A–9710

Sub. Code
4MMA2E4

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Second Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by operation table?

2. Define submonoid. Give an example.

3. Write the statement "If it is raining, then we will not meet today" is symbolic form.

4. Show that $(P \to Q) \wedge (R \to Q)$ and $(P \vee R) \to Q$ are equivalent formulae.

5. Determine whether the following inference pattern is valid or invalid.

   If today is Thursday, then yesterday was Wednesday.

   Yesterday was Wednesday.

   Today is Thursday.

6. Define an open statement.

7. Give any two properties of a lattice.

8. If $D(n)$ denotes the lattice of all positive divisors of the integer n, draw the Hasse diagram of $D(15)$.

9. What is a Boolean polynomial?

10. Give the uses of Karnaugh map.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $(M, *, e)$ be a monoid and $a \in M$. If a is invertible, then prove that its inverse is unique.

Or

(b) Prove that the property of idempotency is preserved under a semigroup homomorphism.

12. (a) Draw the parsing tree for the formula.

$(\neg(\neg(P \wedge (q \rightarrow p))))$.

Or

(b) Construct the truth table for $\neg(\neg P \wedge \neg Q)$.

13. (a) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q.

Or

(b) Prove that the statements

(i) $(\forall x)(P(x)) \rightarrow P(y)$

(ii) $P(y) \rightarrow (\exists x)(P(x))$ are valid statements.

14. (a) Prove that in any lattice $(L, \geq)$ the operations $\vee$ and $\wedge$ are isotone.

Or

(b) Let L be a distributive lattice and $a, b, c, \in L$. If $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$, then prove that $b = c$.

15. (a) In a Boolean algebra *B,* prove that $(a \wedge b)' = a' \vee b', a, b \in B$.

Or

(b) Consider the Boolean function.

$f(x_1, x_2, x_3) = ((x_1 + x_2) + (x_1 + x_2)).x_1.\bar{x}_2$.

Simplify this function and draw the gate circuit diagram.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let T be the set of all even integers. Show that the semigroups $(z, +)$ and $(T, +)$ are isomorphic.

17. (a) Draw the passing tree for the formula.

$((P \rightarrow (\neg q)) \rightarrow (p \wedge q))$

(b) Obtain the $P \subset NF$ of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$.

18. Using indirect method of proof derive $P \rightarrow \neg s$ from $P \rightarrow Q \vee R$, $Q \rightarrow \neg P, S \rightarrow \neg R, P$.

19. Prove that in a distributive lattice the following are equivalent

(a) $a \wedge b \leq x \leq a \vee b$

(b) $x = (a \wedge x) \vee (b \wedge x) \vee (a \wedge b)$

20. Simplify:
$f(a,b,c,d,e) = \Sigma(0,1,3,8,9,13,14,15,16,17,19,24,25,27,31)$ using Karnaugh map.

A–9713

Sub. Code
4MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Fourth Semester

Mathematics

TOPOLOGY – II

(CBCS – 2014 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the one-point compactification.

2. State the countable intersection property.

3. Define the completely regular space.

4. When will you say that two compactification is said to be equivalent?

5. Show that the collection $\mathcal{A} = \{(n, n+2) / n \in z\}$ is locally finite.

6. Define a G_S -set. Give an example.

7. Is R complete? Justify your answer.

8. Define the point-open topology.

9. Define the compact open topology.

10. State Ascoli's theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let x be locally compact Hausdorff space and let A be a subspace of X. If A is closed in X or open in X, then prove that A is locally compact.

Or

(b) Let X be a space. Let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Let $D \in \mathcal{D}$. If $A \supset D$ then prove that $A \in \mathcal{D}$.

12. (a) Show that a product of completely regular spaces is completely regular.

Or

(b) Let $A \subset X$ and let $f : A \to z$ be continuous map of A the Hausdorff space Z. Prove that there is atmost extension of f to a continuous function $g : \overline{A} \to z$.

13. (a) Let $\mathcal{A}$ be a locally finite collection of subsets of X. Prove the following:

(i) Any sub collection of $\mathcal{A}$ is locally finite.

(ii) The collection $\mathcal{B} = \{\overline{A}\}_{A \in \mathcal{A}}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

(iii) $\overline{\bigcup_{A \in \mathcal{A}} A} = \bigcup_{A \in \mathcal{A}} \overline{A}$

Or

(b) Let X be normal and let A be a closed G_δ set in X. Prove that there is a continuous function $f : x \to [0, 1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) Prove that Euclidean space $\mathbb{R}^k$ is complete in either of its usual metrics, the Euclidean metric d or the square metric ρ.

Or

(b) If X is compactly generated, then prove that a function $f : x \to y$ is continuous if for each compact subspace $\mathcal{C}$ of X, the restricted function $f / \mathcal{C}$ is continuous.

15. (a) Let X be locally compact Hausdorff and let $\mathcal{C}(X,Y)$ have the compact open topology. Prove that the map $e : X \times \mathcal{C}(X,Y) \to Y$ defined by the equation $e(x,f) = f(x)$ is continuous.

Or

(b) Let $c_1 \supset c_2 \supset ....$ be a nested sequence of nonempty closed sets in the complete metric space X. If diam $c_n \to 0$, then prove that $\cap c_n \neq \varphi$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that an arbitrary product of compact spaces is compact in the product topology,

17. Let X be a complete regular space. Prove that there exists a compactification Y of X having the property that every bounded continuous map $f : x \to \mathbb{R}$ extends uniquely to a continuous map of Y into $\mathbb{R}$.

18. State and prove the Nagota-Smirnov metrization theorem.

19. Show that a metric space (x,d) is compact if and only if it is complete and totally bounded.

20. State and prove the Baire Category theorem.

A–9783

Sub. Code
4MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Euclidean norm.

2. What is meant by bounded linear map?

3. Define a Banach limit.

4. State Hahn-Banach separation theorem.

5. When will you say that a map F is said to be closed?

6. Write down the geometrical interpretation of uniform boundedness principle.

7. Define normed dual.

8. State the Riesz representation theorem for $C([a, b])$.

9. Define an inner product space.

10. State the Pythagoras theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove the Riesz lemma.

Or

(b) Let X be a normed space. Let E be a convex subset of X. Prove that the interior E^0 of E and the closure $\overline{E}$ of E are also convex. Also prove that $\overline{E} = \overline{E}^0$ of $E_0 \neq \phi$.

12. (a) Let X be a normed space over K, and f be a non-zero linear functional on X. If E is an open subset of X, then prove that $f(E)$ is an open subset of K.

Or

(b) Let X and Y be normed spaces and $X \neq \{0\}$. Prove that $BL(X, Y)$ is a Banach space in the operator norm if and only if Y is a Banach space.

13. (a) State and prove Resonance theorem.

Or

(b) Let X be a linear space over K. Consider subsets U and V of X, and $k \in K$ such that $U \subset V + KU$. Prove that for every $x \in U$, there is a sequence (v_n) in V such that $x - (v_1 + kv_2 + ... + k^{n-1}v_n) \in k^n U$, $n = 1, 2, ...$

14. (a) Let X be a normed space. If X' is separable, then prove that X is also separable.

Or

(b) Let X and Y be normed spaces. Let $F \in BL(X, Y)$. Prove that $\|F'\| = \|F\| = \|F''\|$ and $F''J_x = J_y F$, where J_x and J_y are the canonical embeddings of X and Y into X' and Y', respectively.

15. (a) State and prove Parallelogram law. Also state the polarization identity.

Or

(b) Derive the Bessel's inequality.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let X and Y be normed spaces and $F : X \to Y$ be a linear map. Prove the following conditions are equivalent.

(a) F is bounded on $\overline{U}(0, r)$ for some $r > 0$

(b) F is continuous at 0

(c) F is continuous on X.

(d) $\| F(x) \| \leq \alpha \| x \|$ for all $x \in X$ and some $\alpha > 0$

(e) The zero space $Z(F)$ of F is closed in X and the linear map $\widetilde{F} : x / z(F) \to y$ defined by $\widetilde{F}(x + z(F)) = F(x),\ x \in X$, is continuous

17. State and prove the Hahn-Banach extension theorem.

18. State and prove open mapping theorem.

19. State and prove the Riesz representation theorem for L^p.

20. Discuss Gram-Schmidt orthonormalization process.

A–9784

Sub. Code
4MMA4C3

M.Sc DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Fourth Semester

Mathematics

NUMERICAL ANALYSIS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Find all the critical points of the function

   $F(x_1, x_2) = {x_1}^3/3 + {x_2}^2 x_1 + 3$

2. Write short note on relaxation method.

3. What is meant by chebyshev points?

4. If $p(x)$ is a polynomial of degree <k,then prove that $p(x)$ is orthogonal to $p_k(x)$.

5. What is the need for numerical differential?

6. Write down the formula of composite simpson approximation S_N.

7. Find the general solution of the equation $y' = -2y$.

8. Write down the Taylor's algorithm of order k.

9. Define an interior and exterior mesh points.

10. What is meant by a boundary value problem?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Write down the steepest descent algorithm.

Or

(b) Narrate the fixed - point iteration for linear system algorithm.

12. (a) Calculate a good polynomial approximation of degree n on

$0 \le x \le 1$ to $f(x) = \sqrt{x}$ for $n = 1,2,3.....10$

Or

(b) Find the zeros of the hermit polynomials $H_2(x), H_3(x), H_4(x)$.

13. (a) Write a program for the corrected Trapezoid rule.

Or

(b) Evaluate $\int_0^1 x \cos x \, dx$ by simpson's rule.

14. (a) Find the general solution of the difference equation $y_{n+2} - 4y_{n+1} + 4y_n = n$.

Or

(b) Solve the equation $y' = -2y, 0 \le x \le 1, y(0) = 1$ by Euler's method.

15. (a) Solve by difference methods the boundary value problem $\frac{d^2y}{dx^2} + y = 0,\ y(0) = 0,\ y(1) = 1$ take $h = \frac{1}{4}$.

Or

(b) Write an algorithm for the shooting method for second-order boundary value problem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. The $f(\xi) = 0$ with $f_1(x) = x_1 + 3\ln|x_1| - x_2^2$, $f_2(x) = 2x_1^2 - x_1x_2 - 5x_1 + 1$ has several solutions.Solve the system using damped newton's method.

17. Solve the least square approximation problem if $f(x) = 10 - 2x + \frac{x^2}{10},\ x_n = 10 + \frac{n-1}{5},\ f_n = f(x_n),\ n = 1,2,....6$.

18. Find an approximation to $I = \int_0^3 \frac{(\sin x)^2}{x} dx$,using gaussian quandrature with $k = 3$.

19. Solve the equation, $y' = x + y,\ y(0) = 0$ from $x = 0$ to $x = 1$, using the Adams-Bashforth method.

20. Solve the following problem,using the shooting method:

$y'' = 2y^3,\ y(1) = 1,\ y(2) = \frac{1}{2}$, taking $y'(1) = 0$ as a first guess.

A–9785

Sub. Code
4MMA4E1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Fourth Semester

Mathematics

Elective — ADVANCED STATISTICS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a biased estimator of the parameter.

2. Define the power function of a test of a statistical hypothesis.

3. Define a sufficient statistic.

4. Distinguish between completeness and uniqueness.

5. Define Bayesian statistics.

6. When will you say that a statistic is said to be efficiency?

7. Define a best critical region of size α.

8. Define Wald's sequential probability ratio test.

9. State the Boole's inequality.

10. Write down the formula for correlation coefficient.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Let $\overline{X}$ be the mean of a random sample of size 20 from a distribution that is $N(\mu, 80)$ be 81.2. Find a 95% confidence interval for μ.

Or

(b) Let $\overline{X}$ and $\overline{Y}$ be the means of two independent random samples, each of size n, from the respective distributions $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, where the common variance is known. Find n such that

$$\Pr(\overline{X} - \overline{Y} - \sigma/5 < \mu_1 - \mu_2 < \overline{X} - \overline{Y} + \sigma/5) = 0.90$$

12. (a) Show that the mean $\overline{X}$ of a random sample of size n from a distribution having p.d.f.

$$f(x;\theta) = \left(\frac{1}{\theta}\right) e^{-(x/\theta)},\ 0 < x < \infty,\ 0 < \theta < \infty$$

$= 0$, elsewhere, is an unbiased estimator of θ and has variance $\frac{\theta^2}{n}$.

Or

(b) Let $X_1, X_2, \ldots X_n$ represent a random sample from the discrete distribution having the probability density function

$$f(x;\theta) = \theta^x (1-\theta)^{1-x},\ x = 0, 1,\ 0 < \theta < 1,$$
$$= 0, \text{ elsewhere.}$$

Show that $Y_1 = \sum_1^n x_i$ is a complete sufficient statistic for θ.

13. (a) Let $X_1, X_2, ..., X_n$ denote a random sample from a distribution which is $b(1, \theta), 0 < \theta < 1$. Find the decision function δ which is a Bayes' solution.

Or

(b) Let X be $N(\theta, \sigma^2)$, where $-\infty < \theta < \infty$ and σ^2 is known. Compute Fisher information $I(\theta)$.

14. (a) If $X_1, X_2, ...X_n$ is a random sample from a beta distribution with parameters $\alpha = \beta = \theta > 0$, find a best critical region for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$.

Or

(b) Let X have a p.d.f.

$$f(x, \theta) = \theta^x (1-\theta)^{1-x}, \ x = 0, 1,$$
$$= 0, \text{ elsewhere}$$

Discuss sequential probability ratio test when $H_0 : \theta = \frac{1}{3}$ and $H_1 : \theta = \frac{2}{3}$.

15. (a) Define a non-central chi-square distribution and derive its p.d.f.

Or

(b) A random sample of size $n = 6$ from a bivariate normal distribution yields the value of correlation coefficient to be 0.89. Test the hypothesis $H_0 : \rho = 0$ at 5% level of significances.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let $X_1, X_2, \ldots. X_n$ be i.i.d., each with the distribution having p.d.f. $f(x; \theta_1, \theta_2) = \left(\frac{1}{\theta_2}\right) e^{-(x-\theta_1)/\theta_2}$, $\theta_1 \le x < \infty$, $-\infty < \theta_1 < \infty$, $0 < \theta_2 < \infty$, zero elsewhere. Find the maximum likelihood estimators of θ_1 and θ_2.

17. State and prove the Rao and Blackwell theorem.

18. Suppose that the random sample arises from a distribution with p.d.f.

    $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta \in \Omega = \{\theta : 0 < \theta < \infty\}$ zero elsewhere. Prove that the m.l.e is asymptotically efficient.

19. Define Uniformly most powerful test. Does it exist always? Justify your claim with suitable example.

20. Describe the analysis of variance for two-way classification.

A–10192

Sub. Code
4MMA1C4

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Verify that the function $\phi_1(x) = x, x > 0$ satisfies the equation $x^2 y'' - xy' + y = 0$.

2. Write down the Chebyshev equation.

3. Define indicial polynomial.

4. Write down Bessel function of order α of the first kind.

5. Eliminate the arbitrary function f from the equation $z = f(x - y)$.

6. When will you say that two partial differential equations are said to be compatible?

7. State the telegraphy equation.

8. Classify the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.

9. Show that $r^{-2}\cos\theta$ satisfying the Laplace equation, when r, θ and ϕ are spherical polar coordinates.

10. Define the exterior Neumann problem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that there exist n linearly independent solutions of $L(y) = 0$ on I.

Or

(b) Find two linearly independent power series solution of the equation $y'' + 3x^2y' - xy = 0$.

12. (a) Find all solutions of the equation $x^2y''' + 2x^2y'' - xy' + y = 0$ for $x > 0$.

Or

(b) Obtain two linearly independent solutions of the equation which are valid near $x = 0 : x^2y'' + 2x^2y' - 2y = 0$.

13. (a) Find the general integral of the linear partial differential equation $y^2p - xyq = x(z - 2y)$.

Or

(b) Solve the equation $p^2x + q^2y = z$ using Jacobi's method.

14. (a) Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x}$ is satisfied by $z = \frac{1}{x}\phi(y-x) + \phi'(y-x)$, where ϕ is an arbitrary function.

Or

(b) Solve the one-dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

15. (a) (i) What is meant by a boundary value problem for Laplace's equation?

(ii) Explain about the types of Dirichlet problem.

Or

(b) Enumerate the following terms:

(i) Transverse vibrations of a membrane.

(ii) Sound waves in space.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. With the usual notations, prove that

(a) The coefficient of x^n in $P_n(x)$ is $\frac{(2n)!}{2^n (n!)^2}$.

(b) $\int_{-1}^{1} P_n(x) P_m(x) dx = 0, \ (n \neq m)$.

17. Show that -1 and 1 are regular singular points for the Lagendre equation

$$\left(1 - x^2\right)y'' - 2xy' + \alpha(\alpha + 1)y = 0 .$$

Also find the indicial polynomial and its roots, corresponding to the point $x = 1$.

18. Find the solution of the equation $z = \frac{1}{2}\left(p^2 + q^2\right) + (p - x)(q - y)$ which passes through the x-axis.

19. (a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

(b) Find the particular integral of the equation $\left(D^2 - D^1\right)z = A\cos(lx + my)$ where A, l, m are constants.

20. A uniform insulated sphere of dielectric constant k and radius 'a' carries on its surface a charge of density $\lambda P_n(\cos\theta)$. Prove that the interior of the sphere contributes an amount $\frac{8\pi^2\lambda^2 a^3 kn}{(2n+1)(kn+n+1)^2}$ to the electrostatic energy.

A–10193

Sub. Code
4MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Second Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. If $f(x)=\begin{cases} c\left(\frac{2}{3}\right)^x, & x=1,2,3 \\ 0, & \text{elsewhere} \end{cases}$, then find the constant C so that $f(x)$ satisfies the condition of being a p.d.f of one random variable X

2. Let X have the p.d.f $f(x)=\begin{cases} 2x, & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$. Find the distribution function and p.d.f of $Y=X^2$.

3. Let X and Y have the p.d.f $f(x,y)=\begin{cases} 1, & 0<x<1, 0<y<1 \\ 0, & \text{elsewhere} \end{cases}$ Find the p.d.f of the product $Z=XY$.

4. Show that the random variables x_1 and x_2 with joint p.d.f $f(x_1, x_2) = \begin{cases} (12x_1x_2(1-x_2), & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$ are independent.

5. If X is $n(2,25)$, calculate $\Pr(0 < X < 10)$.

6. Determine the binomial distribution for which the mean is 4 and variance is 3.

7. Show that $S^2 = \frac{1}{n}\sum_{i}^{n}(X_i - \overline{X})^2 = \frac{1}{n}\sum_{i}^{n}(X_i^2 - \overline{X}^{-2})$, where $X = \sum_{1}^{n} X_i / n$

8. Let X have the p.d.f. $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1,2,3,\ldots \\ 0, & \text{elsewhere} \end{cases}$ find the p.d.f of $Y = X^3$.

9. State any two theorems on limiting distributions.

10. Let $\bar{x}$ denote the mean of a random sample of size 100 from a distribution that is $\chi^2(50)$. find an approximate value of $\Pr(49 < \overline{X} < 51)$

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Let as select five cards at random and without replacement from an ordinary deck of playing cards.

(i) Find the p.d.f. of X, the number of hearts in the five cards

(ii) Determine $\Pr(X \le 1)$.

Or

(b) Let $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ be the p.d.f of X. Find the distribution function and the p.d.f of $Y = \sqrt{x}$.

12. (a) Let X_1 and X_2 have the p.d.f. $f(x_1,x_2)=\begin{cases}8x_1x_2 & 0<x_1<x_2<1\\ 0, & \text{elsewhere}\end{cases}$ Find $E(7x_1x_2{}^2+5x_2)$.

Or

(b) Let the random variables X and Y have the joint p.d.f. $f(x,y)=\begin{cases}x+y & 0<x<1,\ 0<y<1\\ 0 & \text{elsewhere}\end{cases}$

Find the correlation coefficient of X and Y.

13. (a) Show that the graph of a p.d.f $N(\mu,\sigma^2)$ has points of inflection at $x=\mu-\sigma$ and $x=\mu+\sigma$.

Or

(b) Let X and Y have a bivariate normal distribution with parameters $\mu_1=5,\ \mu_2=10,\ \sigma_1{}^2=1,\ \sigma_2{}^2=25$ and $p>0$. If $\Pr(4<y<16/X=5)=0.954$, determine P.

14. (a) Let X have the p.d.f. $f(x)=\begin{cases}4x^3, & 0<x<1\\ 0 & ,\text{elsewhere}\end{cases}$ Show that $y=-2$ In X^4 is $\chi^2(2)$.

Or

(b) Show that the t-distribution with r=1 degree of freedom and the cauchy distribution are the same.

15. (a) Let the p.d.f of Y_n be $f_n(1)=1$, $y=n$, zero elsewhere show that y_n does not have a limiting distribution.

Or

(b) Let $\overline{X}$ denote the mean of a random sample of size 100 from a distribution that is $\chi^2(50)$. Compute an approximate value of $\Pr(49<\overline{X}<51)$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) State and prove Chebyshev's inequality.

    (b) If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, determine a lower bound for the probability $\Pr(-2 < X < 8)$ using Chebyshev's inequality.

17. Let $f(x_1, x_2) = 21x_1^{\,2}x_2^{\,3}$, $0 < x_1 < x_2 < 1$, zero elsewhere, be the joint p.d.f of X_1 and X_2.

    (a) Find the conditional mean and variance of X_1 given $X_2 = x_2$ $0 < X_2 < 1$.

    (b) Find the distribution of $y = E(X_1 / X_2)$.

    (c) Determine $E(Y)$ and var(y) and compare these to $E(X_1)$ and var (X_1) respectively.

18. (a) Derive the Poisson distribution as the limiting form of binomial distribution.

    (b) State and prove the recurrence relation for the moments of Poisson distribution.

19. Derive the p.d.f. of F distribution and obtain its mode.

20. State and prove central limit theorem and give its significance.

A–10384

Sub. Code
4MMA1C3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

First Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. With usual notations write the formula for length of a curve.

2. Define the osculating plane.

3. Define an involutes.

4. State the fundamental existance theorem for space curves.

5. Define a surface.

6. What is meant by direction coefficients?

7. State the necessary and sufficient condition for the curve $v = c$ to be geodesic.

8. Give the Christoffel symbols of the record kind.

9. Define an umbilic.

10. Write down the characteristic line.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Determine the function $f(u)$ so that the curve given by $\vec{r} = (a\cos u, a\sin u, f(u))$ shall be plane.

Or

(b) Show that the length of the common perpendicular d of the tangents at two near points distance s apart is approximately given by $d = \frac{k\tau s^2}{12}$.

12. (a) If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma\rho^1) + \frac{\rho}{\sigma} = 0$.

Or

(b) Show that the involutes of a circular helix are plane curve.

13. (a) Find the area of the anchor ring.

Or

(b) Find the coefficients of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

14. (a) Prove that the curves of the family $v^3/u^2 =$ constant are geodesics on a surface with metric.

$$v^2du^2 - 2uvdudv + 2u^2dv^2 \ (u > 0, v > 0).$$

Or

(b) State and prove the Gauss-Bonnet theorem.

15. (a) Derive the second fundamental form of a surface.

Or

(b) Narrate the following terms:

(i) Dupin's indicatrix;

(ii) Osculating developable.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Derive the Serret-Frenet formulae.

17. Show that the intrinsic equations of the curve given by $x = ae^u \cos u, y = ae^u \sin u, z = be^u$ are

$$k = \frac{\sqrt{2}a}{\left(2a^2 + b^2\right)^{\frac{1}{2}}} \cdot \frac{1}{s}, \ \tau = \frac{6}{\left(2a^2 + 6^2\right)^{\frac{1}{2}}} \cdot \frac{1}{s}.$$

18. If θ is the angle at the point (u,v) between the two directions given by $Pdu^2 + 2Qdudv + Rdv^2 = 0$, then prove that $\tan\theta = \frac{2H(Q^2 - PR)^{\frac{1}{2}}}{ER - 2FQ + GP}$.

19. If (λ,μ) is the geodesic curvature vector, then prove that $k_g = \frac{-H\lambda}{Fu' + Gv'} = \frac{H\mu}{Eu' + Fv'}$.

20. State and prove the Radrigue's formula.

A–10195

Sub. Code
4MMA3C4

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Third Semester

Mathematics

NUMBER THEORY

(CBCS – 2014 onwards)

Time : 3 Hours　　　　　　　　　　Maximum : 75 Marks

Part A　　　　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Write any two properties of divisibility.

2. If a prime p does not divide a, then prove that $(p, a) = 1$.

3. Define Lioville's function.

4. If f is multiplicative then prove that $f(1) = 1$.

5. Define Legendre's identity.

6. Define mutually visible lattices.

7. State Little Fermet' theorem.

8. Prove that congruence is transitive.

9. State Reciprocity law for Jacobi symbols.

10. Write Diophantine equation.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the properties of greatest common divisor.

Or

(b) Prove that there are infinitely many prime numbers.

12. (a) Prove that for $n \geq 1$, $\phi(n) = \sum_{d/n} \mu(d)\frac{n}{d}$.

Or

(b) State and prove Selberg identity.

13. (a) Prove that for all $x \geq 1$, $\sum_{n \leq x} \sigma_1(n) = \frac{1}{2}\zeta(2)x^2 + O(x \log x)$.

Or

(b) Prove that for $x > 1$, $\sum \phi(n) = \frac{3}{\pi^2}x^2 + O(x \log x)$.

14. (a) Prove that for any prime p all the coefficient of the polynomial $f(x) = (x-1)(x-2)\ldots(x-p+1) - x^{p-1} + 1$ are divisible by p.

Or

(b) State and prove Chinese remainder theorem.

15. (a) Prove that Legendre's symbols (n/p) is a completely multiplicative function.

Or

(b) If p is an odd positive integer prove that $(-1/p) = (-1)^{(p-1)/2}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

17. If both g and $f * g$ are multiplicative then prove that f is also multiplicative.

18. Prove that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$.

19. State and prove Lagrange's theorem.

20. State and prove Gauss lemma.

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A–10385

Sub. Code
4MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Second Semester

Mathematics

ALGEBRA – II

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a subspace of a vector space.

2. Define the following terms:

   (a) Linearly dependent

   (b) Basis of V

3. Define the annihilator of W.

4. What is meant by orthonormal set?

5. Define algebraic number.

6. Find the degree of the splitting field of $x^4 + 1$ over F.

7. Give an example for a normal extension of F.

8. Express the polynomial $x_1^3 + x_2^3 + x_3^3$ in the elementary symmetric functions in x_1, x_2, x_3.

9. Define a Characteristic vector of T.

10. If $T \in A(v)$ then prove that $(T^*)^* = T$.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Show that the intersection of two subspaces of V is a subspace of V.

Or

(b) If F is the field of real numbers, prove that the vectors (1, 1, 0, 0), (0, 1, -1, 0), (0, 0, 0, 3) in $F^{(4)}$ are linearly independent over F.

12. (a) Prove that A(W) is a subspace of $\hat{v}$.

Or

(b) State and prove the Bessel inequality.

13. (a) State and prove the Remainder theorem.

Or

(b) If F is a field of Characteristic $p \neq 0$, then prove that the polynomial $x^{px} - x \in F[x]$, for $n \geq 1$. has distinct roots.

14. (a) Define the fixed field. prove that the fixed field of G is a subfield of K.

Or

(b) If $x_1, x_{2,} x_3$ are the roots of the cubic polynomial $x^3 + 7x^2 - 8x + 3$, find the cubic polynomial whose roots are $\alpha_1^2, \alpha_2^2, \alpha_3^2$.

15. (a) If V is finite-dimensional over F, then prove that $T \in A(v)$ is regular of and only if T maps V onto v.

Or

(b) Define a unitary transformation. If $(vT, VT) = (v, V)$ for all $v \in V$, then prove that T is unitary.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If V is finite - dimensional and if w is a subspace of v, then prove that w is finite - dimensional, $\dim w \leq \dim v$ and $\dim(x/w) = \dim V - \dim W$.

17. (a) If F is the field of real numbers, find $A(w)$ where w is spanned by (1, 2, 3) and (0, 4, -1).

(b) With the usual notations, prove that

$$|\langle u, v\rangle| \leq \|u\|\|v\| \text{ if } u, v \in V.$$

18. If L us a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F, moreover, $[L:F] = [L:K][K:F]$.

19. State and prove the fundamental theorem of Galois theory.

20. Let $v = F^{(3)}$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of in the basis $v_1 = (1,0,0)$, $v_2(0,1,0)$, $v_3 = (0,0,1)$. Find the matrix of T in the basis $u_1(1,1,0)$, $u_2 = (1,2,0)$, $u_3 = (1,2,1)$.

A–10194

Sub. Code
4MMA2E2

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Second Semester

Mathematics

GRAPH THEORY

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an isomorphism between two graphs with an example.

2. Draw all the tress with 6 vertices.

3. What is meant by cut vertex? Give an example.

4. Write short notes on Konigsberg problem.

5. Define a perfect matching. Give an example.

6. Find the edge chromatic number of a Petersen graph.

7. Prove that $r(2,l) = l$.

8. When will you say that a critical graph is block?

9. State the Jordan curve theorem in the plane.

10. Is the Petersen graph nonplanar? Justify your answer.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that if G is simple and $\varepsilon > \binom{\gamma - 1}{2}$, then prove that G is connected.

Or

(b) Define a center of graph G. Prove that a tree has either exactly one center or two adjacent centers.

12. (a) (i) What is meant by subdivision of an edge?

(ii) If G is a block with $\gamma \geq 3$, then prove that any two edges of G lie on a common cycle.

Or

(b) State and prove the Dirac theorem.

13. (a) State and prove the Berge theorem.

Or

(b) (i) Explain edge colouring of a graph G.

(ii) If G is bipartite then prove that $\psi' = \Delta$.

14. (a) Define an independent set of graph G. Also prove that a set $S \subseteq V$ is an independent set of G if and only if $V - S$ is a covering of G.

Or

(b) Prove that : In a critical graph, no vertex cut is a clique.

15. (a) Let v be a vertex of a planar graph G. Prove that G can be embedded in the plane in such a way that v is on the enterior face of the embedding.

Or

(b) State and prove Euler's formula for connected plane graph.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that an edge e of G is a cut edge of G if and only if e is contained is no cycle of G.

17. With the usual notations, prove that $k \leq k' \leq \delta$.

18. State and prove the Hall's theorem.

19. State and prove the Erdos theorem.

20. Prove that every planar graph is 5-vertex-colourable.

A–10386

Sub. Code
4MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find the radius of convergence of the power series $\Sigma n!\, z^n$.

2. Distinguish between translation, rotation and inversion.

3. State the Cauchy's theorem in a disk.

4. Compute $\int_{|z|=1} e^z z^{-n} dz$.

5. Define zero and pole. Give an example.

6. State the local mapping theorem.

7. Find the residue of the function $\frac{e^z}{(z-a)(z-b)}$ at its poles.

8. State the Rouche's theorem.

9. Obtain the series expansion for $\tan z$ and arc $\sin z$.

10. Define entire function. Give an example.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Derive the complex from of the Cauchy-Riemann equations.

Or

(b) Define the cross ratio. Also find the linear transformation which carries $0, i, -i$ into $1, -1, 0$.

12. (a) Prove that the line integral $\int_{\gamma} pdx + qdy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial u}{\partial x} = p, \frac{\partial \mathrm{u}}{\partial y} = q$.

Or

(b) State and prove the fundamental theorem of algebra. Also state the Cauchy's estimate theorem.

13. (a) State and prove the Weierstrass theorem for an essential singularity.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the argument principle.

Or

(b) Evaluate $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx$, a real.

15. (a) With the usual notations, prove that $\frac{\pi^2}{\sin^2 \pi^2} = \sum_{-\infty}^{\infty} \frac{1}{(z-n)^2}$.

Or

(b) Derive the Jensen's formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Abel's theorem.

17. If the function $f(z)$ is analytic on R, then prove that $\int_{\partial R} f(z) dz = 0$.

18. State and prove the Schwarz lemma.

19. (a) State and prove the residue theorem.

(b) How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$?

20. State and prove the Laurent series.

A–10387

Sub. Code
4MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define topological space.

2. What is meant by the product topology?

3. Define the projection mapping.

4. What is meant by metric topology?

5. Is the rationals Q connected? Justify your answer.

6. State the intermediate value theorem.

7. Is the real line $\mathbb{R}$compact? justify your answer.

8. State the extreme value theorem.

9. Show that the product of two Lindelof spaces need not be Lindelof.

10. Define second countable space with an example.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

All questions carry equal marks.

11. (a) If $\mathcal{B}$ is a basis for the topology of x and $\mathcal{C}$ is a basis for the topology of y, then prove that the collection $D = \{B \times C \mid B \in \mathcal{B}$ and $c \in \mathcal{C}\}$ is a basis for the topology of $x \times y$.

Or

(b) Let y be a subspace of x. Prove that a set A is closed in y if and only if it equals the intersection of a closed set of x with y.

12. (a) Enumerate the following terms. give an example for each:

(i) continuous function

(ii) Homeomorphism

(iii) Quotient map

Or

(b) State and prove the uniform limit theorem.

13. (a) Let A be a connected subspace of x. If $A \subset B \subset \overline{A}$, then prove that B is also connected.

Or

(b) Show that a space x is locally path connected if and only if for every open set U of x, each path component of U is open in x.

14. (a) Prove that the product of finitely many compact spaces is compact.

Or

(b) State and prove the uniform continuity theorem.

15. (a) Show that a subspace of a regular space is regular and a product of regular space is regular.

Or

(b) Suppose that x has a countable basis. prove the following:

(i) Every open covering of x contains a countable subcollection covering x;

(ii) There exists a countable subset of x that is dense in x.

Part C $(3 \times 10 = 30)$

Answer any **THREE** questions.

16. (a) Prove that the collection

$s = \{\pi_1^{-1},(U)/U \text{ open in } x\} \cup \{\pi_2^{-1},(V)/V \text{ open in } y\}$ is a subbasis for the product topology on $x \times y$.

(b) Show that every infinite point set in a Hausdorff space x is closed.

17. Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric p are the same as the product topology on $\mathbb{R}^n$

18. If L is the linear continuous in the order topology then prove that L is connected, and so are intervals and rays in L.

19. State and prove the Lebesgue number lemma.

20. State and prove the Uryshon's metrization theorem.

A–10388

Sub. Code
4MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Third Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a spanning tree with an example.

2. What are the advantages of network analysis?

3. Define setup cost.

4. Write the formula for purchasing cost per unit time in EOQ with price breaks.

5. Identify the customer and the server for the following situations:

   (a) Parking lot operation

   (b) Legal court cases

6. State the forgetfulness property.

7. What is meant by Multichannel Queuing model?

8. Draw the transition - rate diagram.

9. Define the general constrained non linear programming problem.

10. Define the steepest ascent method.

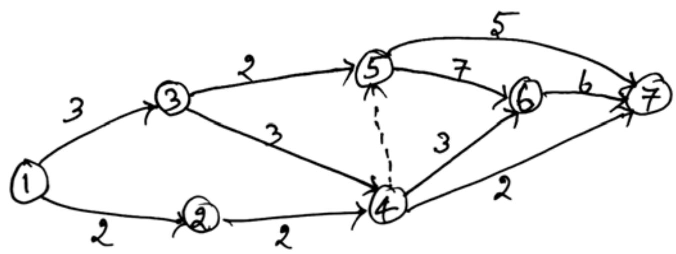
Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Explain Dijkstra's algorithm to find the shortest route.

Or

(b) Determine the critical path for the following project network.



12. (a) Narrate the no-setup model.

Or

(b) A company stocks an item that is consumed at the rate of 50 units per day. It costs the company Rs. 20 each time an order is placed. An inventory unit held in stock for a week will cost Rs. 35.

(i) Find the optimum inventory policy, assuming a lead time of 1 week.

(ii) Determine the optimum number of orders per year (based on 365 days per year)

13. (a) Enumerate the pure birth model in queuing theory.

Or

(b) The time between arrivals at the game room in the student union is exponential with mean 10 minutes.

(i) What is the arrival rate per hour?

(ii) What is the probability that no students will arrive at the game room during the next 15 minutes?

(iii) What is the probability that at least one student will visit the game room during the next 20 minutes?

14. (a) Describe the model $(M/M/1):(GD/\infty/\infty)$.

Or

(b) A telephone exchange has two long distance operators. The telephone company finds that during the peak-load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service an these calls is approximately distributed with mean length 5 minutes.

(i) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day?

(ii) Is the subscribers will not wait and be services in turn. What is the expected waiting time?

15. (a) Find the maximum of the following function by Gradient method:

$$f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

Or

(b) Use Dichotomous method to solve:

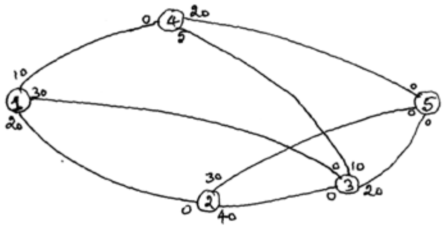
$$f(x) = \begin{cases} 3x, & 0 \le x \le 2 \\ \frac{1}{3}(-x + 20), & 2 \le x \le 3 \end{cases}$$

Given that maximum value of $f(x)$ occurs at $x = 2$ and $\Delta = 0.10$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Determine the maximum flow in the following network.



17. Find the optimal inventory policy for the following five period model. The unit production cost is \$10 for all periods. The unit holding cost is \$1 per period.

Period i	Demand D_i (units)	Setup cost K_i(\$)
1	50	80
2	70	70
3	100	60
4	30	80
5	60	60

18. The florist section in a grocery store stocks 18 dozen roses at the beginning of each week. on the average, the florist sell 3 dozens a day (one dozen at a time), but the actual demand follows a Poisson distribution. Whenever the stock level reaches 5 dozens, a new order of 18 new dozens is placed for delivery at the beginning of the following week. Because of the nature of the item, all roses left at the end of the week are disposed of. Determine the following.

    (a) The probability of placing an order in any one day of the week.

    (b) The average number of dozen roses that will be discarded at the end of the week.

19. For $(M/M/C):(GD/N/\infty)$, $C \le N$, queuing model, show that $\lambda_{eff} = (1 - P_N)\lambda$. Also find W_q, W_s and L_s.

20. Use separable convex programming to solve the NLPP:

Maximize $z = x_1 - x_2$

Subject to:

$3x_1^4 + x_2 \le 243$
$x_1 + 2x_2^2 \le 32$
$x_1 \ge 2.1$
$x_2 \ge 3.5$

A–10196

Sub. Code
4MMA3E3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Third Semester

Mathematics

Elective : FUZZY MATHEMATICS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define L-Fuzzy sets.

2. Define a normalized fuzzy set. Give an example.

3. Define the sugeno class.

4. Give an example of a continuous fuzzy complement which is not involutive.

5. Define the resolution form.

6. Write down the algorithm for transitive closure $R_T(X,X)$.

7. State the axioms for fuzzy measures.

8. Define Bayesian belief measures.

9. Define the maximizing decision.

10. What is meant by fuzzy dynamic programming?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Narrate the following terms. Give an example for each:

(i) Support of a fuzzy set

(ii) Convex fuzzy set

(iii) Extension principle.

Or

(b) Consider the fuzzy sets A, B and C defined on the interval $X = [0,10]$ of real numbers by the membership grade functions $\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = 2^{-x}, \mu_C(x) = \frac{1}{1+10(x-2)^2}$. Determine the following:

(i) $\overline{A}, \overline{C}$

(ii) $A \cup B$

(iii) $A \cup C$

12. (a) If c is a continuous fuzzy complement, then prove that c has a unique equilibrium.

Or

(b) For all $a, b \in [0,1]$, prove that $u(a,b) \le u_{\max}(a,b)$.

13. (a) Consider the sets $X_1 = \{x, y\}, X_2 = \{a, b\}$ and $X_3 = \{*, \$\}$ and the ternary fuzzy relation.

$$R(X_1, X_2, X_3) = \frac{0.9}{(x,a,*)} + \frac{0.4}{(x,b,*)} + \frac{1}{(y,a,*)} + \frac{0.7}{(y,a,\$)} + \frac{0.8}{(y,b,\$)}$$ defined on $X_1 \times X_2 \times X_3$.

Compute the cylindric extensions $[R_{1,2} \uparrow \{x_3\}], [R_{2,3} \uparrow \{x_1\}], [R_{1,3} \uparrow \{x_2\}]$

Or

(b) Describe the concept of fuzzy ordering relations in detail.

14. (a) Show that the function Pl determined by equation $Pl(A) = \sum_{B \cap A \neq \phi} m(B)$, for any given basic assignment m is a plausibility measure.

Or

(b) Prove that every possibility measure π on can $\mathcal{P}(x)$ can be uniquely determined by a possibility distribution function $r : x \to [0,1]$ via the formula $\pi(\mathrm{A}) = \max_{x \in A} r(x)$ for each $A \in \mathcal{P}(x)$

15. (a) Enumerate the following terms:

(i) Optimal values of the objective function.

(ii) The maximizing set over the fuzzy region.

Or

(b) Discuss the vector-maximum problem with an illustration.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Compute the scalar cardinality and the fuzzy cardinality for each of the following fuzzy sets:

(i) $A = \frac{0.4}{v} + \frac{0.2}{w} + \frac{0.5}{x} + \frac{0.4}{y} + \frac{1}{z}$.

(ii) $\mu_C(x) = \frac{x}{x+1}, x \in \{0,1,2,.....10\}$

(b) Show that all α-cuts of any fuzzy set A defined on $\mathbb{R}^n\,(n \geq 1)$ are convex if and only if $\mu_A[\lambda\vec{r} + (1-\lambda)\vec{s}] \geq \min[\mu_A(\vec{r}), \mu_A(\vec{s})]$ for all $\vec{r}, \vec{s} \in \mathbb{R}^n$ and all $\lambda \in [0,1]$.

17. Show that fuzzy set operations of union, intersection and continuous complement that satisfy the law of excluded middle and the law of contradiction are not idem potent or distributive.

18. (a) Explain compatibility relation.

(b) Determine the complete $\alpha -$covers of the compatibility relation whose membership matrix is given below:

	1	2	3	4	5	6	7	8	9
1	1	0.8	0	0	0	0	0	0	0
2	0.8	1	0	0	0	0	0	0	0
3	0	0	1	1	0.8	0	0	0	0
4	0	0	1	1	0.8	0.7	0.5	0	0
5	0	0	0.8	0.8	1	0.7	0.5	0.7	0
6	0	0	0	0.7	0.7	1	0.4	0	0
7	0	0	0	0.5	0.5	0.4	1	0	0
8	0	0	0	0	0.7	0	0	1	0
9	0	0	0	0	0	0	0	0	1

19. Let $x = \{a,b,c,d,e,f,g\}$ and $y = \mathbb{N}_7$. Using joint probability distributions on $X \times Y$, given in terms of the matrix.

	1	2	3	4	5	6	7
a	0.08	0	0.02	0	0	0.01	0
b	0	0.05	0	0	0.05	0	0
c	0	0	0	0	0	0	0.6
d	0.03	0	0	0.3	0	0	0
e	0	0	0.01	0.01	0.2	0.03	0
f	0	0.05	0	0	0	1	0
g	0	0	0	0.02	0	0.01	0

Determine the following:

(a) Marginal probabilities.

(b) Both conditional probabilities.

20. (a) What is meant by fuzzy decision? Also draw a fuzzy decision diagram.

(b) Explain fuzzy constraint with an illustration.

(c) Discuss the fuzzy linear programming.

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A–10431

Sub. Code
4MMA1E1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

First Semester

Mathematics

Elective – MECHANICS

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. State the D'Alembert's principle.

2. Define a scleronomous constraint.

3. What do you mean by the velocity-dependent potential?

4. Write the Lagrangian function for the Atwood's machine.

5. State the Hamilton's principle for a conservative system.

6. Show that the generalized momentum conjugate to a cyclic co-ordinate is conserved.

7. State the condition for a stable orbit.

8. What is called Hooke's law?

9. Find the relation between true anamoly and eccentric anamoly.

10. What is the condition for orbit to be a circle under inverse square law of force?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove principle of virtual work.

Or

(b) Show that Lagrange's equations in the form $\frac{\partial \dot{T}}{\partial \dot{q}_j} - 2\frac{\partial T}{\partial q_j} = Q_j$

12. (a) Write the usual notations, prove that $T = T_0 + T_1 + T_2$.

Or

(b) Obtain the motion of the bead sliding on a uniformly rotating wire in a force - free space.

13. (a) Show that the geodesics of a spherical surface are great circles.

Or

(b) Prove that the central force motion of two bodies about their centre of mass can always be reduced to an equivalent one-body problem.

14. (a) Derive the differential equation for the orbit.

Or

(b) State and prove the Bertrand's theorem.

15. (a) Prove that the orbital equation for motion in a central inverse-square law force is

$$t = \frac{l^3}{mk^2} \int_{\theta_0}^{\theta} \frac{d\theta}{[1 + e\cos(\theta - \theta')]^2}.$$

Or

(b) Derive the equation of the orbit for the Kepler problem using Laplace-Runge-Lenz vector.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Derive the Lagrange equations of motion from D'Alembert's principle.

17. Derive the expression for the Lagrangian in the form $L = T - q\phi + \frac{q}{e}\overline{A}\cdot\overline{V}$ for a charged particle charge q moving with velocity $\overline{V}$ in an electromagnetic field.

18. State and prove the Euler-Lagrange differential equations.

19. State and prove the virial theorem. Also prove that $\overline{T} = -\frac{1}{2}\overline{V}$.

20. Using Kepler's equation, prove that $\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{\psi}{2}$. Also prove that $E = -\frac{mk^2}{2l^2}$.

A–10432

Sub. Code
4MMA2C2

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Second Semester

Mathematics

ANALYSIS - II

(CBCS – 2014 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   ($10 \times 2 = 20$)

Answer **all** questions.

1. Define a refinement of a partition.

2. Define a curve in R^k.

3. Define pointwise convergence of a sequence.

4. Define an algebra of a family $\mathcal{A}$ of complex functions defined on a set F.

5. Define analytic functions.

6. Define an orthogonal system of functions.

7. Define outer measure.

8. If $M^*E = 0,$ then prove that E is measurable.

9. Define simple function.

10. State Bounded convergence theorem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that $f \in \mathcal{R}(\alpha) on [a,b]$ if and only if for every $\varepsilon > 0$ there exits a partition P such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon.$

Or

(b) State and prove the theorem for change of variable.

12. (a) Prove that the sequence of functions $\{f_n\}$ defined on E converges uniformly on E if and only if for every $\varepsilon > 0$ there exists an Integer N such that $m \geq N,$ $n \geq N,\ x \in E$ implus $|f_n(x) - f_m(x)| \leq \varepsilon.$

Or

(b) If K is a compact metric space, if $f_n \in \mathcal{C}(k)$ for $n = 1, 2, 3, ...$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

13. (a) If $\sum C_n$ converges and if $f(x) = \sum_{n=0}^{\infty} C_n x^n (-1 < x < 1)$ then prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} C_n.$

Or

(b) If for some x, there are constants $\delta > 0$ and $M < \infty$ such that $|f(x+t) - f(x)| \leq M|t|$ for all $t \in (-\delta, \delta)$ then prove that $\lim_{N \to \infty} \delta_N(f;x) = f(x).$

14. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers then prove that $m^*(\cup A_n) \le \sum m^* A_n$.

Or

(b) Let A be any set, and $E_1, ...E_n$ a finite sequence of dispoint measurable sets. Then prove that

$$m^*\left(A \cap \left[\bigcup_{i=1}^{n} E_i\right]\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$$

15. (a) State and prove Fatou's lemma.

Or

(b) Let $< f_n >$ be a sequence of measurable functions that converges in measure to f. Then prove that there is a subsequence $< f_{nk} >$ that converges to f almost everywhere.

Part C (3 × 10 = 30)

Answer any **THREE** questions.

16. Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Then prove that $f \in \mathscr{R}(a)$ if and only if $f\alpha' \in \mathscr{R}$. Also prove that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.

17. State and prove Stone-Weierstrass theorem.

18. State and prove Taylor's theorem.

19. Let C be a constant and f and g are two measurable real-valued functions defined on the same domain. Then prove that the functions $f+c, cf, f+g, g-f$ and fg are measurable.

20. Let f be defined and bounded on a measurable set E with mE finite. Prove that the condition $\inf_{f \leq \psi} \int_E \psi(x)dx = \sup_{f \geq \varphi} \int_E \varphi(x)dx$ for all simple functions φ and ψ is the necessary and sufficient for f to be measurable.

A–9711

Sub. Code
4MMA4E3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Fourth Semester

Mathematics

Elective : AUTOMATA THEORY

(CBCS – 2014 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When a string is accepted by a NDFA?

2. Draw a block diagram of a finite automation.

3. Define one-step derivation.

4. What is meant by type 3 production?

5. Consider the grammar G given by

   $S \to OSA_1 2,\ S \to 012,\ 2A_1 \to A_1 2,\ 1A_1 \to 11$. Test whether $001122 \in L(G)$.

6. Is two grammars of different types can generate the same language? Justify your answer.

7. Describe the following set by regular expression:

L_2 = the set of all strings of 0's and 1's beginning with 0 and ending with 1.

8. Give any two applications of pumping lemma.

9. Give an example for Parse tree.

10. If G is the grammar $S \rightarrow sbs|a$, then prove that G is ambiguous.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that for any transition function δ and for any two input strings x and y, $\delta(q, xy) = \delta(\delta(q, x), y)$.

Or

(b) Construct a deterministic automation equivalent to $M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$, where δ is defined by the following state table:

State/Σ	0	1
$\rightarrow$ (q_0)	q_0	q_1
q_1	q_1	q_0, q_1

12. (a) Let L be the set of all palindromes over $\{a, b\}$. Construct a grammar G generating L.

Or

(b) Prove that every monotonic grammar G is equivalent to a type 1 grammar.

13. (a) Construct context-free grammars to generate the following:

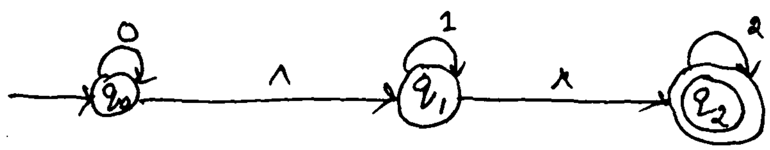
(i) $\{a^{2n} / n \geq 1\}$.

(ii) The set of all strings over $\{a, b\}$ ending in a.

Or

(b) Show that the class L_0 is closed under concatenation.

14. (a) Consider a finite automation, with $\wedge$-moves, given below. Obtain an equivalent automation without $\wedge$-moves.



Or

(b) If X and Y are regular sets over Σ, then prove that $X \cap Y$ is also regular over Σ.

15. (a) If $A \overset{*}{\Rightarrow} W$ in G, then prove that there is a leftmost derivation of W.

Or

(b) Construct a reduced grammar equivalent to the grammar $S \rightarrow aAa$, $A \rightarrow Sb | bcc | DaA$, $C \rightarrow abb | DD$, $E \rightarrow ac$, $D \rightarrow aDA$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If L is the set accepted by NDFA, then prove that there exists a DFA which also accepts L.

17. Construct a grammar G generating $\{a^n b^n c^n / n \geq 1\}$.

18. Show that there exists a recursive set which is not a context-sensitive language over $\{0,1\}$

19. (a) State and prove the Arden's theorem.

    (b) With the usual notations, prove that $L = \{a^{i^2} | i \geq 1\}$ is not regular.

20. State and prove Reduction to Chomsky normal form.