write down the space form of the wave equation.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Show that there exist n linearly independent solutions of $L(y) = 0$.

Or

(b) State the Legendre equation. Also prove the following :

(i) $P_n(-x) = (-1)^n P_n(x)$;

(ii) $P_n(-1) = (-1)^n$.

12. (a) Find all solutions of the equation

$x^2 y'' + xy' + 4y = 1$ for $|x| > 0$.

Or

(b) Obtain two linearly independent solutions of the following equation which are valid near $x = 0$: $x^2 y'' + 3xy' + (1 + x)y = 0$.

13. (a) Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$.

Or

(b) Solve : $Z^2 = pqxy$ using Charpit's method.

14. (a) Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x}$ is satisfied by $Z = \frac{1}{x}\phi(y-x) + \phi'(y-x)$, where ϕ is an arbitrary function.

Or

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

15. (a) Show that $r^{-2}\cos\theta$ satisfying the Laplace equation, where r, θ and ϕ are spherical polar coordinates.

Or

(b) A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the distributed stream.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find two linearly independent power series solutions of the equation $y'' + y = 0$.

17. Determine the solutions of the Bessel equation $x^2 y'' + xy' + (x^2 - \alpha^2)\, y = 0$.

18. Find the solution of the equation

$Z = \frac{1}{2}(p^2 + q^2) + (p - x)\,(q - y)$ which passes through the x-axis.

19. Solve the one-dimensional diffusion equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\,\frac{\partial z}{\partial t}.$$

20. Prove that the solutions of a certain Neumann problem can differ from one another by a constant only.

Wk ser

E–0424

Sub. Code
1MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

ALGEBRA – II

(CBCS – 2011 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a vector space over a field.

2. Define basts of a vector space over a field.

3. Prove that $A(W)$ is a subspace of a vector space V.

4. Define orthogonal and orthonormal set in an inner product space.

5. Define a splitting field.

6. Define algebraic over a field.

7. Show that the fixed field of G is a sub field of K.

8. Define a normal extension.

9. Define a regular element in $A(V)$.

10. If $T \in A(V)$ and if $S \in A(V)$ is regular, then prove that $r(T) = r(STS^{-1})$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that the sum of two subspaces of a vector space V is also a subspace of V.

Or

(b) Prove that $L(S)$ is a subspace of V.

12. (a) Prove that $A(A(W)) = A(W)$.

Or

(b) If V is an inner product space, then prove that $|(u,v)| \leq \|u\|.\|v\|$, for all $u, v \in V$.

13. (a) For any $f(x), g(x) \in F[x]$ and only $\alpha \in F$, prove that $(f(x).g(x))' = f'(x)g(x) + f(x).g'(x)$.

Or

(b) State and prove the transitivity property of algebraic extension.

14. (a) If K is a finite extension of K, then prove that $G(K,F)$ is a finite group and its order satisfies, $O(G(K,F)) \leq [K:F]$.

Or

(b) Let K be the field of complex numbers and F be the field of real numbers, then compute $G(K,F)$ and fixed field of $G(K,F)$.

15. (a) If A is an algebra with unit element over F, then prove that A is isomorphic to a subalgebra of $A(V)$, for some vector space V over F.

Or

(b) If V is a finite dimensional over F then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If V is a finite dimensional vector space and W is a subspace of V over F, then prove that W is finite dimensional, $\dim W \leq \dim V$ and $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

17. Establish Gram-Schmidt's orthogonalization process.

18. State and prove the transitivity property of a finite extension.

19. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

20. If $T \in A(V)$, then prove that

(a) $T^* \in A(V)$

(b) $(T^*)^* = T$

(c) $(S+T)^* = S^* + T^*$

(d) $(\lambda S)^* = \bar{\lambda} S^*$

(e) $(ST)^* = T^* S^*$ for all $S \in A(V)$ and $\lambda \in F$.

E–0425

Sub. Code
1MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

NUMERICAL ANALYSIS

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Find all the critical points of the equation.

$$F(x_1, x_2) = \frac{x_1^3}{3} + x_2^2\, x_1 + 3 .$$

2. Find the spectral radius of the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$.

3. Find the zeros of the Ledendre polynomial $P_2(x)$.

4. Define orthogonal polynomial with an example.

5. Find $\frac{dy}{dx}$ at $x = 1.2$ from the following data.

x :	1.2	1.3	1.4	1.5	1.6
y :	1.5095	1.6984	1.9043	2.1293	2.3756

6. Evaluate $\int_0^6 \frac{dx}{1+x}$ using Trapezoidal rate taking $h = 1$.

7. Find the general solution of the difference equation $y_{n+3} - 2y_{n+2} - y_{n+1} + 2y_n = 0$.

8. Using Euler's method, find $y(0.1)$, given that $y' = 1 + xy$ with $y(0) = 1$.

9. Write the Milne's predictor and corrector formulae.

10. What do you mean by a boundary value Probe? Give an example.

Part B (5 × 5 = 25)

Answer **all** questions by choosing either (a) or (b).

11. (a) Use the steepest descent method to find the maxima and minima of the function $F(x_1, x_2) = \frac{x_1^3}{3} + x_2^2 x_1 + 3$.

Or

(b) Solve the system $x^2 + xy^3 = 9$, $3x^2 y - y^3 = 4$ by fixed-point iteration.

12. (a) Verify directly that the Legendre polynomial $P_3(x)$ is orthogonal to any polynomial of degree 2.

Or

(b) Derive the normal equations for the best c_1^*, c_2^*, when $F(x) = F(x\,;\, c_1, c_2) = c_1 e^{c_2 x}$.

13. (a) The function $f(x)$ is defined on $[0, 1]$ as follows

$$f(x) = \begin{cases} x & 0 \le x \le 1/2 \\ 1 - x & 1/2 \le x \le 1 \end{cases}$$

Calculate $\int_0^1 f(x)$ using (i) Trapezoidal rule (ii) Simpson's 1/3 rule.

Or

(b) Find an approximation to $I = \int_1^3 \frac{(\sin x)^2}{x} dx$ using Gaussian quadrature with $k = 3$.

14. (a) Find the general solution of the difference equation $y_{n+2} - (2 + h^2)\, y_{n+1} + y_n = h^2$.

Or

(b) Determine an upper bound for the discretization error of Euler's method in solving the equation $y' = y,\ y(0) = 1$ from $x = 0$ to $x = 1$.

15. (a) Solve by shooting method $y'' = 2y^3$, $y(1) = 1$, $y(2) = \frac{1}{2}$, taking $y'(1) = 0$ as a first guess.

Or

(b) Solve by finite difference method $\frac{d^2y}{dx^2} + y = 0$, $y(0) = 0$, $y(1) = 1$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Apply damped Newton's method to solve the system.

$f_1(x) = x_1 + 3 l_n |x_1| - x_2^2$; $f_2(x) = 2x_1^2 - x_1 x_2 - 5x_1 + 1$ starting with $x^{(0)} = [2,1]^T$.

17. Using the appropriate recurrence relation, generate the first five Hermite Polynomials. Are the polynomials orthogonal? Justify.

18. Derive the Simpson's rule together with its error term. Hence, Solve $\int_0^6 \frac{dx}{1+x^2}0$ taking $h = 0.5$.

19. Use Runge-Kutta method of order 4 to solve $y' = y - x^2$, $y(0.6) = 1.7379$ and find $y(0.8)$ by taking $h = 0.1$.

20. Determine the value of $y\,(0.4)$ by Milne's method given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$; Use Taylor series to get the values of $y\,(0.1)$, $y(0.2)$ and $y(0.3)$.

———————

E–0426

Sub. Code
1MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an analytic function with an example.

2. What is meant by conformal mapping?

3. Compute $\int\limits_{|Z|=2} \frac{dz}{Z^2 - 1}$ for the positive sense of the circle.

4. State the fundamental theorem of algebra.

5. When will you say a function is meromorphic?

6. Define an essential isolated singularity with an example.

7. Determine the poles and the corresponding residue for $\frac{1}{(Z^2-1)^2}$.

8. State the Rouche's theorem.

9. Obtain the series expansion for arc $\cos z$.

10. Define a entire function.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that a harmonic function satisfies the formal differential equation $\frac{\partial^2 u}{\partial z\, \partial \bar{z}} = 0$.

Or

(b) Distinguish between translation, rotation and inversion. Also prove that the reflection $Z \to \bar{z}$ is not a linear transformation.

12. (a) Prove that the line integral $\int_\gamma p dx + q dy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial u}{\partial x} = p$ and $\frac{\partial u}{\partial y} = q$.

Or

(b) State and prove the Cauchy's integral formula.

13. (a) State and prove the Taylor's theorem.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the Residue theorem.

Or

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

15. (a) State and prove the Hurwitz theorem.

Or

(b) Write the usual notations, prove that

$$\sin \pi z = \pi z \prod_{1}^{\infty}\left(1-\frac{z^2}{n^2}\right) .$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the Lucas's theorem.

(b) Prove that the limit function of a uniformly convergent sequence of Continuous functions is itself continuous.

17. State and prove the Cauchy's theorem for rectangle.

18. State and prove the schwarz lemma.

19. Prove that $\int_{0}^{\infty} \log \sin x \, dx = -\pi \log 2$.

20. Derive the Jensen's formula.

E–0427

Sub. Code
1MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2011 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. State the Jensen's inequality.

2. Define a bounded linear map.

3. Let X be a linear space over C. Regarding X as a linear space over R. Consider a real-linear functional $u : X \to R$. Define $f(x) = u(x) - iu(ix)$, $x \in X$. Prove that f is a complex-linear functional on X.

4. Define a Schauder basis for normed space.

5. State the Resonance theorem.

6. When will you say that a map F is said to be closed?

7. Define the dual of normed space.

8. State the Riesz representation theorem for $C([a, b])$.

9. Define an orthonormal set.

10. What is meant by the orthogonal complement of the closed subspace?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove Riesz lemma.

Or

(b) Let X and Y be normed spaces. If X is finite dimensional, then prove that every linear map from X to Y is continuous. Also if X is infinite dimensional and $Y \neq \{0\}$, then prove that there is a discontinuous linear map from X to Y.

12. (a) State and prove Hahn-Banach separation theorem.

Or

(b) Let X be a normed space and Y be a closed subspace of X. Prove that X is a Banach space if and only if Y and X/Y are Banach space in the induced norm and the quotient norm, respectively.

13. (a) State and prove the Banach-Steinhaus theorem.

Or

(b) State and prove open mapping theorem.

14. (a) Let X, Y and Z be normed spaces. Let F_1 and F_2 be in $BL(X, Y)$ and $k \in K$. Prove that $(F_1 + F_2)' = F_1' + F_2'$ and $(KF_1)' = KF_1'$. Also prove that $(GF)' = F'G'$ if $F \in BL(X, Y)$ and $G \in BL(Y, Z)$.

Or

(b) Let $1 \le p \le \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. For a fixed $y \in L^q$, define $f_y : L^p \to K$ by $f_y(x) = \int_a^b xy \, dm$, $x \in L^p$. Prove that $f_y \in (L^p)$ and $\|f_y\| = \|y\|_q$.

15. (a) State and prove Schwarz inequality.

Or

(b) Let X be an inner product space and $f \in X'$. Let $\{u_1, u_2, \ldots\}$ be an orthonormal set in X. Prove that $\sum_n |f(u_n)|^2 \leq \|f\|^2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let Y be a closed subspace of a normed space X. For $X+Y$ in the quotient space X/Y and let $|\|X+Y\|| = \inf\{\|x+y\| : y \in Y\}$. Prove that $|\| \||$ is a norm on X/Y.

17. State and prove the Taylor-Foguel theorem.

18. State and prove Uniform boundedness principle.

19. State and prove the Riesz representation theorem for L^p.

20. (a) Derive Bessel's inequality.

(b) With the usual notations, prove that for all $x, y \in X$, $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$.

E–0428

Sub. Code
1MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

NUMBER THEORY

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. If $(a,b) = (a,c) = 1$, then prove that $(a, bc) = 1$.

2. What is meant by a reduced fraction?

3. Define the Mobius function $\mu(n)$.

4. State the mangoldt function $\wedge(n)$.

5. Define the Riemann Zeta function.

6. If n is a positive integer and x is a real number, then prove that $[x/n] = [[x]/n]$ if $n \geq 1$.

7. What are the solutions of the congruence $x^2 \equiv 1 \pmod 8$?

8. State the Wilson's theorem.

9. Define the Legendre's symbol. Give an example.

10. State the quadratic reciprocity law.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Given any two integers a and b, prove that there is a common divisor d of a and b of the form $d = ax + by$, where x and y are integers.

Or

(b) Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

12. (a) If $n \geq 1$, prove that $\varphi(n) = \sum_{d/n} \mu(d) \frac{n}{d}$.

Or

(b) Derive the selberg identity.

13. (a) Show that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$.

Or

(b) For $x \geq 1$, prove that

(i) $\sum_{n \leq x} \mu(n) \left[\frac{x}{n}\right] = 1$

(ii) $\sum_{n \leq x} \wedge(n) \left[\frac{x}{n}\right] = \log\,[x]!$.

14. (a) State and prove the Euler-Fermat theorem.

Or

(b) State and prove the Chinese remainder theorem.

15. (a) State and prove the Euler's criterion theorem.

Or

(b) Determine whether 219 is a quadratic residue or non residue mod 383.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Define compositive number with an example.

(b) State and prove the fundamental theorem of arithmetic.

17. If both g and $f * g$ are multiplicative, then prove that f is also multiplicative.

18. State and prove the Dirichlet's asymptotic formula.

19. State and prove the Lagrange theorem.

20. State and prove Gauss lemma.

E–0429

Sub. Code
1MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

TOPOLOGY – II

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Is the real line R locally compact? Justify your answer.

2. Define the one-point compactification.

3. Give an example for a complete regular space.

4. When will you say two compactifications are equivalent?

5. Define locally finite in the topological space. Give an example.

6. Define a G_δ-set. Give an example.

7. When will you say the metric space is complete?

8. Define the sub metric.

9. What do you mean by the compact open topology?

10. Define a Baire space.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that the rationals Q are not locally compact.

Or

(b) Let X be a set and $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. If A is a subset of X that intersects every element of $\mathcal{D}$, then prove that A is an element of $\mathcal{D}$.

12. (a) Define completely regular space. Prove that a product of completely regular space is completely regular.

Or

(b) Let $A \subset X$ and let $f : A \to Z$ be continuous map of A into the Hausdorff space Z. Prove that there is at most one extension of f to a continuous function $g : \overline{A} \to Z$.

13. (a) Let $\mathcal{A}$ be a locally finite collection of subsets of X. Prove the following :

(i) Any subcollection of $\mathcal{A}$ is locally finite.

(ii) The collection $\mathcal{B} = \{\overline{A}\}_{A \in \mathcal{A}}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

Or

(b) Let X be normal and let A be a closed G_δ set in X. Prove that there is a continuous function $f : X \to [0, 1]$. Such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) Let X be the product space $X = \pi X_\alpha$ and let x_n be a sequence of points of X. Prove that $x_n \to x$ if and only if $\pi_\alpha(x_n) \to \pi_\alpha(x)$ for each α.

Or

(b) If X is locally compact or if X satisfies the first countability axion, then prove that X is compactly generated.

15. (a) Let X be a locally compact Hausdorff and let $\mathcal{C}(X, Y)$ have the compact open topology. Prove that the map $\mathcal{C}: X \times \mathcal{C}(X,Y) \to Y$ defined by the equation $\mathcal{C}(x,f) = f(x)$ is continuous.

Or

(b) Prove that any open subspace y of a Baire space X is itself a Baire space.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that an arbitrary product of compact spaces is compact in the product topology.

17. Let X be a completely regular space. Prove that there exists a compactification Y on X having the property that every bounded continuous map $f: X \to \mathbb{R}$ extends uniquely to a contribution map of Y into $\mathbb{R}$.

18. Let X be a metrizable space. If $\mathcal{A}$ is an open covering of X, then prove that there is an open covering ε of X refining $\mathcal{A}$ that is countably locally finite.

19. Let $I = [0, 1]$. Prove that there exists a continuous map $f : I \to I^2$ whose image fills up the entire square I^2.

20. State and prove the Baire category theorem.

E–0430

Sub. Code
1MMA4E1

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

Elective — ADVANCED STATISTICS

(CBCS – 2011 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an unbiased estimator.

2. Define a simple and a composite statistical hypothesis.

3. State the minimax principle.

4. Define the likelihood function of the random sample.

5. What is meant by a Baye's solution?

6. Define an efficient estimator of a parameter.

7. Define a uniformly most powerful test.

8. Define a non control t-distribution with degrees of freedom.

9. Define a real quadratic form with an example.

10. Give examples for a linear and not a linear model.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Let $X_1, X_2, ..., X_n$ denote a random sample from the distribution with p.d.f.

$$f(x) = \begin{cases} \theta^x \ (1-\theta)^{1-x}, & x = 0, 1 \\ 0 & \text{elsewhere} \end{cases} \quad \text{where} \quad 0 \le \theta \le 1.$$

Find the m.l.e. $\hat{\theta}$ of θ.

Or

(b) One of the first six positive integers is to be chosen by a random experiment by costing a die. Let $A_i = \{x; x = i\}$; $i = 1$ to 6. Test the hypothesis $H_0 : p_i = p_{io} = \frac{1}{6}$, $i = 1$ to 6 at the 5% significance level by repeating the random experiment 60 independent times. The observed values of $x_1, x_2, ..., x_6$ are 13, 19, 11, 8, 5 and 4.

12. (a) Show that the mean $\overline{X}$ of a random sample of size n from a distribution having p.d.f

$$f(x, \theta) = \begin{cases} \left(\frac{1}{\theta}\right) e^{-\left(\frac{x}{\theta}\right)}, & 0 < x < \infty, \ 0 < \theta < \infty \\ \text{zero} & \text{elsewhere} \end{cases}$$

is an unbiased estimator of θ and has variance $\frac{\theta^2}{n}$.

Or

(b) Let $X_1, X_2, ..., X_n$ denote a random sample of size $n > 1$ from a distribution with p.d.f $f(x, \theta) = \theta e^{-\theta x}$, $0 < x < \infty$, zero elsewhere, and $\theta > 0$. Then $Y = \sum_{i=1}^{n} X_i$ is a sufficient statistic for θ. Prove that $(n-1)/Y$ is the unbiased minimum variance estimator of θ.

13. (a) Explain about Bayesian estimation.

Or

(b) Let X be a binomial $b(1, \theta)$. Compute the fisher information $I(\theta)$.

14. (a) Let $X_1, X_2, ..., X_n$ denote a random sample from a distribution that is $N(0, \theta)$. Where the variance θ is an unknown positive number. Show that there exists a uniformly most powerful test with significance level α for testing the simple hypothesis $H_0 : \theta = \theta'$, where θ' is a fixed positive number, against the alternative composite hypothesis $H_1 = \theta > \theta'$.

Or

(b) Let X be $N(\theta, 100)$. Find the sequential probability ratio test for testing $H_0 : \theta = 75$ against $H_1 : \theta = 78$ such that each of α and β is approximately equal to 0.10.

15. (a) Consider the variance s^2 of the random sample of size $n = ab$. Prove that

$$abs^2 = \sum_{i=1}^{a}\sum_{j=1}^{b}(X_{ij} - \overline{X}_i)^2 + b\sum_{i=1}^{a}(\overline{X}_i - \overline{X}....)^2$$

Or

(b) Let Y_i, $i-1, 2,....,n$ denote independent random variables that are, respectively, $\chi^2(r_i, \theta_i)$, $i=1, 2,.....,n$. Prove that $z=\sum_1^n Y_i$ is $\chi^2\left(\sum_1^n r_i \sum_1^n \theta_i\right)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Determine the confidence interval for the unknown parameters p of a binomial distribution when the parameter n is known.

17. State and prove the factorization theorem of Neyman.

18. Derive the Rao-Cramer in equality.

19. State and prove Neyman-Pearson theorem.

20. Show that a random variable that has an m.g.f. of the functional form $M(t)=\frac{1}{(1-2t)^{r/2}}e^{t\theta/(1-2t)}$ where $t<\frac{1}{2}$, $0<\theta$ and r is a positive integer, is a non control chi-square distribution with r degrees of freedom and noncentrality parameters θ.