A–9023

Sub. Code
4MMA1C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

First Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define arc length.

2. What is meant by binormal line?

3. Define osculating sphere.

4. Define the pitch of the helix.

5. What is meant by right helicoid?

6. Find H for the paraboloid $x = u$, $y = v$, $z = u^2 - v^2$.

7. Define geodesic parallels.

8. Define Gaussian curvature.

9. State the Meusnier's theorem.

10. Define the polar developable.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that if a curve is given in terms of a general parameter *u,* then the equation of the osculating plane is $[\vec{R}-\vec{r},\ \dot{\vec{r}},\ \ddot{\vec{r}}]=0$.

Or

(b) Find the curvature and the torsion of the curve $\vec{r}=\{a(3u-u^3),\ 3au^2,\ a(3u+u^3)\}$.

12. (a) If the radius of spherical curvature is constant, prove that the curve either lies on a sphere or has constant curvature.

Or

(b) Show that the involutes of a circular helix are plane curves.

13. (a) For the anchor ring $\vec{r}=(b+a\cos u)\cos v$, $(b+a\cos u)\sin v, a\sin u)$. Calculate the area corresponding to the domain $0\le u\le 2\pi$, $0\le v\le 2\pi$.

Or

(b) Find the surface of revolution which is isometric with a region of the right helicoid.

14. (a) Prove that the curves of the family $\dfrac{v^3}{u^2}$ = constant are geodesics on a surface with metric $v^2\,du^2-2uv\,du\,dv+2u^2\,dv^2(u>0,\ v>0)$.

Or

(b) Derive the Liouville's formula for kg.

15. (a) Enumerate the following terms :

(i) Principal curvatures

(ii) Dupin indicatrix.

Or

(b) Prove that the edge of regression of the rectifying developable has equation

$$\vec{R} = \vec{r} + k\frac{(\tau\bar{t} + k\bar{b})}{k'\tau - k\tau'}.$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that the length of the common perpendicular d of the tangents at two near points distance s, apart is approximately given by $d = \frac{k\tau s^3}{12}$.

17. Show that the intrinsic equations of the curve given by $x = ae^u \cos u$, $y = ae^u \sin u$, $z = be^u$ are

$$k = \frac{\sqrt{2}\,a}{(2a^2 + b^2)^{1/2}} \cdot \frac{1}{s}, \quad \tau = \frac{b}{(2a^2 + b^2)^{1/2}} \cdot \frac{1}{s}.$$

18. If θ is the angle at the point (u, v) between the two directions given by $pdu^2 + 2Q\,du\,dv + R\,dv^2 = 0$ then prove that $\tan\theta = \frac{2H(Q^2 - PR)^{1/2}}{ER - 2FQ + GP}$.

19. Prove that if (λ, μ) is the geodesic curvature vector, then

$$k_g = \frac{-H\lambda}{Fu' + Gv'} = \frac{H\mu}{Eu' + Fv'}.$$

20. Derive the Rodrigue's formula.

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A–9024

Sub. Code
4MMA1C4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS – 2014 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Verify that $\phi_1(x) = e^x$, $(x > 0)$ is a solution of $xy'' - (x+1)y' + y = 0$

2. Prove that $P_n(-1) = (-1)^n$.

3. Define singular point an regular singular point.

4. Compute the indical polynomial and its roots of the equation $x^2 y'' + xy' + \left(x^2 + \frac{1}{4}\right)y = 0$.

5. Form a partial differential equation, by eliminating the function F from $z = xy + f\left(x^2 + y^2\right)$.

6. Find the complete integral of the equation of $pq = p + q$.

7. Write down the telegraphy equation.

8. If u is the complementary function and $z_1 a$ particular integral of a linear partial differential equation, then prove that $(u + z_1)$ is a general solution of the equation.

9. Show that $r \cos\theta$ satisfying the Laplace equation. Where r, θ and ϕ are spherical polar coordinates.

10. Define the exterior Dirchlet problem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $L(y) = 0$ denote an n[th] order differential equation on an interval I. Show that there exist n linearly independent solution of $L(y) = 0$ on I.

Or

(b) Show that $\int_{-1}^{1} P_n^{\,2}(x)dx = \dfrac{2}{2n+1}$.

12. (a) Find all solutions of the equation $x^2 y'' - (2+i)xy' + 3iy = 0$ for $/x/ > 0$.

Or

(b) Find the singular points of the equation $(1-x^2)y'' - 2xy' + 2y = 0$ and determine whether regular singular point or not.

13. (a) Fine the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)\sum = (x^2 - y^2)Z$ which contains the straight line $x + y = 0,\ z = 1$

Or

(b) Solve $p^2 x + q^2 y = z$ using Charpit's methods.

14. (a) Solve the equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}.$$

Or

(b) Find the complementary function of $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

15. (a) Show that the solutions of a certain Neumann problem can differ from one another by a constant only.

Or

(b) A right sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find the two linearly independent power series solutions of the equation $y'' - xy' + y = 0$.

17. With the usual notations, prove that

$$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(x/2\right)^{2m}$$

18. Show that the equation $xp = yq,\ z(xp + yq) = 2xy$ are compatible and solve them.

19. Reduce the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to Canonical form and hence solve it.

20. Show that $y = A(p)e^{ip\left(t \pm x/c\right)}$ is a solution of the wave equation for arbitrary form of the functions A which depends only on p. Interpret these solutions physically.

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A–9025

Sub. Code
4MMA1E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

First Semester

Mathematics

Elective – PROGRAMMING IN C++

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. List down the basic concepts of OOP.

2. What are the differences between "break" and "continue" statement?

3. Give the syntax of Array Initialization with an example.

4. List the properties of static members.

5. What are the characteristics of constructor?

6. Define operator overloading.

7. What are the benefits of inheritance?

8. What is an abstract class?

9. What are C++ Streams?

10. What are the types of formatted console i/o operations?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain the structure of C++ program.

Or

(b) Define the following terms related to object oriented Paradigm

(i) Abstract data type

(ii) Polymorphism.

12. (a) Write a program to explain the inline function.

Or

(b) Write a program to illustrate the use of call by reference.

13. (a) Write a program to demonstrate the use of Copy constructor.

Or

(b) What is a destructor? Explain it with an example.

14. (a) Write a C++ program to illustrate the virtual function in hierarchical inheritance.

Or

(b) With an example, explain multiple inheritance.

15. (a) Write note on Unformatted Console I/O Operations.

Or

(b) Explain the following output manipulators :

(i) setω( )

(ii) setprecision( )

(iii) setfill( ).

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Explain in detail about control structures with example.

17. Describe the concept of function overloading with suitable examples.

18. Explain about Unary Operator and Binary Operator Overloading with program.

19. Explain Single and Hybrid inheritance each with suitable example.

20. Discuss in detail about Exception handling with example.

A–9026

Sub. Code
4MMA2C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Second Semester

Mathematics

ALGEBRA – II

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. When will you say that a vector space V is said to be finite dimensional?

2. Define a linear functional on V in to F.

3. Define the orthogonal complement of W.

4. Define the annihilator of W.

5. Find the degree of the splitting field of $x^4 - 2$ over F.

6. Is any finite extension of a field of characteristic 0 is a simple extension ? Justify.

7. Define the fixed field of G.

8. Express the polynomial $x_1^3 + x_2^3 + x_3^3$ in the elementary symmetric functions in x_1, x_2, x_3.

9. Define the range of T.

10. Define the following terms:

    Hermitian and Skew–Hermitian

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If U and W are subspaces of V, prove that U + W = $\{v \in V / v + w, u \in U, w \in W\}$ is a subspace of V.

Or

(b) If $v_1, v_2, \ldots v_n$ is a basis of V over F and if $w_1, w_2, \ldots . w_m$ in V are linearly independent over F, then prove that $m \leq n$.

12. (a) If V is finite–dimensional, then prove that ψ is an isomorphism of V onto $\hat{\hat{V}}$.

Or

(b) With the usual notations, prove that

$$\|u+v\|^2 + \|u-v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$$

13. (a) State and prove the Remainder theorem.

Or

(b) Prove that $(f(x)+g(x)) = f'(x)+g'(x)$ and $(\alpha f(x))' = \alpha f'(x)$ for $f(x), g(x) \in F[x]$ and $\alpha \in F$.

14. (a) Prove that K is a normal extension of F it and only if K is the splitting field of some polynomial over F.

Or

(b) If $\alpha_1, \alpha_2, \alpha_3$ are the roots of the Cubic polynomial $x^3 + 7x^2 - 8x + 3$, find the cubic polynomial whose roots are $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3}$,

15. (a) If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

Or

(b) If $(vT, vT) = (v, v)$ for all $v \in V$ then prove that T is unitary.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If V is finite–dimensional and if W is a subsequence of V, then prove that W is finite-dimensional, $\dim w \leq \dim v$ and $\dim {}^{v}\!/\!_{w} = \dim v - \dim w$.

17. (a) If V is a finite-dimensional inner product space and W is a subspace of V then prove that $(W^{\perp})^{\perp} = W$.

(b) State and prove the Bessel inequality.

18. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F, Moreover $[L:F] = [L:K][K:F]$

19. If K is a finite extension of F, then prove that $G(K,F)$ is a finite group and its order $O(G(K,F))$ satisfies $O(G(K,F) \leq [K:F]$.

20. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristics roots in F, then prove that T satisfies a polynomial of degree n over F.

A–9027

Sub. Code
4MMA2C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Second Semester

Mathematics

ANALYSIS — II

(CBCS – 2014 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the Stieltjes integral.

2. If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a, b]$, then prove that $fg \in \mathcal{R}(\alpha)$.

3. Define equicontinuous family of functions.

4. State the stone's generalization of the Weierstrass theorem.

5. Let e^x be defined on R^1 by $E(x) = e^x$ and $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$.

   Prove that $(e^x)' = e^x$.

6. When will you say a function is orthonormal?

7. If $m^* E = 0$ then prove that E is measurable.

8. Define the simple function.

9. State the bounded convergence theorem.

10. Define the following terms :

(a) $f^{+}(x)$

(b) $f^{-}(x)$

(c) $|f|$.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) If f is continuous on $[a,b]$ then prove that $F \in \mathcal{R}(\alpha)$ on $[a,b]$.

Or

(b) State and prove the fundamental theorem of calculus.

12. (a) Show that compactness is really needed in Dini's theorem. Justify your answer by means of example.

Or

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

13. (a) Given a double sequence $\{a_{ij}\}$, $i = 1, 2, 3.......$, $j = 1, 2, 3,......$ suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i$ $((i = 1, 2, 3,....)$ and Σb_i converges prove that $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}$.

Or

(b) Derive the Stirling's formula.

14. (a) Define a Borel set. Also prove that every Borel set is measurable.

Or

(b) State and prove the Lusin's theorem.

15. (a) If f and g are bounded measurable functions defined on a set E of finite measure. Prove that following :

(i) $\int_E (af + bg) = a\int_E f + b\int_E g$

(ii) If $f = g$ a.e. then $\int_E f = \int_E g$.

Or

(b) State and prove the Fatou's lemma.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

17. State and prove Arzele-Ascoli theorem.

18. Define the gamma function. Prove the following :

(a) The functional equation $\Gamma(x+1) = x\Gamma(x)$ holds if $0 < x < \infty$;

(b) $\Gamma(n+1) = n!$ for $n = 1, 2, 3, \ldots.$

(c) $\text{Log}\ \Gamma$ is convex on $(0, \infty)$

19. Show that the outer measure of an interval is its length.

20. (a) Let f and g be integrable over E. Prove the following :

    (i) If $f \leq g$ a.e., then $\int_E f \leq \int_E g$;

    (ii) If A and B are disjoint measurable sets contained in E, then $\int_{A \cup B} f = \int_A f + \int_B f$.

    (b) State and prove the Lebesgue convergence theorem.

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A–9028

Sub. Code
4MMA2C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Second Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define a probability set function.

2. Prove that the distribution function $f(x)$ is a non decreasing function.

3. Define the conditional p.d.f. of the discrete type of ransom variable.

4. Define the m.g.f of the joint distribution of X and Y.

5. What is meant by a Bernouli experiment?

6. Suppose that X has a Poisson distribution with $\mu = 2$. Then find the mean and the variance..

7. Let $\overline{X}$ be the mean of a random sample of size 25 from a distribution that is $N(75{,}100)$ Find $pr(71 < \overline{X} < 79)$.

8. If $X_1, X_2, \ldots\ldots, X_n$ are observations of a random sample from a distribution with m.g.f. $m(t)$ then find the m.g.f of $\overline{X} = \sum_{i=1}^{n} \frac{\text{x}i}{n}$.

9. When we say that the sequence of random variables $Y_1, Y_2 \ldots.$ converges in distribution to a random variable with distribution $F(y)$?

10. Let U_n and V_n converse in probability to C and d, respectively, prove that the sum $U_n + V_n$ converges in probability to $C+d$.

Part B (5 × 5 = 25)

Answer **all the** questions, choosing either (a) or (b).

11. (a) Let $f(x) = x/_{15}, x = 1,2,3,4,5,$ zero else where be the p.d.f of X. Find $pr(x = 1 \text{ or } 2), \Pr\left(\frac{1}{2} < X < \frac{5}{2}\right)$ and $pr(1 \leq X \leq 2.)$

Or

(b) Let the subsets $A_1 = \left\{x : \frac{1}{4} < X < \frac{1}{2}\right\}$ and $A_2 = \left\{x : \frac{1}{2} \leq X < 1\right\}$ of the space $A = \{x : 0 < x < 1\}$ of the random variable X be such that $P(A_1) = \frac{1}{8}$ and $P(A_2) = \frac{1}{2}$. Find $P(A_1 \cup A_2), P(A_1{*})$ *and* $P(A_1{*} \cap A_2{*})$.

12. (a) Let $F(x, y)$ be the distribution function of X and Y. Show that $\Pr(a < X \leq b, C < Y \leq d) = F(b,d) - F(b,c) - F(a,d) + F(a,c),$ for all real constants $a < b, c < d.$

Or

(b) If the random variables X_1 and X_2 have the joint p.d.f $f(x_1, x_2) = 2e^{-x_1-x_2}, 0 < x_1 < x_2, 0 < x_2 < \infty$, zero elsewhere. Show that X_1 and X_2 are dependent.

13. (a) If X is $b(n, p)$, then show that $E\left(\frac{x}{n}\right) = p$ and $\left[\left(\frac{X}{n} - p\right)^2\right] = \frac{p(1-p)}{n}$

Or

(b) If X is N(75,100). Find $\Pr(X < 60)$ and $\Pr(70 < X < 100)$.

14. (a) If $x_i = i, i = 1,2,\ldots,n$, Compute the values of $\bar{x} = \sum x_i/n$ and $S^2 = \sum(x_i - \bar{x})^2 / n$.

Or

(b) Find the mean and variance of the beta distribution.

15. (a) Let $\overline{X}_n$ have the distribution function $F_n(\bar{x}) = \int_{-\infty}^{\bar{x}} \frac{1}{\sqrt{1/n}\sqrt{2\pi}} e^{-nw^2/2} dw$. Find the limiting distribution of $\overline{X}_n$

Or

(b) Let $F_n(u)$ denote the distribution function of a random variable U_n whose distribution depends upon the positive integer n. Further, let U_n converge in probability to the positibe constant C and let $\Pr(U_n < 0) = 0$ for every n. Then prove that the ransom variable $\sqrt{U_n}$ Converges in probability to $\sqrt{C}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X have the *p.d.f*$(x)=2(1-x), 0<x<1$, zero elesewhere. Compute $E(6X+3X^2)$ and $E[(X+2)^2]$.

17. Let X_1 and X_2 have the p.d.f $f(x_1, x_2) = 8x_1x_2, 0 < x, < x_2 < 1,$ zero elsewhere compute $E(7X_1X_2^2 + 5X_2)$.

18. Find the m.g.f of a gamma distribution and hence find mean and variance.

19. If X have the p.d.f $f(x) = x^2/9, 0 < x < 3,$ zero elsewhere. Find the p.d.f. of $Y = X^3$.

20. State and prove the central limit theorem.

A–9029

Sub. Code
4MMA2E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Second Semester

Mathematics

Elective – GRAPH THEORY

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an induced subgraph of g. Give an example.

2. Draw all the trees with six vertices.

3. Define a block of a graph. Give an example.

4. What is meant by Hamilton path of G?

5. Find the number of different perfect matching in K_{2n}.

6. Draw 3-edge chromatic graph.

7. Define an independent set of a graph.

8. Draw a 4–critical graphs.

9. If G is a simple planar graph, then prove that $\delta \leq 5$.

10. Define the dual of a plane graph.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) With the usual notation, prove that

(i) $\sum_{v \in v} d(v) = 2 \in$

(ii) $\delta \leq \frac{2 \in}{v} \leq \Delta$

Or

(b) Prove that an edge e of g is a cut edge of G if and only if e is contained in no cycle of G.

12. (a) Define the following terms. Give an illustration for each:

(i) Edge connectivity;

(ii) Subdivision of an edge;

(iii) Euler tour

Or

(b) Prove that C(G) is well defined.

13. (a) If G is a k-regular bipartite graph with k>0, then prove that G has a perfect matching.

Or

(b) If g is bipartite, then prove that $\chi' = \Delta$.

14. (a) Prove that a set $S \subseteq V$ is an independent set of G if and only if $v - s$ is a covering of G.

Or

(b) Show that every critical graph is a block.

15. (a) Prove that a graph G is embeddable in the plane if and only if it is embeddable on the sphere.

Or

(b) If G is a connected plane graph, then prove that $\nu - \varepsilon + \phi = 2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. With the usual notations, prove that $\tau(K_n) = n^{n-2}$.

17. Prove that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

18. State and prove that Hall's theorem.

19. Show that $r(k,k) \geq 2^{k/2}$.

20. State and prove that the five-colour theorem.

A–9030

Sub. Code
4MMA3C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a power series. Also find the radius of convergence of the power series $\sum n!z^n$.

2. What is meant by the Jordan curve?

3. State the Cauchy's theorem in a disk.

4. Compute $\int_{|z|=1} \frac{e^z}{z} dz$.

5. Define removable singularity with an example.

6. Find the poles of $\frac{1}{(z^2-1)^2}$.

7. Determine the residue of $e^z/(z-a)(z-b)$.

8. State the Rouche's theorem.

9. State the Weierstrass's theorem for power series.

10. Define the genus.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove Lucas's theorem.

Or

(b) Derive the complex form of the Cauchy-Riemann equations.

12. (a) Define the index of the point a with respect to the curve γ. If the piecewise differentiable closed curve γ does not pass through the point a, then prove that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

Or

(b) State the Liouville's theorem. Also state and prove the fundamental theorem of algebra.

13. (a) State and prove the Taylor's theorem.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the argument principle theorem.

Or

(b) Evaluate : $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\, dx$.

15. (a) With the usual notations, prove that

$$\sin \pi z = \pi z \prod_{1}^{\infty}\left(1 - \frac{z^2}{n^2}\right).$$

Or

(b) Derive the Jensen's formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) If $T_1(z) = \frac{z+2}{z+3}$, $T_2(z) = \frac{z}{z+1}$, find $T_1 T_2(z)$, $T_2 T_1(z)$ and $T_1^{-1} T_2(z)$.

(b) If Z_1, Z_2, Z_3, Z_4 are distinct points in the extended plane and T any linear transformation, then prove that $\left(T_{z_1}, T_{z_2}, T_{z_3}, T_{z_4}\right) = \left(Z_1, Z_2, Z_3, Z_4\right)$.

17. State and prove Cauchy's theorem for a rectangle.

18. State and prove the schwarz lemma.

19. Show that $\int_0^{\pi} log\ sin\ x\ dx = \pi\ log(1/2)$.

20. (a) State and prove Hurwitz theorem.

(b) State the Laurent series. Also prove that the Laurent development is unique.

A–9031

Sub. Code
4MMA3C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the standard topology. Give an example.

2. What is meant by the subspace topology?

3. State the pasting lemma.

4. When will you say that a topological space is said to be metrizable?

5. Is the rationals Q connected ? Justify.

6. Define the term linear continuum.

7. Prove that the real line $\mathbb{R}$ is not compact.

8. Define a Lebesgue number.

9. Prove that the product of two Lindelof spaces need not be Lindelof.

10. State the Urysohn lemma.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then prove that the collection $D = \{B \times C / B \in \mathcal{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

Or

(b) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inhertis as a subspace of $X \times Y$.

12. (a) Narrate the following terms:

(i) Product topology;

(ii) Standard bounded metric;

(iii) Quotient map.

Or

(b) Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric P are the same as the product topology on $\mathbb{R}^n$.

13. (a) Show that the image of a connected space under a continuous map is connected.

Or

(b) If X is a topological space, each path component of X lies in a component of X. If X is locally path connected, then prove that the components and the path components of X are the same.

14. (a) Prove that every compact subspace of a Housdorff space is closed.

Or

(b) State and prove uniform continuity theorem.

15. (a) Suppose that X has a countable basis. Prove that every open covering of X contains a countable subcollection covering X.

Or

(b) Prove that a product of Hausdorff spaces is Hausdorff.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X be a topological space. Prove the following:

(a) ϕ and X are closed.

(b) Arbitrary intersections of closed sets are closed.

(c) Finite unions of closed sets are closed.

17. Let X and Y be topological spaces and let $f : x \to y.$ prove that the following are equivalent:

(a) f is continuous.

(b) For every subset A of X, one has $f(\overline{A}) \subset \overline{f(CA)}.$

(c) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

18. If L is a linear continuum in the order topology, then prove that L is connected, and so are intervals and rays in L.

19. (a) State the tube lemma.

(b) Prove that compactness implies limit point compactness, but not conversely.

(c) Show that [0, 1] is not limit point compact as a subspace of $\mathbb{R}_l$.

20. State and prove the Urysohn metrization theorem.

A–9032

Sub. Code
4MMA3C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Third Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2014 onwards)

Time : Three Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define a spanning tree. Give an example.

2. What is meant by optimistic time estimate?

3. Define shortage cost.

4. Write down the general assumptions of the no-setup model.

5. Identify the customer and the server for the following situations:

   (a) Registration for classes in a university.

   (b) Planes arriving at an airport.

6. Define balk and priority in queuing system.

7. State the Little's formula.

8. Write down the formula for L_q of the model $(M/M/C):(GD/\infty/\infty)$.

9. Write a short notes on steepest ascent method.

10. Define the general constrained non linear programming problem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Enumerate the Floyd's algorithm.

Or

(b) Consider a project with the following WBS for building a site preparation. Determine the precedence relationship, draw the network and number the events.

(i) Clear the site

(ii) Survey and Layout

(iii) Rough grade

(iv) Exavacate the sewer

(v) Exavacate for electric manholes

(vi) Install sewer and backfill

(vii) Install electrical manholes

(viii) Construct boundary wall

12. (a) Explain classic EOQ model.

Or

(b) Find the optimal order quantity for a product when the annual demand for the product is 500 units, the cost of storage per unit per year is 10% of the unit cost and ordering cost per order is Rs. 180. The unit costs are given below:

Quantity	Unit cost (Rs.)
$0 \leq y_1 < 500$	25
$500 \leq y_2 < 1500$	24.80
$1500 \leq y_3 < 3000$	24.60
$3000 \leq y_4$	24.40

13. (a) What is a quene? Give an illustration. Also describe the basic elements of a queuing model.

Or

(b) Babies are born in sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following :

(i) The average number of births per year

(ii) The probability that no births will occur in any one day.

(iii) The probability of issuing 50 birth certificates in 3 hours given that 40 certificates were issued during the first 2 hours of the 3-hour period.

14. (a) For $(M/M/1):(GD/N/\infty)$ queuing model, show that the two expressions for λ_{eff} are equivalent namely $\lambda_{eff} = \lambda(1-P_N)$.

Or

(b) Show that the $p-k$ formula reduces to L_S of the $(M/M/1):(GD/\infty/\infty)$. When the service time is exponential with a mean of $\frac{1}{\mu}$ time units.

15. (a) Determine the maximum of the function $f(x) = -(x-3)^2$, $2 \le x \le 4$ by dichotomous search by taking $\Delta = 0.05$.

Or

(b) Prove that in general, the Newto-Raphson method when applied to a strictly convave quadratic function will converge in exactly one step. Apply the method to the maximization of

$$F(x) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2 .$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. A small project is composed of activities whose time estimate are listed in the table below: Activities are identified by their beginning (a) and ending (b) node numbers.

Activity i-j	Estimated Duration (Weeks)		
	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(i) Determine the project network.

(ii) Find the expected duration and variance for each activity. What is the expected project length?

(iii) Calculate the variance and standard deviation of the project length. What is the probability that the project will be completed.

(1) At least 4 weeks earlier than expected?

(2) No more than 4 weeks later than expected time?

17. An item is manufactured to meet known demand for four periods according to the following data :

Production range (units)	Unit production cost ($) for period			
	1	2	3	4
1-3	1	2	2	3
4-11	1	4	5	4
12-15	2	4	7	5
16-25	5	6	10	7
Unit holding cost to next period ($)	0.30	0.35	0.20	0.25
Total demand (units)	11	14	17	29

(a) Find the optimal solution, indicating the number of units to be produced in each period.

(b) Suppose that 10 additional units are needed in period 4. Where should they be produced?

18. A service machine always has a standby unit for immediate replacement upon failure. The time to failure of the machine (or its stand by unit) is exponential and occurs every 5 hours, on the average. The machine operator claims that the machine "has the habit" of breaking down every night around 8:30 P.M. Analyze the operators' claim. Also determine the average number of failures in 1 week, assuming the service is offered 24 hours a day, 7 days a week.

19. Automata car wash facility operates with only one bay. Cars arrive according to a poisson distribution with a mean of 4 cars per hour, and may wait in the facility parking lot if the bay is busy. The time for washing and cleaning a cannot part in the lot can wait in the street bordering the was facility. The means that, for all practical purposes, there is no limit on the size of the system. The manager of the facility wants to determine the size of the parking lot.

20. Solve the following non-linear programming problem using restricted basis method of separable programming.

$$Maximize\, z = x_1 + x_2^4$$

Subject to

$$3x_1 2x_2^2 \le 9 \quad and \quad x_1, x_2 \ge 0.$$

A–9033

Sub. Code
4MMA3C4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Third Semester

Mathematics

NUMBER THEORY

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define composite number. Give an example.

2. When will you say that the numbers are said to be relatively prime?

3. Write down the Euler totient function $p(n)$.

4. State the generalized Mobius inversion formula.

5. Write down the Euler's summation formula.

6. If $0 < y < 1$, what are the possible values of $[x]-[x-y]$?

7. Define a complete residue system modelo m.

8. State the Little Fermat theorem.

9. Fine the value of $\left(7/11\right)$ and $\left(22/11\right)$.

10. State the reciprocity law for Jacobi symbols.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Given x and y, let $m = ax + by$, $n = cx + dy$, where $ab - bc = \pm 1$ prove that $(m, n) = (x, y)$.

Or

(b) Prove that the infinite series $\sum_{n=1}^{\infty} 1/p_n$ diverges.

12. (a) For $n \geq 1$, prove that $\phi(n) = n \pi_{p/n} \left(1 - \frac{1}{p}\right)$.

Or

(b) If $n \geq 1$, then prove that $\log_n = \sum_{d/n} \wedge(d)$.

13. (a) Show that $[-x] = \begin{cases} -[x] & if \quad x = [x], \\ -[x] - 1 & if \quad x \neq [x] \end{cases}$.

Or

(b) For all $x \geq 1$, prove that $\left| \sum_{n \leq x} \frac{\mu(n)}{n} \right| \leq 1$, with equality holding only if $x < 2$.

14. (a) State and prove that Wolstenholme's theorem.

Or

(b) State any prove the Chinese remainder theorem

15. (a) Let P be an odd prime. Prove that $\left(n/p \right) \equiv n^{(p-1)/2} (\text{mod } p)$ for all n.

Or

(b) Determine whether 219 is a quadratic residue or nonresidue mod 383.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the division algorithm.

17. (a) For every $n > 1$, prove that $\sum_{d/n} \lambda(d) = \begin{cases} 1 \text{ if n is a square} \\ 0 \text{ otherwis.} \end{cases}$. Also prove $\lambda^{-1}(n) = |\mu(n)|$ for all n

(b) State and prove the Selberg identity.

18. For all $x > 1$, prove that $\sum_{n \le x} d(n) = x \log x + (2c - 1)x + 0(\sqrt{x})$ where c is Euler's constant.

19. State and prove that Lagrange theorem.

20. State and prove the Gauss' lemma.

A–9034

Sub. Code
4MMA3E3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Third Semester

Mathematics

Elective — FUZZY MATHEMATICS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say that a fuzzy set is said to be normalized?

2. Define the fuzzy cardinality.

3. Write down the axiomatic skeleton for fuzzy complements.

4. Define the intersection of two fuzzy sets. Give an example.

5. Write a short notes on sagittal diagram.

6. What is meant by tolerance relation?

7. Define a fuzzy measure.

8. Define a possibility distribution associated with the function r.

9. Define maximizing decision. Give an example.

10. Give an illustration for fuzzy constraint.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Order the fuzzy sets defined by the following membership grade functions (assuming $x \geq 0$) by the inclusion (subset) relation :

$$\mu_A(x) = \frac{1}{1+20x}, \quad \mu_B(x) = \left(\frac{1}{1+10x}\right)^{1/2},$$

$$\mu_c(x) = \left(\frac{1}{1+10x}\right)^2.$$

Or

(b) Show that all α-cuts of any fuzzy set A defined on $\mathbb{R}^n$ $(n \geq 1)$ are convex if and only if

$\mu_A[\lambda r + (1-\lambda)s] \geq \min[\mu_A(r), \mu_A(s)]$ for all $r, s \in \mathbb{R}^n$ and $\lambda \in [0, 1]$.

12. (a) If c is a continuous fuzzy complement, then prove that c has a unique equilibrium.

Or

(b) For all $a, b \in [0, 1]$, prove that $i(a, b) \geq i_{\min}(a, b)$.

13. (a) The fuzzy relation R is defined on sets $x_1 = \{a, b, c\}$, $X_2 = \{s, t\}$, $X_3 = \{x, y\}$, $X_4 = \{i, j\}$ as follows.

$$R(X_1, X_2, X_3, X_4) = \frac{0.4}{(b, t, y, i)} + \frac{0.6}{(a, s, x, i)} + \frac{0.9}{(b, s, y, i)} + \frac{1}{(b, s, y, j)} + \frac{0.6}{(a, t, y, j)} + \frac{0.2}{(c, s, y, i)}.$$

Compute the projections $R_{1, 2, 3}$ and $R_{1, 3}$.

Or

(b) Describe the concept of fuzzy ordering relations in detail.

14. (a) Let $X = \{a, b, c, d\}$. Given the belief measure Bel $(\{b\}) = 0.1$, Bel $(\{a, b\}) = 0.2$, Bel $(\{b, c\}) = 0.3$, Bel $(\{b, d\}) = 0.1$, Bel $(\{a, b, c\}) = 0.4$, Bel $(\{a, b, d\}) = 0.2$, Bel $(\{b, c, d\}) = 0.6$ and $\text{Bel}(x) = 1$. Determine the corresponding basic assignment.

Or

(b) State and prove a necessary and sufficient condition for a belief measures Bel on a finite power set to be a probability measure.

15. (a) What is meant by fuzzy dynamic programming? Explain with suitable example.

Or

(b) Enumerate the vector-maximum problem with an example.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Consider the fuzzy sets A, B and C defined on the interval $x = [0, 10]$ of real numbers by the membership grade functions $\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = 2^{-x}$, $\mu_c(x) = \frac{1}{1+10(x-2)}$ and let $f(x) = x^2$ for all $x \in X$. Use the extension principle to derive $f(A), f(B)$ and $f(C)$.

17. Prove the following :

(a) $\lim\limits_{w \to \infty} \min[1, (a^w + b^w)^{1/w}] = \max(a, b)$

(b) $\lim\limits_{w \to \infty} i_w = \lim\limits_{w \to \infty} (1 - \min[1, ((1-a)^w + (1-b)^w)^{1/w}])$

$= \min(a, b)$

18. Let $M_P = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}$ and $M_Q = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{bmatrix}$

(a) Determine $M_{P \circ Q}$.

(b) Draw the sagittal diagram and also find $P \odot Q$.

19. Prove : Given a consonant body of evidence ($\mathfrak{F}$, m), the associated consonant belief and plausibility measures posses the following properties :

(a) $\text{Bel}(A \cap B) = \min[\text{Bel}(A), \text{Bel}(B)]$ for all $A, B \in \mathfrak{P}(x)$

(b) $Pl(A \cup B) = \max[Pl(A), Pl(B)]$ for all $A, B \in \mathfrak{P}(x)$.

20. Consider the following problem :

Minimize $Z = 4x_1 + 5x_2 + 2x_3$

Subject to

$$3x_1 + 2x_2 + 2x_3 \leq 60$$
$$3x_1 + x_2 + x_3 \leq 30$$
$$2x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

Determine the optimal solution.

A–9035

Sub. Code
4MMA4C1

M.Sc DEGREE EXAMINATION, NOVEMBER 2019

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define the Euclidean norm.

2. Define the operator norm.

3. What is meant by a support hyperplane?

4. Define a Banach limit.

5. State the Banach-steinhaus theroem.

6. When will you say that a map F is said to be closed?

7. What is meant by dual of a normed space?

8. Let X and Y be normed spaces. Let F_1 and F_2 be in BL(X,Y), and Prove that $(F_1 + F_2)' = F'_1 + F'_2$ $F'_1 + F'_2$.

9. State the polarization identify.

10. Define an orthonormal set.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let Y be a closed subspace of a normed space X. For $x+y$ in the quotient space x/y, let $\|x+y\| = \inf\{\|x+y\| : y \in Y\}$. prove that $\| \ \|$ is a norm on x/y

Or

(b) Let X and Y be normed spaces and F: $x \to y$ be a linear map such that the range R(F) of F is finite dimensional. Prove that F is continous if and only if the zero space Z(F) of F is closed in X.

12. (a) State and prove Hahn-Banach entension Theorem.

Or

(b) Prove that a normed space X is a Bauach space if and only if every absolutely summable series of elements in X is summable in X.

13. (a) State and prove the uniform boundedness principle theorem.

Or

(b) Let X be a normed space and $P : x \to x$ be a projection. Prove that P is a closed map if and only if the subspaces R(P) and Z(P) are closed in X.

14. (a) Let X be a normed space and $A \in BL(x)$. Prove that $\sigma(A') \subset \sigma(A)$.

Or

(b) Let X be a normed space. If X' is separable, then prove that X is separable.

15. (a) State and prove Bessel's inequality.

Or

(b) State and prove projection theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X be a normed space. Prove the following conditions are equivalent:

(a) Every closed and bounded subset of X is compact

(b) The subset $\{x \in x : \|x\| \leq 1\}$ of X is compact

(c) X is finite dimensional

17. State and prove the Hahn-Banach separation theorem.

18. State and prove open mapping theorm.

19. State and prove the Riesz representation theorem for $\subset ([a, b])$.

20. Derive Gram-Schmidt orthonormalization process.

A–9036

Sub. Code
4MMA4C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Fourth Semester

Mathematics

TOPOLOGY – II

(CBCS – 2014 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part A　　　　(10 × 2 = 20)

Answer **all** questions.

1. What is meant by one-point compactification?

2. Define subnet of (x_α).

3. What is meant by completely regular space?

4. Define the stone-cech compactification.

5. Define countably locally finite.

6. Define a $G_{\delta-}$ set. Give an example.

7. When will you say that the metric space is said to be totally bounded?

8. Define the point-open topology.

9. Define the compact open-topology.

10. When will you say that the space is said to be Baire space?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Define locally compact. Also prove that the rationals Q are not locally compact.

Or

(b) Let X be a space and $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection. property. Let $D \in \mathcal{D}$. Show that if $A \supset D$, then $A \in \mathcal{D}$.

12. (a) Show that every locally compact Hausdorff space is completely regular.

Or

(b) Let $A \subset X$ and let $f : A \to Z$ be a continuous map of A into the Hausdorff space Z. Prove that there is at most one extension of f to a continuous function $g : \overline{A} \to Z$.

13. (a) Let A be a locally finite collection of subsets of X. Prove that the collection $\mathcal{B} = \{\overline{A}\}_{A \in \mathcal{A}}$ of closures of the elements of $\mathcal{A}$ is locally finite.

Or

(b) Determine a nondiscrete space that has a countably local finite basis but does not have a countable basis.

14. (a) Prove that Euclidean space $\mathbb{R}^K$ is complete in either of its usual metrics, the Euclidean metric d or the square metric p.

Or

(b) If X is locally compact, or if X satisfies the first countability axiom, then prove that X is compactly generated.

15. (a) Let X be locally compact Hausdorff and let $\mathcal{C}(X, Y)$ have the compact open topology. Prove that the map $\mathcal{C}$: $X \times \mathcal{C}(X, Y) \rightarrow Y$ defined by the equation $\mathcal{C}(x, f) = f(x)$ is continuous.

Or

(b) Let $C_1 \supset C_2 \supset .....$ be a nested sequence of nonempty closed sets in the complete metric space X. If $\operatorname{diam} C_n \rightarrow 0$, then prove that $\cap C_n \neq \phi$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Tychonoff theorem.

17. Let X be a completely regular space. If Y_1 and Y_2 are two compactifications of X satisfying the extension property. Prove that Y_1 and Y_2 are equivalent.

18. Prove that a space X is metrizable if and only if X is regular and has basis that is countably locally finite.

19. Show that a metric space (x, d) is compact if and only if it is complete and totally bounded.

20. State and prove that Ascoli's theorem.

A–9037

Sub. Code
4MMA4C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Fourth Semester

Mathematics

NUMERICAL ANALYSIS

(CBCS – 2014 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part A　　　　(10 × 2 = 20)

Answer **all** questions.

1. Find the spectral radius of the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$.

2. Define damped Newton's method.

3. What do you meant by least-square approximation?

4. Find the zeros of the Legendre polynomial $p_2(x)$.

5. What is meant by discretization error?

6. For which polynomial, Simpson's rule is exact?

7. Define Wronskian.

8. Find the general solution of $y''-a^2y=0,\ y(o)=0,\ y'(o)=1$.

9. What do you mean by multistep methods?

10. Write the Milen's predictor and corrector formulae.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find all the critical points of the function $F(x_1,x_2)=\frac{x_1^{\ 3}}{3}+x_2^{\ 2}x_1+3.$

Or

(b) Find the real root of $xe^x-2=0$ correct to three places of decimal using Newton's method.

12. (a) Prove that the orthogonal polynomials satisfy a three-term recurrence relation given by,

$$P_{i+1,}\ (x)=A_i(x-B),P_i(x)=C_iP_{i-1}(x)\quad i=0,1,....k-1.$$

Or

(b) Express the polynomial $p(x)=x^4+2x^3+x^2+2x+1$ as a sum of Legendre polynomials.

13. (a) Use Simpson's rule to estimate the value of the integral $I=\int_0^1(1-x^2)dx,$ taking $h=0.1$.

Or

(b) Evaluate $\int_0^1 e^{-x^2dx}$ by corrected Trapezoidal rule.

14. (a) Find the solution of $y''-4y'+4y=x$ subject to $y(0)=0, y'(0)=1$.

Or

(b) Find the general solution of the difference equation

$y_{n+2} - y_{n+1} - y_n = 0.$

15. (a) Solve by difference methods, the boundary-value problem

$$\frac{d^2y}{dx^2} + y = 0;\ y(0) = 0;\ y(1) = 1.$$

Or

(b) Solve the following using the shooting method,

$y'' = e^y,\ y(0) = y(1) = 0,$ taking $y'(0) = 0.$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Apply damped Newton's method to solve the system

$f_1(x) = x_1 + 2\log|x_1| - {x_2}^2,\ f_2(x) = 2x_1^2 - x_1x_2 - 5x_1 + 1$ starting with $x^{(0)} = [2,1]^T$.

17. Find the zeros of the Hermite polynomials

$H_2(x), H_3(x), H_4(x).$

18. Apply each of the five standard rules of integration to find an approximation to $I = \int_0^1 x \sin x\, dx$. Compare the results with the correct value $I = \sin 1 - x\cos 1 = 0.301169.$

19. Compute $y(0.2)$ given $\frac{dy}{dx}+y+xy^2=0 \quad y(0)=1$ by taking $h=0.1$ using Runge-kutta method of order 4, correct to 4 decimals.

20. Solve $y'=x^2+y, y(0)=1,$ from $x=0$ to $x=0.5,$ using Euler's method as a predictor-corrector method. Determine the step h so that four decimal places of accuracy are obtained at $x=0.5$. Start with $h=0.05$.

A–9038

Sub. Code
4MMA4E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

Fourth Semester

Mathematics

Elective – ADVANCED STATISTICS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define biased and unbiased estimator for the parameter.

2. What is meant by composite statistical hypothesis?

3. Define sufficient statistic.

4. Define complete family of p.d.f.

5. Define the fisher-information.

6. What is meant by asymptotically efficient?

7. Define likelihood ratio test.

8. Define non central t-distribution.

9. What is analysis of variance? What purpose does the technique serve?

10. Write down the correlations coefficient of the random sample.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Derive the confidence intervals for difference of means.

Or

(b) Let the observed value of the mean $\overline{X}$ of a random sample of size 20 from a distribution that is $N(\mu, 80)$ be 81.2. Find 95% confidence interval for μ.

12. (a) Let $X_1, X_2, ...X_n$ denote a random sample from a normal distribution with mean zero and variance θ, $0 < \theta < \infty$. Show that $\sum_1^n \frac{X_i^2}{n}$ is an unbiased estimator of θ and has variance $\frac{2\theta^2}{n}$.

Or

(b) Let $X_1, X_2 ... X_n$ denote a random sample from a distribution that is $N(0, \theta)$. Prove that $Y = \Sigma x_i^2$ is a complete sufficient statistic for θ. Also find the unbiased minimum variance estimator of θ^2.

13. (a) Discuss the Bayesian estimation.

Or

(b) Let X have a gamma distribution with $\alpha = 4$ and $\beta = \theta > \alpha$. Find the fisher information $I(\theta)$.

14. (a) Enumerate the uniformly most powerful test.

Or

(b) Narrate the sequential probability ratio test.

15. (a) Compute the mean and the variance of a random variable that is $\chi^2(r, \theta)$.

Or

(b) Explain the analysis of variance for two-way classification.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Explain the maximum likelihood estimator.

(b) Let $X_1, X_2 ... X_n$ denote a random sample from the distribution with p.d.f

$$f(x; \theta) = \theta^x (1-\theta)^{1-x}, x = 0, 1$$
$$= 0 \text{ elsewhere, where } 0 \le \theta \le 1$$

17. State and prove the Neyman factorization theorem.

18. State and prove the Rao-Cramer inequality.

19. State and prove the Neyman-Pearson theorem.

20. (a) With the usual notations, prove that $Q = Q_2 + Q_4 + Q_5$.

(b) Show that $\sum_{j=1}^{b}\sum_{i=1}^{a}\left(X_{ij} - \overline{X}_{i.}\right)^2 =$

$$\sum_{j=1}^{b}\sum_{i=1}^{a}\left(X_{ij} - \overline{X}_{i.} - \overline{X}_{.j} + \overline{X}_{..}\right)^2 + a\sum_{j=1}^{b}\left(\overline{X}_{.j} - \overline{X}_{..}\right)^2.$$