F–4660

Sub. Code
7MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Give an example of a linear map which is not continuous.

2. State Jensen's inequality.

3. Let X be a linear space over C. Regarding X as a linear space over R, consider a real-linear functional $u : X \to R$. Define $f(x) = u(x) - iu(x), x \in X$. Prove that f -is a complex linear functional on X.

4. State Hahn-Banach extension theorem.

5. Prove $Gr(F^{-1})$ is closed in $Y \times X$ whenever $Gr(F)$ is closed in $X \times Y$.

6. State Banach-Steinhaus theorem.

7. Define the term normed dual.

8. Define Dual basis.

9. State polarization identity.

10. Define orthonormal set.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let X be a normed space and f be a non-zero linear functional on X. Then prove that f is discontinuous if and only if $z(f)$ is dense in X.

Or

(b) Let Y be a closed subspace of normed space X. For $x+Y$ in the quotient space X/Y, let $|||x+Y||| = \inf\{\|x+y\| : y \in Y\}$. Prove that $|||\ \ |||$ is a norm on X/Y.

12. (a) Prove that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.

Or

(b) Let X be a normed space over K, and f be a nonzero linear functional on X. If E is an open subset of X, then prove that $f(E)$ is an open subset of K.

13. (a) State and prove closed graph theorem.

Or

(b) State and prove Resonance theorem.

14. (a) Prove that if χ' is separable, then so is χ.

Or

(b) If X is a Banach space and $A \in BL(X)$, then prove that $\sigma(A) = \sigma_a(A) \cup \sigma_e(A') = \sigma(A')$.

15. (a) Prove that $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$, for all $x, y \in X$..

Or

(b) State and prove Bessel's inequality.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X denote a subspace of $B(T)$ with the sup norm, $1 \in X$ and f be a linear functional on X. If f is continuous and $\|f\| = f(1)$, then prove that f is positive. Conversely, if $\operatorname{Re} x \in X$ whenever $x \in X$ and if f is positive, then f is continuous and $\|f\| = f(1)$.

17. State and prove Taylor-Foguel theorem.

18. State and prove uniform boundedness principle.

19. Let X and Y be Banach spaces and $F \in BL(X,Y)$, then prove that $R(F) = Y$ if and only if F' is bounded below.

20. State and prove Riesz representation theorem on Hilbert space.

F–4661

Sub. Code
7MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define spanning tree with suitable example.
2. What is the difference between PERT and CPM?
3. Define Lead time.
4. Write the formula for purchasing cost per unit time in EOQ with price breaks.
5. Draw the diagram for cost based queing decision model.
6. Define truncated poisson distribution.
7. Define balance equation.
8. Define self service model.
9. Define steepest ascent method.
10. Define Golden section method

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain Three-Jug puzzle with an illustration.

Or

(b) Explain in detail about minimal spanning tree algorithm.

12. (a) Explain in detail about multi-item EOQ with storage limitation.

Or

(b) Discuss in detail about general dynamic programming algorithm.

13. (a) Describe the basic elements of queing model.

Or

(b) A service machine always has a standy unit for immediate replacement upon failure. The time to failure of the machine is exponential and occurs every 5 hours, on the average. The machine operator claims that the machine is "in the habit" of breaking down every night around 8.30 PM. Analyze the operator's claim.

14. (a) Explain in detail about machine servicing model.

Or

(b) Explain in detail about cost model.

15. (a) Solve Maximize $f(x)=\begin{cases} 3x & 0 \le x \le 2 \\ \frac{1}{3}(-x+20) & 2 \le x \le 3 \end{cases}$ using golden section method. Given the maximum value of $f(x)$ occurs at $x=2$ and $\Delta = 0.10$.

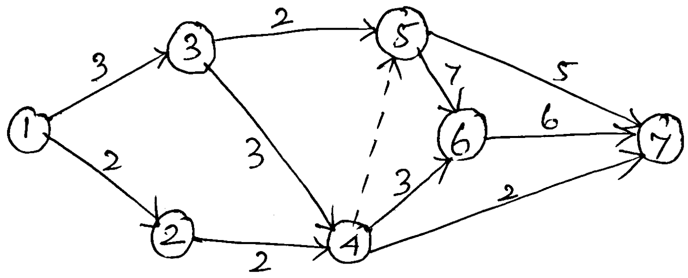
Or

(b) Discuss briefly about separable convex programming.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Determine the critical path for the following project network.



17. Explain in detail about no-setup EOQ model.

18. Explain in detail about Pure Birth Model.

19. Explain in detail about Machine servicing model.

20. Solve the following problem using restricted basis method

Maximize $z = x_1 + x_2^4$

Subject to $3x_1 + 2x_2^2 \le 9$

$x_1, x_2 \ge 0$.

F–4662

Sub. Code
7MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

Fourth Semester

Mathematics

TOPOLOGY – II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define locally compact space.
2. State the countable intersection property.
3. Define completely regular space.
4. Define compactification of a space.
5. Define locally finite set.
6. What is meant by locally discrete set?
7. Define complete metric space.
8. Define totally bounded metric space.
9. Define compact-open topology.
10. Define Baire space.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let X be a Hausdroff space. Prove that X is locally compact at x if and only if for every neighborhood U of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$.

Or

(b) Let X be a space. Let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Led $D \in \mathcal{D}$. If $A \supset \mathcal{D}$ then prove that $A \in \mathcal{D}$.

12. (a) A subspace of a completely regular space is completely regular. Prove that a product of completely regular spaces is completely regular.

Or

(b) Let X be completely regular, let $\beta(X)$ be its Stone-Cech compactification. Then prove that every bounded continuous real-valued function on X can be uniquely extended to a continuous real-valued function on $\beta(X)$.

13. (a) Let $\mathcal{A}$ be a locally finite collection of subsets of X. Then prove that

(i) Any sub collection of $\mathcal{A}$ is locally finite.

(ii) The collection $\mathcal{B} = \{\overline{A}\}_{A\in\mathcal{A}}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

(iii) $\overline{\bigcup_{A\in\mathcal{A}} A} = \bigcup_{A\in\mathcal{A}} \overline{A}$

Or

(b) Let X be normal and Let A be a closed G_S set in X. Prove that there is a continuous function $f : x \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) Prove that a metric space X is complete if every Cauchy sequence in X has a convergent sequence.

Or

(b) Let X be a compactly generated space and let (Y,d) be a metric space. Then prove that $\mathcal{C}(X,Y)$ is closed in Y^X in the topology of compact convergence

15. (a) Let X be locally compact Hausdroff space and let $\mathcal{C}(X,Y)$ have the compact open topology. Prove that the map $e : X \mathcal{C}(X,Y) \to \mathrm{Y}$ defined by the equation $e(x, f) = f(x)$ is continuous.

Or

(b) Let $C_1 \supset C_2 \supset ....$ be a nested sequence of non-empty closed sets in the complete metric space X. If diam $C_{n \to 0}$, then prove that $\cap C_n \neq \phi$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X be a locally compact Hausdroff space which is not compact and Y be the one-point compactification of X. Then prove that Y is a compact Hausdroff space, X is a subspace of Y, the set $Y \to X$ consists of a single point and $\overline{X} = Y$.

17. Let X be a completely regular space. If Y_1 and Y_2 are two compactifications of X satisfying the extension property. Prove that Y_1 and Y_2 are equivalent.

18. Prove that a space X is metrizable if and only if X is regular and has a countably locally finite basis.

19. Let (X,d) be a metric space. Prove that there is an isometric imbedding of X into a complete metric space.

20. Let X be a space and let (Y,d) be a metric space. Prove that for the space $\mathcal{C}(X,Y)$, the compact-open topology and the topology of compact convergence coincide.

F–4663

Sub. Code
7MMA4E1

M.Sc DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Fourth Semester

Mathematics

Elective: ADVANCED STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a consistent estimator of a parameter.

2. Write down the confidence intervals for differences of means.

3. Define a loss function and a risk function.

4. Write down the likelihood principle.

5. Define on efficiency of the statistic.

6. Define a Baye's solution.

7. When we say that a critical region is a uniformly most powerful critical region of sized?

8. Define a sequential probability ratio test.

9. Define a central chi-square variable.

10. Define a non central F-variable.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $X_1, X_2, ...X_n$ denote a random sample from the distribution with *p.d.f* $f(x) = \theta^x (1-\theta)^{1-x}$, $x = 0,1$ and zero elsewhere where $0 \le \theta \le 1$. Find the *m.l.e.* $\hat{\theta}$. of θ.

Or

(b) Let two independent random samples, each of size 10, from two normal distributions $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\bar{x} =$ 4.8, $s_1^{\ 2} = 8.64$, $\bar{y} = 5.6$, $s_2^{\ 2} = 7.88$. Find a 95 percent confidence interval for $\mu_1 = \mu_2$

12. (a) Let $X_1, X_2, ...X_n$ denote a random sample from a distribution that is $N(\theta, 1), -\infty < \theta < \infty$ Find the unbiased minimum variance estimator of θ^2.

Or

(b) Let $X_1, X_2, ..., X_n$ be a random sample form the normal distribution $N(0, \theta), 0 < \theta < \infty$. Show that $\sum_1^n xi^2$ is a sufficient statistic for θ.

13. (a) Show that the mean $\overline{X}$ for a random sample of size n from a distribution which is $b(1, \theta), 0 < \theta < 1$, is an efficient estimator of θ.

Or

(b) Show that $I(\theta)=\int_{-\infty}^{\infty}\left[\frac{\delta \ln f(x;\theta)}{\delta\theta}\right]^2 f(x;\theta)dx$

14. (a) Consider the one random variable X that has a binomial distribution with $n = 5$ and $p = \theta$. Let $f(x;\theta)$ denote the $p.d.f$ of X and let $H_0 : \theta = \frac{1}{2}$ and $H_1 : \theta = \frac{3}{4}$. Find the values of $f\left(x;\frac{1}{2}\right)$, $f\left(x;\frac{3}{4}\right)$ and $f\left(x;\frac{1}{2}\right) / f\left(x;\frac{3}{4}\right)$.

Or

(b) Let X have the $P.d.f$ $f(x;\theta)=\theta^x(1-\theta)^{1-x}$ $x = 0,1$ zero elsewhere. We test the simple hypothesis $H_0 : \theta = \frac{1}{4}$ against the alternative composite hypothesis $H_1 : \theta < \frac{1}{4}$ by taking a random sample of size 10 and rejecting $H_0 : \theta = \frac{1}{4}$ if and only if the observed values $x_1, x_2, \ldots\ldots\ldots x_{10}$ of the sample observations are such that $\sum_1^{10} xi \le 1$. Find the power function $k(\theta) 0 < \theta \le \frac{1}{4}$ of this test.

15. (a) Define a non-central χ^2 variate and obtain its p.d.f.

Or

(b) Show that the square of a non-central T random variable is a non-central F random variable.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. A number is to be selected from the interval $\{x : 0 < x < 2\}$ by a random process. Let $Ai = \left\{x : (i-1)/2 < x \le \frac{i}{2}\right\}$, $i = 1,2,3$ and let $A_4 = \left\{x : \frac{3}{2} < x < 2\right\}$.A certain hypothesis assigns probabilities P_{jo} to these sets in accordance with $P_{io} = \int_{Ai} \left(\frac{1}{2}\right)(2-x)dx, i = 1,2,3,4$ this hypothesis is to be tested, at the 5 percent level of significance, by a chi-square test. If the observed frequencies of the sets Ai, $i = 1,2,3,4$ are respectively $30, 30, 10, 10$ would H_0 be accepted at the 5 percent level of significance.

17. State and prove Rao and Black well theorem.

18. State and prove the Rao-Cramer inequality.

19. State and prove Neyman-pearson theorem.

20. Let $Y_i, i = 1,2,....n$ denote independent random variables that are respectively, $\chi^2(ri, \theta i) i = 1,2,...n$ prove that $Z = \sum_1^n Yi$ is $\chi^2\left(\sum_1^n ri, \sum_1^n \theta i\right)$

F–4664

Sub. Code
7MMA4E3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

Fourth Semester

Mathematics

Elective – NUMERICAL METHODS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by iteration function?

2. Define efficiency index of an iterative method.

3. When will iteration method succeed?

4. What is meant by partial pivoting?

5. State the properties of cubic spline.

6. Write the condition for a spline to be cubic.

7. State three point Gaussian Quadrature formula.

8. Define error of approximation.

9. What is the truncation error of Taylor's method?

10. Is Euler's method formula, a particular case of second order Runge-Kutta method?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the number of real and complex roots of the polynomial $p(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$.

Or

(b) State and prove Sturm's theorem.

12. (a) Determine the Euclidean and the maximum absolute row sum norms of the matrix.

$$A = \begin{bmatrix} 1 & 7 & -4 \\ 4 & -4 & 9 \\ 12 & -1 & 3 \end{bmatrix}.$$

Or

(b) State and prove Gerschgorin theorem.

13. (a) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data:

x	1	2	4	8
$f(x)$	3	7	21	73

Hence, estimate the values of $f(3)$ and $f(7)$.

Or

(b) The following data for a function $f(x, y)$ is given

y/x	0	1
0	1	1.414214
1	1.732051	2

Find $f(0.25,\ 0.75)$ using linear interpolation.

14. (a) Find the Jacobian matrix for the system of equations $f_1(x,y) = x^2 + y^2 - x = 0$

$f_2(x,y) = x^2 - y^2 - y = 0$ at the point $(1,1)$, using the methods

$\left(\frac{\partial t}{\partial x}\right)_{(x_i y_j)} = \frac{f_{i+1,j} - f_{i-1,j}}{2h}, \left(\frac{\partial f}{\partial y}\right)_{(x_i, y_j)} = \frac{f_{i,j+1} - f_{i,j-1}}{2k}$ with $h = k = 1$.

Or

(b) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss-Legendre three point formula.

15. (a) Find the singlestep method for the differential equation $y' = f(t,y)$, which produce exact results for $y(t) = a + b\cos t + c\sin t$.

Or

(b) Find the three term Taylor series solution for the third order initial value problem.

$W''' + WW'' = 0, W(0) = 0$

$W'(0) = 0, W''(0) = 1$. Find the bound on the error for $t \in [0,0.2]$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Perform two iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $p_3(x) = x^3 + x^2 + x^2 - x + 2 = 0$. Use the initial approximations $p_0 = -0.9$, $q_0 = 0.9$.

17. Using the Jacobi method find all the eigenvalues and the corresponding eigenvectors of the matrix $A\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$.

18. The following data for a function $f(x,y)$ is given

y/x	0	1	3
0	1	2	10
1	2	4	14
3	10	14	28

Construct the bivariate interpolating polynomial and hence find $(0.5,0.5)$

19. Find the quadrature formula $\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$

Which is exact for polynomials of highest possible degree.

20. Use the Euler method to solve numerically the initial value problem $u^1 = -2tu^2, u(0) = 1$ with $h = 0.2, 0.1$ and 0.05 on the interval $[0,1]$.

F–4983

Sub. Code
7MMA1C3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

First Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the arc length.

2. Define a cylindrical helix.

3. What is meant by right helicoid?

4. Write short note an isometric correspondence between two surfaces.

5. Define geodesics.

6. Write down the christoffel symbols of the first kind.

7. Define the geodesic curvature Kg.

8. State the Minding's theorem.

9. Define an umbilic.

10. What is meant by the osculating developable?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that $[\dot{\vec{r}}, \ddot{\vec{r}}, \dddot{\vec{r}}] = 0$ is a necessary and sufficient condition that the curve be plane.

Or

(b) If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$.

12. (a) For the anchor ring

$\bar{r} = ((b + a\cos u)\cos v, (b + a\cos u)\sin v, a\sin u)$ calculate the area corresponding to the domain $0 \le u \le 2\pi,\ 0 \le v \le 2\pi$.

Or

(b) Find the coefficient of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

13. (a) Prove that the curves of the family $\frac{v^3}{u^2} = \text{constant}$ are geodesics on a surface with metric.

$v^2 du^2 - 2uv\,du\,dv + 2v^2 dv^2 \quad (u > 0, v > 0)$.

Or

(b) Show that every helix on a cylinder is a geodesic.

14. (a) Prove that the components λ, μ of the geodesic curvature vector are given by the following formulae, with s ads parameter.

$$\lambda = \frac{1}{H^2}\frac{u}{v'}, \frac{\partial T}{\partial v'} = -\frac{1}{H^2}\frac{v}{u'}, \frac{\partial T}{\partial v'}$$

$$\mu = \frac{1}{H^2}\frac{v}{u'}, \frac{\partial T}{\partial u'} = -\frac{1}{H^2}\frac{u}{v'}, \frac{\partial T}{\partial u'}$$

Or

(b) Calculate the circumference of a geodesic circle of small radius r and to see how it differs from the Euclidean formula $2\pi r$.

15. (a) Derive the second fundamental form of a surface.

Or

(b) Show that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normals along the curve form a developable.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that the intrinsic equations of the curve given by $x = ae^u \cos u$, $y = ae^u \sin u$, $z = be^u$ are $\kappa = \frac{\sqrt{2}a}{(2^2+6^2)^{1/2}}.\frac{1}{s}$, $\tau = \frac{b}{(2a^2+2^2)^{1/2}}.\frac{1}{s}$.

17. If θ is the angle at the point (u,v) between the two directions given by $pdu^2 + 2\theta dudv + Rdv^2 = 0$, then prove that $\tan\theta = \frac{2H(Q^2 - PR)^{\frac{1}{2}}}{ER - 2FQ + GP}$.

18. Establish the differential equations for a geodesic using the normal property.

19. Derive the Liouville's formula for Kg.

20. State and prove the Rodrigue's formula.

———————

F–4984

Sub. Code
7MMA1C4

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say that two functions ϕ_1 and ϕ_2 are said to be linearly dependent on I?

2. Find the Wronskian of the set $\{e^x, \cos x, \sin x\}$.

3. Verify that $\phi_1(x) = e^x$, $(x > 0)$ is a solution of $xy'' - (x+1)y' + y = 0$.

4. Write down the Hermit equation.

5. Define a singular point.

6. Show that $x^{1/2} J_{1/2}(x) = \dfrac{\sqrt{2}}{\Gamma(1/2)} \sin x$.

7. Find an integrating factor of $(2y^3 + 2)dx + 3xy^2 dy = 0$.

8. Compute the first four successive approximations of $y' = y^2$, $y(0) = 1$.

9. Consider the problem $y' = y + \lambda x^2 \sin y$, $y(0) = 1$, where λ is some real parameter, $|\lambda| \le 1$. Prove that the solution ψ of this problem exists for $|x| \le 1$.

10. Consider the initial value problem $y_1' = y_2^2 + 1$, $y_2' = y_1^2$, $y_1(0) = 0$, $y_2(0) = 0$. If this problem is denoted by $y' = f(x, y)$. $y(0) = y_0$ what are f and y_0?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find alls solutions of the equation $y'' - 7y' + 6y = \sin x$.

Or

(b) Find the solution ϕ of the initial –value problem $y''' + y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$.

12. (a) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

Or

(b) Find two linearly independent power series solutions of the equation $y'' - xy' + y = 0$.

13. (a) Find the singular points of the equation $(1 - x^2)y'' - 2xy' + 2y = 0$ and determine whether regular singular point or not.

Or

(b) Consider the equation $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$, where α is a non negative constant. Compute the indicial polynomial and its two roots.

14. (a) Show that the solution ϕ of $y' = y^2$ which passes through the point (x_0, y_0) is given by $\phi(x) = \dfrac{y_0}{1 - y_0(x - x_0)}$.

Or

(b) By computing appropriate Lipschitz constant, show that the following function satisfy Lipschitz condition on the set S indicated : $f(x, y) = x^2 \cos^2 y + y \sin^2 x$ on $S : |x| \le 1, |y| < \infty$.

15. (a) Let f be continuous and satisfy a Lipschitz condition R. If ϕ and ψ are two solution of $y' = f(x, y)$, $y(x_0) = y_0$, on and interval I containing x_0, then prove that $\phi(x) = \psi(x)$ for all x in I.

Or

(b) Consider the system $y_1' = y_1 + \in y_2$, $y_2' = \in y_1 + y_2$, where $\in$ is a positive constant. Find all solutions of the original system.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Consider the equation $y'' + a_1y' + a_2y = 0$, where a_1, a_2 are real constants such that $4a_2 - a_1^2 > 0$. Let $\alpha + i\beta$, $\alpha - i\beta$ $(\alpha, \beta \text{ real})$ be the roots of the characteristics polynomial.

(a) Show that ϕ_1, ϕ_2 defined by $\phi_1(x) = x^{\alpha x} \cos \beta x$, $\phi_2(x) = e^{\alpha x} \sin \beta x$ are solutions of the equation.

(b) Compute $w(\phi_1, \phi_2)$ and show that ϕ_1, ϕ_2 are linearly independent on any interval I.

17. With the usual notations, prove that

(a) $\int_{-1}^{1} P_n(x) P_m(x) dx = 0 \ (n \neq m)$.

(b) $\int_{-1}^{1} P_n^2(x) dx = \frac{2}{2n+1}$.

18. Derive Bessel function of zero order of the first kinds $J_0(x)$.

19. Let M, N be two real valued functions which have continuous first partial derivative on some rectangle $R : |x - x_0| \le a$, $|y - y_0| \le b$. Prove that the equation $M(x, y) + N(x, y) y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

20. Let f be a real valued continuous function on the strip $S : |x - x_0| \le a$, $|y| < \infty$, $(a > 0)$, and suppose that f satisfies on S a Lipschitz condition with constant $k > 0$. Show that the successive approximations $\phi_0(x) = y_0$,

$$\phi_{K+1}(x) = y_0 + (x - x_0) y_1 + \int_{x_0}^{x} (x - t) f(t, \phi_k(t)) dt,$$

$(k = 0,1,2,\ldots)$, exist as continuous functions on the whole interval $I : |x - x_0| \le a$, and converge on I to a solution ϕ of the initial value problem $y'' = f(x, y)$, $y(x_0) = y_0$, $y'(x_0) = y_1$.

F–4985

Sub. Code
7MMA1E1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

First Semester

Mathematics

Elective : NUMBER THEORY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define greatest common divisor.

2. Write any two properties of divisibility.

3. Define Mobius function.

4. Define Liouville's function.

5. Define asymptotic function.

6. Define Legendre's identity.

7. State Wilson's theorem.

8. Prove that congruence is transitive.

9. State Little Fermat's theorem.

10. Define Jacobi's symbol.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the properties of greatest common divisor.

Or

(b) Prove that every integer $n > 1$ is either a prime number or a product of prime numbers.

12. (a) Prove that for $n \geq 1$, $\phi(n) = \sum_{d/n} \mu(d)\frac{n}{d}$.

Or

(b) If f and g are arithmetical functions, then prove that $(f * g)' = f' * g + f * +g'$.

13. (a) If $x \geq 1$, then prove that $\sum_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$.

Or

(b) If $x \geq 1$, then prove that $\sum_{n \leq x} \phi(n) = \frac{3}{\pi^2} x^2 + O(x \log x)$, and average order of $\phi(n)$ is $3n/\pi^2$.

14. (a) Solve the congruence

(i) $5x \equiv 3 \pmod{24}$

(ii) $25x \equiv 15 \pmod{120}$.

Or

(b) State and prove Chinese remainder theorem.

15. (a) Determine whether 888 is a quadratic residue or nonresidue of the prime 1999.

Or

(b) State and prove Reciprocity law for Jacobi's symbols.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the infinite series $\sum_{n=1}^{\infty} 1/p_n$ diverges.

17. For $n \geq 1$, prove that $\phi(n) = n \underset{p/n}{\pi} \left(1 - \frac{1}{p}\right)$.

18. State and prove Euler's summation formula.

19. State and prove Wolstenholme's theorem.

20. State and prove Euler's criterion.

F–4986

Sub. Code
7MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS –2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the complex form of the Cauchy-Reimann equations.

2. Define a parallel translation.

3. Compute $\int\limits_{|z|=2} \frac{dz}{z^{2-1}}$ for the positive sense of the circle.

4. Define the winding number.

5. Show that the function $\sin z$ have essential singularity at ∞.

6. State the maximum principle theorem.

7. Find the residue of $\frac{1}{(z^2-1)^2}$ at its poles.

8. How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$?

9. State the Hurwitz theorem.

10. When will you say that a function is said to be entire?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove Lucas's theorem.

Or

(b) If $T_1(z) = \dfrac{z+2}{z+3}, T_2(z) = \dfrac{z}{z+1}$, find $T_1T_2(z)$, $T_2T_1(z)$ and $T_1^{-1}T_2(z)$.

12. (a) Prove that the line integral $\int_\gamma pdx + qdy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x,y)$ in Ω with the partial derivatives $\dfrac{\partial u}{\partial x} = p, \dfrac{\partial u}{\partial y} = q$

Or

(b) State and prove Liouville's theorem. Also state the fundamental theorem of algebra.

13. (a) State and prove the Taylor's theorem.

Or

(b) State and prove the Schwarz lemma.

14. (a) State and prove the argument principle

Or

(b) Evaluate : $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

15. (a) Prove that for $|z| < 1$

$$(1+z)(1+z^2)(1+z^4)(1+z^8)\ldots. = \frac{1}{1-z}.$$

Or

(b) With the usual notations, prove that

$$\sin \Pi z = \Pi z \prod_{1}^{\infty} \left(1 - \frac{z^2}{n^2}\right).$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

17. State and prove that Cauchy's theorem for a rectangle.

18. State and prove the local mapping theorem.

19. (a) State and prove the residue theorem.

(b) State and prove the Rouche's theorem.

20. Derive the Jensen's formula.

———————

F–4987

Sub. Code
7MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define standard topology and lower limit topology.

2. Prove that every finite point set in a Hausdorff space X is closed.

3. State the pasting lemma.

4. Define quotient topology with an example.

5. Let A be connected subset of X. If $A \subset B \subset \overline{A}$, then prove that B is also connected.

6. Define linear continuum.

7. Show that the image of a compact space under a continuous map is compact.

8. State maximum and minimum value theorem.

9. Define countable basic and first countability axiom.

10. Show that a closed subspace of a normal space is normal.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

Or

(b) Let X be a Hausdorff space, let A be a subset of X. Prove that the point x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A.

12. (a) Write any five rules for constructing continuous functions.

Or

(b) State and prove sequence lemma.

13. (a) Is the space R_ℓ connected? Justify your answer.

Or

(b) Show that R^n and R are not homeomorphic if $n > 1$.

14. (a) Prove that a subset A of R^n is compact if and only if it is closed and is bounded in the Euclidean metric d or the square metric ρ?

Or

(b) Show that $[0,1]^{\omega}$ is not limit point compact in either the box or the uniform topologies.

15. (a) Show that every compact metrizable space X has a countable basis.

Or

(b) Show that every well-ordered set X is normal in the order topology.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies J and J' respectively, on X. Then prove that the following are equivalent:

(i) J' is finer than J.

(ii) For each $x \in X$ and each basis element $B' \in \mathcal{B}'$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.

(b) Prove that the collection $S = \{\pi_1^{-1}(U) \mid U \text{ open in } X\}$ $U\{\pi_2^{-1}(V) \mid V \text{ open in } Y\}$ is a subbasis for the product topology on $X \times Y$.

17. (a) Show that the topologies on R^n induced by the Euclidean metric d and the square metric ρ are the same as the product topology on R^n.

(b) Show that $R \times R$ in the dictionary order topology is metrizable.

18. Prove that the Cartesian product of connected spaces is connected.

19. Let X be a simply ordered set having the least upper bound property. Prove that in the order topology each closed interval in X is compact.

20. State and prove Urysohn lemma.

———————

F–4988

Sub. Code
7MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

Third Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define a mode of a distribution.
2. Show that a distribution function $F(x)$ is a nondecreasing function of x.
3. Define the marginal p.d.f. of a random variable.
4. Define the conditional expectation of a function.
5. Define a negative binomial distribution.
6. If the random variable X has a poisson distribution such that $\Pr(X = 1) = \Pr(X = 2)$, find $\Pr(X = 4)$.
7. Define the order statistic.
8. Define a statistic.

9. When we say that a sequence of random variables $X_1, X_2, ...$ converges in probability to a random variable X?

10. What is meant by degenerate distribution?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove Baye's theorem.

Or

(b) Let X have the p.d.f

$$f(x) = 2(1-x), \quad 0 < x < 1$$
$$= 0 \quad \text{elsewhere}$$

Find $E(6X + 3X^2)$.

12. (a) Let $F(x, y)$ be the distribution function of X and Y. Show that $\Pr(a < X \le b, c < y \le d)$ $= F(b,d) - F(b,c) - F(a,d) + F(a,c)$, for all real constants $a < b, c < d$.

Or

(b) Let the random variables X and Y have the joint p.d.f.

$$f(x, y) = x + y \quad 0 < x < 1, 0 < y < 1$$
$$= 0 \quad \text{elsewhere}$$

Compute the correlation coefficient of X and Y.

13. (a) If the random variable X is $N(\mu, \sigma^2)$, $\sigma^2 > 0$ then prove that the random variable $V = {(X-\mu)^2}/{\sigma^2}$. Is $\chi^2(1)$.

Or

(b) Find the m.g.f of a binomial distribution and hence find mean and variance of a binomial distribution.

14. (a) Let X have the p.d.f. $f(x) = \left(\frac{1}{2}\right)^x, x = 1,2,.....$ zero elsewhere. Find the p.d.f of $Y = X^3$.

Or

(b) Show that the mean and the variance of a beta distribution is $\mu = \frac{\alpha}{\alpha + \beta}$ and $\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta + 1)(\alpha + \beta)^2}$.

15. (a) Let X be $\chi^2(50)$. Approximate $\Pr(40 < X < 60)$.

Or

(b) Let $\overline{X}$ denote the mean of a random sample of size 100 from a distribution that is χ^2 (50). Compute an approximate value of $\Pr(49 < \overline{X} < 51)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. For each of the following p.d.f. of X, compute $\Pr(|X| < 1)$ and $\Pr(X^2 < 9)$.

(a) $f(x) = x^2/18, -3 < x < 3$, zero elsewhere.

(b) $f(x) = (x + 2)/18, -2 < x < 4$, zero elsewhere.

17. Let X, and X_2 denote random variables that have the joint p.d.f. $f(x_1, x_2)$ and the marginal p.d.f. $f_1(x_1)$ and $f_2(x_2)$, respectively. Furthermore, let $M(t_1, t_2)$ denote the m.g.f of the distribution. Then prove that X_1 and X_2 are independent if and only if $M(t_1, t_2) = M(t_1, 0)M(0, t_2)$.

18. Find the m.g.f. of a gamma distribution and hence find the mean and the variance of a gamma distribution.

19. Derive the p.d.f. of F-distribution.

20. State and prove the central limit theorem.

F–4989

Sub. Code
7MMA3E1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Third Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define a subsemigroup and a submonoid.

2. Find whether * defined on the set Z is binary where $a*b=a-b$.

3. Define a recurrence relation on a sequence with an example.

4. Write $P(x)=x^5+3x^4-15x^3+x-10$ is telescopic form.

5. If A denotes Ackermann's function, evaluate A (1, 1).

6. Write down the procedure for finding the generating function of a given recurrence relation.

7. Show that the poset $P(\{1, 2, 3\}, \subseteq)$ is not a chain.

8. Prove that every lattice with 0 and 1 is not complemented.

9. Define Not-gate and AND-gate.

10. Define a fundamental product of the n-variables.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that for a finite monoid (M, *), no two rows or columns of the Cayley table are identical.

Or

(b) Prove that the property of idempotency is preserved under a semigroup homomorphism.

12. (a) Prove that $\sqrt{2}$ is irrational.

Or

(b) Show that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}.$$

13. (a) Find T (27) where T (n) denotes the worst time for binary search of a file with n records.

Or

(b) Show that $f(x, y) = x^y$ is primitive recursive.

14. (a) Let $(L, \leq)$ be a lattice. Then prove that L satisfies the following laws :

(i) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ and

(ii) $(a \vee b) \vee c = a \vee (b \vee c)$ for all $a, b, c, \in L$.

Or

(b) Show that a lattice L is distributive if and only if for all a, b, c in L $(a \vee b) \wedge c \leq a \vee (b \wedge c)$.

15. (a) Simplify the following using Karnaugh diagrams :

$$f(x_1, x_2, x_3, x_4) = x_1 x_3 + x_1' x_3 x_4 + x_2 x_3' x_4 + x_2' x_3 x_4 .$$

Or

(b) Write down the minterm normal form of

$$f(x_1, x_2) = \overline{x}_1 \vee \overline{x}_2 .$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that the set operations $\cup, \cap$ (union and intersection) are both commutative, associative, idempotent and each one is distributive over the other.

17. Solve $D(K) - 8D(K-1) + 16D(K-2) = 0$ where $D(2) = 16$, $D(3) = 80$.

18. If $P(K) - 6P(K-1) + 5P(K-2) = 0$, $P(0) = 2$, $P(1) = 2$. What is the generating function of P?

19. Prove that $(L \times M, \wedge, \vee)$ is a lattice.

20. Let B be a finite Boolean algebra and let A be the set of all atoms of B. Then prove that Boolean algebra B is isomorphic to the Boolean algebra $P(A)$.

———————

F–4990

Sub. Code
7MMA3E4

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Third Semester

Mathematics

Elective – FUZZY MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. What is meant by the support of a fuzzy set?

2. Define the extension principle.

3. Write down the axiomatic skeleton for fuzzy complements.

4. Define the intersection of two fuzzy sets.

5. What is meant by fuzzy relation?

6. Define a fuzzy partial ordering.

7. Define a fuzzy measure. Give an example.

8. Define a lattice of possibility distributions of length n.

9. Write a short note on Shannon entropy.

10. When will you say that the function is said to be information transmission?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that a fuzzy set A is convex if and only if $\mu_A(\lambda\vec{r} + (1-\lambda)\vec{s}) \geq \min[\mu_A(\vec{r}), \mu_A(\vec{s})]$, for all $\vec{r}, \vec{s} \in \mathbb{R}^n$ and $\lambda \in [0,1]$.

Or

(b) Order the fuzzy sets defined by the following membership grade functions (assuming $x \geq 0$) by the inclusion (subset) relation :

$$\mu_A(x) = \frac{1}{1+20x}, \qquad \mu_B(x) = \left(\frac{1}{1+10x}\right)^{1/2},$$

$$\mu_C(x) = \left(\frac{1}{1+10x}\right)^2.$$

12. (a) For each $a \in [0,1]$, prove that ${}^d a = c(a)$ if and only if $c(c(a)) = a$, that is, when the complement is involutive.

Or

(b) For all $a, b \in [0,1]$, prove that $i(a,b) \leq \min(a,b)$.

13. (a) Consider the sets $X_1 = \{x, y\}$, $X_2\{a, b\}$ and $X_3 = \{*, \$\}$ and the ternary fuzzy relation

$$R(X_1, X_2, X_3) = \frac{0.9}{(x,a,*)} + \frac{0.4}{(x,b,*)} + \frac{1}{(y,a,*)} + \frac{0.7}{(y,a,\$)} + \frac{0.8}{y,b,\$}$$

define on $X_1 \times X_2 \times X_3$. Compute the projections $R_1, R_2, R_3, R_{12}, R_{13}, R_{23}$.

Or

(b) What is meant by a sagittal diagram? Explain with suitable illustration.

14. (a) Let $X = \{a, b, c, d\}$. Given the basic assignment $m(\{a, b, c\}) = 0.5$, $m(\{a, b, d\})0.2$ and $m(X) = 0.3$, determine the corresponding belief and plausibility measures.

Or

(b) Prove that every possibility measure π on $\mathcal{P}(X)$ can be uniquely determined by a possibility distribution function $r : X \to [0,1]$ via the formula $\pi(A) = \max_{x \in A} r(x)$ for each $A \in \mathcal{P}(X)$.

15. (a) Enumerate the following terms:

(i) Measure of fuzziness

(ii) Index of fuzziness.

Or

(b) With the usual notations, prove that $H(X, Y) \le H(X) + H(Y)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let the membership grade functions of sets A, B, C defined on the set $X = \{0,1,2, \ldots 10\}$ by $\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = 2^{-x}$, $\mu_C(x) = \frac{1}{1 + 10(x-2)^2}$. Determine mathematical formulas and graphs of the membership grade functions of each of the following:

(i) $\overline{A}, \overline{B}, \overline{C}$; (ii) $A \cup B$, $B \cup C$; (iii) $A \cap B$, $A \cap C$.

17. (a) Prove that $\lim_{w \to \infty} \min\left[1, \left(a^w + b^w\right)^{1/w}\right] = \max(a, b)$.

(b) Let $h_\alpha(a_1, a_2, \ldots a_n) = \left(\frac{a_1^\alpha + a_2^\alpha + \ldots + a_n^\alpha}{n}\right)^{1/\alpha}$. Prove that $\lim_{\alpha \to 0} h_\alpha = (a_1, a_2 \ldots a_n)^{1/n}$.

18. (a) Explain tolerance relation.

(b) Determine the complete α-covers of the compatibility relation whose membership matrix is given below.

	1	2	3	4	5	6	7	8	9
1	1	0.8	0	0	0	0	0	0	0
2	0.8	1	0	0	0	0	0	0	0
3	0	0	1	1	0.8	0	0	0	0
4	0	0	1	1	0.8	0.7	0.5	0	0
5	0	0	0.8	0.8	1	0.7	0.5	0.7	0
6	0	0	0	0.7	0.7	1	0.4	0	0
7	0	0	0	0.5	0.5	0.4	1	0	0
8	0	0	0	0	0.7	0	0	1	0
9	0	0	0	0	0	0	0	0	1

19. (a) What is meant by plausibility measure?

(b) Prove that a belief measure Bel on finite power set $\mathcal{P}(X)$ is a probability measure if and only if its basic assignment m is given by $m(\{x\}) = Bel(\{x\})$ and $m(A) = 0$ for all subsets of X that are not singletons.

20. Let m_X and m_Y be marginal basic assignments on set X and Y respectively, and let m be a joint basic assignment on $X \times Y$ such that for all $A \in \mathcal{P}(X)$ and $B \in \mathcal{P}(Y)$. Prove that $E(m) = E(m_X) + E(m_Y)$.

F–5118

Sub. Code
7MMA1C1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

First Semester

Mathematics

ALGEBRA – I

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. State the Fermat theorem.

2. Define an automorphism of a group.

3. Define a conjugate of a in G.

4. What is meant by the external direct product of groups?

5. Define a zero-divisor.

6. Define a homomorphism of rings.

7. Define two-sided ideal of a ring.

8. When will you say that a ring R is said to be imbedded in a ring R'?

9. Define relatively prime element.

10. What is meant by primitive polynomial?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If H and K are subgroups of G and $O(H) > \sqrt{O(G)}$, $O(K) > \sqrt{O(G)}$, then prove that $H \cap K \neq (e)$.

Or

(b) State and prove that Cauchy's theorem for abelian groups.

12. (a) Define the normalizer of an element in G. Prove that $N(a)$ is a subgroups of G.

Or

(b) Suppose that G is the internal direct product of $N_1, N_2, \ldots. N_n$. Prove that for $i \neq j$, $N_i \cap N_j = (e)$, and if $a \in N_i$, $b \in N_j$ then prove that $ab = ba$.

13. (a) Define an integral domain. Also prove that a finite integral domain is a field.

Or

(b) Define a field. Prove that any field is an integral domain.

14. (a) If U is an ideal of the ring R, then prove that R/U is a ring and is a homomorphic image of R.

Or

(b) Prove that the mapping $\phi : D \to F$ defined by $\phi(a) = [a,1]$ is an isomorphism of D into F.

15. (a) State and prove the Unique factorization theorem.

Or

(b) If R is a Unique factorization domain, then prove that $R[x]$ is also an Unique factorization domain.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that every group is isomorphic to a subgroup of $A(S)$ for some appropriate S.

17. State and prove first part of Sylow's theorem.

18. If ϕ is a homomorphism of R into R', then prove the following

(a) $\phi(0) = 0$; (b) $\phi(-a) = -\phi(a)$ for every $a \in R$.

19. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

20. (a) State and prove the Gauss' lemma.

(b) Prove that the Polynomial $1 + x + x^2 + ... + x^{p-1}$, where p is a prime number, is irreducible over the field of rational numbers.

———————

F–5119

Sub. Code
7MMA1C2

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

First Semester

Mathematics

ANALYSIS – I

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define an isolated point of E.

2. Define connected set. Give an example.

3. If $p > 0$, then prove that $\lim_{n \to \infty} \frac{1}{n^p} = 0$.

4. State the Roof test.

5. Find the radius of convergence of the power series $\sum \frac{2^n}{n!} z^n$.

6. State the Merten's theorem.

7. What is the difference between continuity and uniform continuity?

8. Give an example of the two kinds of discontinuous of a function.

9. Define a local maximum.

10. State the Taylor's theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that closed subsets of compact sets are compact.

Or

(b) Let p be a nonempty perfect set in R^k prove that p is uncountable.

12. (a) Prove that the subsequential limits of a sequence $\{p_n\}$ in a metric space X form a closed subset of X.

Or

(b) With the usual notations, prove that $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

13. (a) State and prove partial summation formula.

Or

(b) If $\sum a_n = A$ and $\sum b_n = B$, then prove that $\sum (a_n + b_n) = A + B$ and $\sum ca_n = cA$, for any fixed c.

14. (a) Let f and g be complex continuous functions on a metric space X. Prove that $f + g, fg$ and f / g are continuous on X.

Or

(b) State and prove the intermediate value theorem.

15. (a) Prove that the function $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0 \quad , & x=0 \end{cases}$ is differentiable at all points of x and f' is not continuous function.

Or

(b) Suppose f is a continuous mapping of $[a,b]$ into R^{κ} and f is differentiable in (a,b). Prove that there exists $x\in(a,b)$ such that $|f(b)-f(a)|\leq(b-a)|f'(x)|$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that a subset E of the real line R^1 is connected if and only if it has the following property: If $x\in E, y\in E$, and $x<z<y$, then $z\in E$.

17. (a) If E is the closure of a set E in a metric space x, then prove that diam $\overline{E}=\operatorname{diam} E$.

(b) For any sequence $\{c_n\}$ of positive numbers, prove the following:

(i) $\lim_{n\to\infty}\inf\frac{c_n+1}{c_n}\leq\lim_{n\to\infty}\inf\sqrt[n]{c_n}$;

(ii) $\lim_{n\to\infty}\sup\sqrt[n]{c_n}\leq\lim_{n\to\infty}\sup\frac{c_n+1}{c_n}$.

18. Let $\sum a_n$ be a series of real numbers which converges, but not absolutely. Suppose $-\infty\leq\alpha\leq\beta\leq\infty$. Prove that there exists a rearrangement $\sum a_n'$ with partial sums s_n' such that $\lim_{n\to\infty}\inf s_n'=\alpha$, $\lim_{n\to\infty}\sup s_n'=\beta$.

19. Show that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(v)$ is open in X for every open set V in Y.

20. State and prove that L' Hospital's rule.

F–5396

Sub. Code
7MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Second Semester

Mathematics

ALGEBRA – II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Prove that the Kernel of T is a subspace.

2. In a vector space show that $\alpha(v - w) = \alpha v - \alpha w$.

3. Prove that $A(w)$ is a subspace of $\hat{V}$.

4. Define an inner product space.

5. When we say that an element is algebraic of degree n over a field?

6. What is the degree of $\sqrt{2} + \sqrt{3}$ over Q?

7. Show that the fixed field of G is a subfield of k.

8. State the fundamental theorem of Galois theory.

9. Give an example for an element in $A(V)$ is right-invertible but is not invertible.

10. If $T \in A(V)$ then prove that $T^* \in A(V)$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Define a linear span of a non empty subset of the vector space V. Also prove that $L(S)$ is a subspace of V.

Or

(b) If $v_1, \dots v_n$ are in V then prove that either they are linearly independent of some v_k is a linear combination of the proceeding ones, $v_1, \dots, v_{k-1}$.

12. (a) If V is finite-dimensional and $v \neq o \in V$, then prove that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

Or

(b) State and prove that Schwarz inequality.

13. (a) Prove that a polynomial of degree n over a field can have atmost n roots in any extension field.

Or

(b) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

14. (a) Let K be the field of complex numbers and let F be the field of real numbers. Compute $G(K, F)$.

Or

(b) If $P(x) = x^n - 1$ prove that the Galois group of $P(x)$ over the field to rational numbers is abelian.

15. (a) If V is an n-dimensional vector space over F, then prove that, given any element T in $A(V)$, there exists a nontrivial polynomial $q(x) \in F[x]$ of degree atmost n^2, such that $q(T) = 0$.

Or

(b) If V is finite-dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If V is finite-dimensional over F then prove that any two bases of V have the same number of elements.

17. Let V be a finite-dimensional inner product space. Then prove that V has an orthonormal set as a basis.

18. If L is a finite extension of F and K is a subfield of L which contains F, then prove that $[K : F]/[L : F]$.

19. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

20. Prove that a change of basis from one orthonormal basis to another is accomplished by a unitary linear transformation.

F–5397

Sub. Code
7MMA2C2

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement / Arrear Examinations

Second Semester

Mathematics

ANALYSIS – II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the upper and lower Riemann integrals of f over $[a, b]$.

2. State the fundamental theorem of calculus.

3. Define an equicontinuous on a set E.

4. Define an algebra.

5. Give an example for an orthonormal basis.

6. Prove that the functions C and S are periodic with period 2π.

7. If $m * E = 0$, then prove that E is measurable.

8. When we say that a real-valued function is simple?

9. Define the positive and the negative part of a function f.

10. Define the upper and lower Riemann integral of f.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If P^* is a refinement of P, then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ and

$U(P^*, f, \alpha) \leq U(P, f, \alpha)$.

Or

(b) If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)| dt$.

12. (a) State and prove that Cauchy criterion for uniform convergence theorem.

Or

(b) Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$, for $n = 1,2,3\ldots$ and suppose $f_n \to f$ uniformly on $[a, b]$. Then prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$ and $\int_a^b f d\alpha \lim_{n\to\infty} \int_a^b f_n d\alpha$.

13. (a) State and prove Taylor's theorem.

Or

(b) State and prove Bessel's in equality.

14. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Then prove that $m^*(\bigcup A_n) \leq \sum m^* A_n$.

Or

(b) Let $\langle En \rangle$ be an infinite decreasing sequence of measurable sets. Let mE_1, be finite. Then prove that $m\left(\bigcap_{i=1}^{\infty} Ei\right) = \lim_{n\to\infty} mEn$.

15. (a) State and prove monotone convergence theorem.

Or

(b) Let f be a nonnegative measurable function. Show that $\int f = 0$ implies $f = 0$ a.e.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Assume α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then prove that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$. In that case $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.

17. Suppose $fn \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that $\lim_{t\to x} fn(t) = A_n (n = 1,2,\ldots)$. Then prove that $\{A_n\}$ convergence and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.

18. State and prove Parseval's theorem.

19. Let C be a constant and f and g two measurable real-valued functions defined on the same domain. Then prove that the functions $f + c$, cf, $f + g$, $g - f$ and fg are also measurable.

20. State and prove the bounded convergence theorem.

———————

F–5398

Sub. Code
7MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Solve : $\frac{dx + dy}{(x+y)^2} = \frac{dz}{z(x+y)}$.

2. Define a Pfaffian differential equation.

3. Form a partial differential equation, by eliminating the constants a and b from $z = (x+a)(y+b)$.

4. Define complete solution and general solution of the first order partial differential equation.

5. Find the complete integral of the equation $p = (z + qy)^2$.

6. Write down the Clairaut equations.

7. Classify the equation $x^2 \frac{\partial^2 z}{\partial x^2} + 2x \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.

8. Find the particular integral of the equation $(D^2 - D')z = 2y - x^2$.

9. Define the interior Neumann problem.

10. Write down the one dimensional diffusion equation.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Solve the equation $\frac{dx}{y + \alpha z} = \frac{dy}{z + \beta x} = \frac{dz}{x + \gamma y}$.

Or

(b) Find the orthogonal trajectories on the sphere $x^2 + y^2 + z^2 = a^2$ of its intersections with the paraboloids $xy = cz$, c being a parameter.

12. (a) If u is a function of x, y and z which satisfies the partial differential equation

$$(y - z)\frac{\partial u}{\partial x} + (z - x)\frac{\partial u}{\partial y} + (x - y)\frac{\partial u}{\partial z} = 0.$$

Show that u contains x, y and z only in combinations $x + y + z$ and $x^2 + y^2 + z^2$.

Or

(b) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 - a^2$, $z = 0$.

13. (a) Show that the equations $xp = yq$, $z(xp + yq) = 2xy$ are compatible and solve them.

Or

(b) Solve the equation $p^2x + q^2y = z$ using Jacobi's method.

14. (a) Find the solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y.$$

Or

(b) By separating the variables, show that the one-dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ has solutions of the form $z(x, t) = c_n \cos(nx + \varepsilon_n) e^{-n^2kt}$, where c_n is a constant. Hence show that functions of the form $z(x, t) = \sum_{n=0}^{\infty} c_n \cos(nx + \varepsilon_n) e^{-n^2kt}$.

15. (a) Show that $r^{-2}\cos\theta$ satisfying othe Laplace equation, when r, θ and ϕ are spherical polar co-ordinates.

Or

(b) Derive D'Alembert's solution of the one-dimensional wave equation.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that a necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$, not involving x or y explicitly is that $\frac{\partial(u, v)}{\partial(x, y)} = 0$.

17. Find the solution of the equation

$z = \frac{1}{2}(p^2 + q^2) + (p - x)\ (q - y)$ which passes through the x-axis.

18. Prove that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$, which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$.

19. Derive the solution of the equation :

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} = 0.$$

for the region $r \geq 0, z \geq 0$, satisfying the conditions :

(a) $v \to 0$ as $z \to \infty$ and as $r \to \infty$

(b) $v = f(r)$ on $z = 0, r \geq 0$.

20. The points of trisection of a string are pulled aside through a distance ε on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest.

F–5399

Sub. Code
7MMA2C4

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

Second Semester

Mathematics

MECHANICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by degrees of freedom? Give an example.

2. Define a virtual displacement.

3. Write down the standard from of Lagrange's equation for a nonholonomic system.

4. Define an ignorable coordinates.

5. Write down the Euler-Lagrange equation.

6. Define the Hamiltonian function.

7. What do you mean by Pfaffian differential forms?

8. State the Stackel's theorem.

9. What is meant by a point transformation?

10. Write down the Poisson bracket.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Narrate the following terms:

(i) Holonomic constraints;

(ii) Nonholonomic constraints.

Or

(b) State and prove D' Alembert's principles.

12. (a) Find the differential equations of motion for a spherical pendulum of length l.

Or

(b) Define the Routhian function R, and show that

$$\frac{d}{dt}\left(\frac{\partial R}{\partial \dot{q}_i}\right) - \frac{\partial R}{\partial q_i} = 0, i = k+1, \ldots\ldots.. n.$$

13. (a) Obtain the solution of Brachistochrone problem by the method of Calculus of variations.

Or

(b) Find the equation of motion for a charged particle in an electromagnetic field using Hamiltonian H.

14. (a) Use Hamilton Jacobi method to analyse the Kepler problem.

Or

(b) Define Lioville's system and show that the Lioville's conditions are sufficient for the separability of an orthogonal system.

15. (a) Prove that the transformation $Q = \frac{1}{2}\left(q^2 + p^2\right)$,

$P = -\tan^{-1}\left(q/p\right)$ is canonical.

Or

(b) Consider the transformation $Q = q - tp + \frac{1}{2}gt^2$, $Q = p - gt$. Find $K - H$ and the generating functions.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. A particle of mass m is suspended by a Massless wire of length $\vec{r} = a + b\cos wt (a > b > 0)$ to form a spherical pendulum. Find the equations of motion.

17. Derive the Lagrange's equations of motion from D' Alembert's principle.

18. Establish the principle of least action. Deduce the Jacobi's form of the principle of least action.

19. Derive the Hamilton-Jacobi equation in the form $\frac{\partial s}{\partial t} + H\left(q, \frac{\partial s}{\partial q}, t\right) = 0$.

20. Define Lagrange bracket $[u,v]$ and show that $[q_j, q_k] = [p_j, p_k] = 0$, while $[q_j, p_k] = \delta_{jk}$.

F–5400

Sub. Code
7MMA2E1

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary / Improvement/ Arrear Examinations

Second Semester

Mathematics

Elective : GRAPH THEORY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a spanning subgraph of G. Give an example.

2. Prove that $\delta \leq 2\, {}^{\in}\!/_{\gamma} \leq \Delta$.

3. Define a block of a graph. Give an example.

4. If G is Hamiltonian then prove that for every non empty proper subset S of V, $w(G-S) \leq |S|$.

5. Define a perfect matching. Give an example.

6. Is the Petersen graph 4-edge chromatic? Justify your answer.

7. Prove that $r(2,l) = l$.

8. Define a k-critical graph.

9. Draw a graph an embedding of $k_{3,3}$ on the Mobius band.

10. If G is a simple planar graph then prove that $\delta \leq 5$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that a graph is bipartite if and only if it contains no odd cycle.

Or

(b) Show that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

12. (a) With the usual notations, prove that $k \leq k' \leq \delta$.

Or

(b) Let G be a simple graph and let u and v be nonadjacent vertices in G such that $d(u) + d(v) \geq \gamma$. Prove that G *is* Hamiltonian if and only if $G + uv$ is Hamiltonian.

13. (a) State and prove Berge theorem in a matching.

Or

(b) What is meant by edge colouring? Also show, by finding an appropriate edge colouring that $\chi'(k_{m,n}) = \Delta(k_{m,n})$.

14. (a) For any two integers $k \geq 2$ and $l \geq 2$, prove that $r(k,l) \leq r(k,l-1) + r(k-1,l)$.

Or

(b) For any graph G, prove that $\chi \leq \Delta + 1$.

15. (a) With the usual notations, prove that k_5 is nonplanar.

Or

(b) State and prove Euler's formula for connected plane graph.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. With the usual notations, prove that $\tau(k_n) = n^{n-2}$.

17. Prove that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

18. State and prove Vizin'g theorem.

19. State and prove Brook's theorem.

20. (a) Explain the dual graphs with an illustration.

(b) If two bridges overlap, then prove that they are skew or else they are equivalent 3-bridges.

F–5404

Sub. Code
7MMA3E5

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Third Semester

Mathematics

Elective : STOCHASTIC PROCESSES

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define state space.
2. Define second order processes.
3. Define transition matrix.
4. Define Markov chain.
5. Define ergodic.
6. Explain null persistent.
7. Explain regularity.
8. Write down the parameters of Poisson process.
9. Define renewal event.
10. Define a periodic.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Classify the process with examples.

Or

(b) If a Gaussian process $X(t)$ is co-variance stationary, then prove it is strictly stationary.

12. (a) Explain Markov chain as graph.

Or

(b) Explain Markov-Bernoulli chain.

13. (a) Prove the following theorem: state j is Persitent iff $\sum_{n=0}^{\infty} p_{ij}^{(n)} = \infty$.

Or

(b) State and prove the Basic limit theorem of renewal theory.

14. (a) Derive Poisson process.

Or

(b) Prove that the differences of two independent Poisson process in not a Poisson process.

15. (a) State and prove Renewal theorem.

Or

(b) Explain renewal theory in discrete time.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Consider the process $X(t) = A_1 + A_2 t$ where A_1, A_2 are independent $r - v'$ s with $E(A_i) = a_i$, Var $(A_i) = \sigma_i^2$, $i = 1, 2$ prove the process in evolutionary.

17. State and prove Chapman- Kolmogorov equation.

18. Discuss generalisation of independent Bernoulli trials.

19. Prove the following theorem.

    If the intervals between successive occurrences of an event E are independently distributed with a common exponential distribution with mean $\frac{1}{\lambda}$, then the events E form a Poisson process with mean λt.

20. State and prove Wald's equation.

F–5405

Sub. Code
7MMA3E6

M.Sc. DEGREE EXAMINATION, APRIL 2021 &

Supplementary/Improvement/Arrear Examinations

Third Semester

Mathematics

Elective – COMBINATORIAL MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the exponential enumerator for the permutations of none, one, two,…, p of p identical objects.

2. Define a Ferrers graph.

3. Define a pattern with an example.

4. Write down the characteristics equation of $a_n = a_{n-1} + a_{n-2}$.

5. Define a derangement of the integers.

6. How many ways to place two nontaking rooks on a regular 8×8 chessboard?

7. Define the difference two sets with an example.

8. Define an equivalence relation.

9. Define a block design.

10. Define a symmetric balanced incomplete block design.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that the number of ways of selecting an even number of objects is equal to the number of ways of selecting an odd number of objects from n distinct objects.

Or

(b) Find the number of r–digit Quantenary sequences in which each of the digits 1,2 and 3 appears at least once.

12. (a) Let there be n ovals drawn on the plane. If an oval intersects each of the other ovals at exactly two points and no three ovals meet at the same point. How many regions do these ovals divide the plane?

Or

(b) State and solve the tower of Hanoi problem.

13. (a) Find the number of n–digit ternary sequences that have an even number of 0's.

Or

(b) Consider the permutations of the n–integers 1, 2,….n. Find the number of permutations in which no two adjacent integers are consecutive integers.

14. (a) Show that the binary relation on a set induced by a permutation group of the set is a equivalence relation.

Or

(b) Find the number of ways of painting the four faces a, b, c and d of the pyramid with two colors of paints x and y.

15. (a) If a set of r orthogonal latin squares if order n and a set of r orthogonal latin squares of order n' exists, then prove that there exists a set of r latin squares of order nn'.

Or

(b) Prove that the Kronecker product of two Hadamard matrices is a Hadamard matix.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Derive the number of ways of distributing r distinct object into n distinct cells so that no cell is empty and the order of objects within a cell is not important.

17. Find the number of n–digit binary sequences that have exactly r pairs of adjacent 1's and no adjacent 0's.

18. State an prove the principle of inclusion and exclusion.

19. State and prove Polya's fundamental theorem.

20. In a (b, v, r, k, λ) – configuration, prove that $b \geq v$.