F–6314

Sub. Code
7MMA1C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

First Semester

Mathematics

ALGEBRA – I

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a subgroup of G. Give an example.

2. When will you say that two groups are said to be isomorphic?

3. Define the normalizer of an element in group.

4. State the second part of Sylow's theorem.

5. State the pigeonhole principle.

6. Define a Boolean ring.

7. If U is an ideal of R and $1 \in U$, then prove that $U = R$.

8. Define a maximal ideal of ring R.

9. Define a Euclidean ring.

10. Define a unique factorization domain.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) State the Euler theorem. If G is a finite group whose order is a prime number p, then prove that G is a cyclic group.

Or

(b) Define a permutation group. Prove that every permutation is the product of its cycles.

12. (a) State and prove the Cauchy theorem.

Or

(b) Let G be a group and suppose that G is the internal direct product of $N_1, N_2, \ldots. N_n$. Let $T = N_1 \times N_2 \times \ldots. \times N_n$. Prove that G and T are isomorphic.

13. (a) If R is a ring, then prove that for all $a, b \in R$,

(i) $a0 = 0a = 0$;

(ii) $a(-b) = (-a)b = -(ab)$;

(iii) $(-a)(-b) = ab$

Or

(b) Prove that the homomorphism ϕ of R into R' is an isomorphism if and only if $I(\phi) = (0)$.

14. (a) Define a left-ideal of R. For $a \in R$ and let $Ra = \{xa \mid x \in R\}$. Prove that Ra is a left-ideal of R.

Or

(b) If $[a,b]=[a',b']$ and $[c,d]=[c',d']$ then prove that $[a,b][c,d]=[a',b']\ [c',d']$

15. (a) Prove that $J[i]$ is a Euclidean ring.

Or

(b) State and prove the Eisenstein criterion for polynomials.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove fundamental theorem of homomorphism of groups.

17. State and prove third part of Sylow's theorem.

18. Let C be the set of all symbols (α,β) where α and β are real numbers and define $(\alpha,\beta)=(\gamma,\delta) \Leftrightarrow \alpha=\gamma$ and $\beta=\delta$. Prove that C is a filed.

19. Show that every integral domain can be imbedded in a field.

20. (a) State and prove the unique factorization theorem.

(b) If $f(x)$, $g(x)$ are two nonzero elements of $F[x]$, then prove that $\deg(f(x)g(x)) = \deg f(x) + \deg g(x)$.

F–6315

Sub. Code
7MMA1C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021.

First Semester

Mathematics

ANALYSIS – I

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define open cover.

2. What do you mean by denumerable set?

3. If $\lim_{n\to\infty} S_n = s_1, \lim_{n\to\infty} t_n = t$. Prove that $\lim_{n\to\infty}(s_n + t_n) = s + t$.

4. Define Cauchy sequence.

5. Define power series.

6. What do you mean by rearrangement?

7. Define continuous on *E*.

8. State characterization of continuity theorem.

9. Suppose f is a differentiable in (a,b) and if $f(x) = 0 \forall x \in (a,b)$. Prove that f is constant.

10. Define local maximum at P.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that every neighborhood is an open set.

Or

(b) If E is an infinite subset of a compact set K, prove that E has a limit point in K.

12. (a) Prove that the subsequential limits of a sequence $\{P_n\}$ in a metric space X form a closed subset of X.

Or

(b) Prove that

(i) $\lim_{n \to \infty} \sqrt[n]{p} = 1$ if $p > 0$

(ii) $\lim_{n \to \infty} \sqrt[n]{n} = 1$

13. (a) If Σa_n converges absolutely, prove that Σa_n converges.

Or

(b) If Σa_n = A and $\Sigma b_n = B$, prove that $\Sigma(a_n + b_n)$ = $A + B$ and $\Sigma c a_n = CA$, for any fixed C.

14. (a) Let f be monotonic on (a,b) , prove that the set of point of (a,b) at which f is discontinuous is at most countable.

Or

(b) Prove that continuous image of a connected set is connected.

15. (a) State and prove chain rule for differentiation.

Or

(b) Let f be defined on $[a,b]$, if f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists. Prove that $f'(x) = 0$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If a set E in R^k has one of the following three properties, Prove that the following are equivalent.

(a) E is closed and bounded

(b) E is compact

(c) Every infinite subset of E has a limit point in E.

17. Prove that $\lim_{n \to 8} \left(1 + \frac{1}{n}\right)^n = e$.

18. Let Σa_n be a series of real numbers which converges, but not absolutely. Suppose $-\infty \leq \alpha \leq \beta \leq \infty$. Prove that there exists a rearrangement $\Sigma a'_n$ with partial sums s'_n such that $\lim_{n \to \infty} \inf s'_n = \alpha, \lim_{n \to \infty} \inf s'_n = \beta$.

19. Let E be a non compact set in R'. Prove that

(a) there exists a continuous function on E which is not bounded.

(b) there exists a continuous and bounded function on E which has no maximum.

20. State and prove L'Hospital's Rule.

F–6316

Sub. Code
7MMA1C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

First Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by inflexional?

2. Define an osculating circle.

3. Define the anchor ring.

4. When will you say that the components are said to be direction coefficient?

5. Write down the canonical equations for geodesics.

6. Define geodesic parallels.

7. Define the normal curvature.

8. What is meant by pseudo – sphere?

9. Define a hyperbolic point.

10. Write down the characteristic line, corresponding to the plane u.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Obtain the equations of the circular helix $\vec{r} = (a\cos u, a\sin u, bu)$, $-\infty < u < \infty$ where $a > 0$, referred to s as parameter, and show that the length of one complete turn of the helix is $2\pi c$, where $c = \sqrt{a^2 + b^2}$.

Or

(b) Show that the torsion of an involute of a curve is equal to $\frac{\rho(\sigma\rho' - \sigma'\rho)}{(\rho^2 + \sigma^2)(c - s)}$.

12. (a) Explain the following terms:

(i) Surface of Revolution; (ii) Helicoids.

Or

(b) Calculate the fundamental coefficients E, F, G and H for the paraboloid $\vec{r} = (u, v, u^2 - v^2)$.

13. (a) Prove that on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesics is $EF_2 + FE_1 - 2EF_1 = 0$.

Or

(b) Discuss the normal property of geodesics in detail.

14. (a) Derive the Liouville's formula of Kg.

Or

(b) State and prove Gauss – Bonnet theorem.

15. (a) Narrate the following terms.

    (i) Umbilic;

    (ii) Gaussian curvature;

    (iii) Dupin's indicatrix.

Or

(b) Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Establish Serret – Frenet formulae.

17. Find a surface of revolution which is isometric with a region of the right helicoid.

18. Determine the geodesics on a surface of revolution.

19. State and prove the Minding's theorem.

20. (a) State and prove the Meusnier's theorem.

    (b) State and prove the Euler's theorem.

F–6317

Sub. Code
7MMA1C4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the Wronskian of $\phi_1, \phi_2, ...\phi_n$.

2. State the existence theorem for linear equation with constant coefficients $L(y) = 0$.

3. Verify that the function $\varphi_1(x) = x^3, x > 0$ Satisfies the equation $x^2 y'' - 7xy' + 15y = 0$

4. State the existence theorem for analytic coefficients.

5. Define indicial polynomial.

6. Show that $x^{\frac{1}{2}} J_{-\frac{1}{2}}(x) = \frac{\sqrt{2}}{\Gamma_{\left(\frac{1}{2}\right)}} \cos x$.

7. Find an integrating factor of $(e^y + xe^y)dx + xe^y dy = 0$.

8. Write down the Lipschitz condition.

9. State the uniqueness theorem for solutions to first order equations.

10. Consider the system $y_1' = y_1 + \in y_2$, $y_2' = \in y_1 + y_2$ where $\in$ is a positive constant. Prove that every solution exists for all real x.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0, then prove that

$$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0).$$

Or

(b) Find all solution of the equation $y^{(4)} + 16y = 0$

12. (a) One solution of $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

Or

(b) Define the n[th] Legendre polynomial $P_n(x)$. Also prove that the coefficient of x^n in $P_n(x)$ is $(2n)!/2^n (n!)^2$.

13. (a) Find all solutions of the equation

$$x^2 y'' + xy' + 4y = 1 \quad \text{for } |x| > 0.$$

Or

(b) Obtain two linearly independent solutions of the following equation which are valid near $x = 0$; $x^2 y'' + 3xy' + (1 + x)y = 0$.

14. (a) Find all real-valued solutions of the equation $y' = \frac{e^{x-y}}{1+e^x}$.

Or

(b) Solve: $Cosx \cos^2 ydx - Sin\, xSin\, 2y\, dy = 0$.

15. (a) Suppose f is a real-valued continuous functions on the plane $|x| < \infty$, $|y| < \infty$, which satisfies a Lipschitz condition on each strip $S_\alpha : |x| \le a$, $|y| < \infty$, where a is any positive number. Prove that every initial value problem $y' = f(x, y)$, $y(x_0) = y_0$, has a solution which exists for all real x.

Or

(b) Consider the initial value problem $y' = xy + y^{10}$, $y(0) = \frac{1}{10}$. Show that a solution ψ of this problem exists for $|x| \le \frac{1}{2}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Consider the equation $y'' + w^2 y = A\cos wx$, where A, w are positive constants.

(a) Find all solutions on $0 \le x < \infty$

(b) Show that every solution ϕ is such that $|\phi(x)|$ assumes arbitrarily large values as $x \to \infty$.

(c) Sketch the graph of that solution φ satisfying $\varphi(0) = 0$, $\varphi'(0) = 1$.

17. Find the solution φ of $y''+(x-1)^2 y'-(x-1)y=0$ in the form $\varphi(x)=\sum_{k=0}^{\infty} C_k (x-1)^k$, which satisfies $\varphi(1)=1$, $\varphi'(1)=0$.

18. With the usual notations prove that

$$J_\alpha(x)=\left(\frac{x}{2}\right)^{\alpha}\sum_{m=0}^{\infty}\frac{(-1)^m}{m!\Gamma(m+\alpha+1)}\left(\frac{x}{2}\right)^{2m}, (\mathrm{Re}\,\alpha \ge 0).$$

19. Show that a function φ is a solution of the initial value problem $y'=f(x,y), y(x_0)=y_0$, on an interval I if and only if it is a solution of the integral equation $y=y_0+\int_{x_0}^{x} f(t,y)dt$ on I.

20. Let f be a continuous vector-valued function defined on $R:|x-x_0|\le a, |y-y_0|\le b, (a,b>0)$, and suppose f satisfies a Lipschitz condition on R. If M is a constant such that $|f(x,y)|\le M$ for all (x,y) in R. Prove that the successive approximations $\{\varphi_k\},(k=0,1,2,...)$ given by

$$\phi_0(x)=y_0, \phi_{k+1}(x)=y_0+\int_{x_0}^{x} f(t,\phi_k(t))dt, \qquad (k=0, 1, 2, ...)$$

converge on the interval $I:|x-x_0|\le \alpha = Minimum\{a, b/M\}$ to a solution ϕ of the initial value problem $y'=f(x,y), y(x_0)=y_0$ on I.

F–6318

Sub. Code
7MMA1E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

First Semester

Mathematics

Elective : NUMBER THEORY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. State and prove Euclid's lemma.

2. If a prime p does not divide a then prove that $(p, a) = 1$.

3. Define $\varphi(n)$.

4. If f is multiplicative then prove that $f(1) = 1$.

5. Define Euler's constant

6. Define mutually visible lattices.

7. Define Little Fermat's theorem.

8. Define Fermat number.

9. State Reciprocity law for Jacobi symbols.

10. Determine whether –104 is quadratic residue or non residue of the prime 997.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove the Euclidean algorithm.

Or

(b) Prove that there are infinitely many prime numbers.

12. (a) If $n \geq 1$ then prove that $\sum_{d/n} \mu(d) = \left[\frac{1}{n}\right] = \begin{cases} 1 & if \ \ n = 1 \\ 0 & if \ \ n > 1 \end{cases}$.

Or

(b) State and prove the Selberg identity.

13. (a) Prove that for all $x \geq 1$, $\sum_{n \leq x} \sigma_1(n) = \frac{1}{2}\zeta(2)x^2 + O(x \log x)$.

Or

(b) If $x \geq 2$ then prove that $\log[x]! = x \log x - x + O(\log x)$.

14. (a) Prove that for any prime p all the coefficients of the polynomial $f(x) = (x-1)(x-2)....(x-p+1) - x^{p-1} + 1$ are divisible by p

Or

(b) State and prove the Chinese remainder theorem.

15. (a) Prove that Legendre's symbol $\left(\frac{n}{p}\right)$ is a completely multiplicative function of n.

Or

(b) If p is an odd prime and $\chi(r) = (r/p)$ then prove that $G(1,\chi)^2 = (-1/p)p$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. For any two integers a and b then prove that there is a common divisor d of a and b of the form $d = ax + by$, x and y are integers.

17. If both g and $f * g$ are multiplicative then prove that f is also multiplicative.

18. Prove that the set of lattice points visible from the origin has density $6/\pi^2$.

19. State and prove Wolstenholme's theorem.

20. State and prove the Gauss lemma.

F–6320

Sub. Code
7MMA2C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Second Semester

Mathematics

ANALYSIS – II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** the questions.

1. Define the Riemann – Stieltjes integral of f with respect to α over $[a,b]$.

2. Define a rectifiable curve.

3. Show by an example that $\lim_{n\to\infty}\lim_{m\to\infty} S_{m,n} \neq \lim_{m\to\infty}\lim_{n\to\infty} S_{m,n}$.

4. Give an example of an algebra which does not separate points.

5. Prove that the function E is periodic, with period $2\pi i$.

6. Show that $E(z+w) = E(z)E(w) \forall z, w \in \mathbb{C}$.

7. Write down any one of the Littlewood's three principles.

8. Define a Borel set.

9. Define the Lebesgue integral of f over E.

10. Define a step function.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that $f \in \mathscr{R}(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$ there exists a partition p such that $U(p,f,\alpha) - L(p,f,\alpha) < \varepsilon$.

Or

(b) State and prove integration by parts theorem.

12. (a) Prove that a sequence $\{f_n\}$ converges to f with respect to the metric of $\mathscr{C}(X)$ if and only if $f_n \to f$ uniformly on X.

Or

(b) If K is a compact metric space, if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3,\ldots$ and if $\{f_n\}$ converges uniformly on K. Then prove that $\{f_n\}$ is equicontinuous on K.

13. (a) Prove that every non constant polynomial with complex coefficients has a complex root.

Or

(b) State and prove Stirling's formula.

14. (a) If E_1 and E_2 are measurable then prove that $E_1 \cup E_2$ is measurable.

Or

(b) Le $E \subset [0,1)$ be a measurable set. Then prove that for each $y \in [0,1)$ the set $E \dot{+} y$ is measurable and $m\left(E \dot{+} y\right) = mE$.

15. (a) State and prove Fatou's lemma.

Or

(b) State and prove Lebesgue convergence theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a,b]$, then prove that

(a) $fg \in \mathcal{R}(\alpha)$

(b) $|f| \in \mathcal{R}(\alpha)$ and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$.

17. Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$. If $\{f_n'\}$ converges uniformly on $[a,b]$, then prove that $\{f_n\}$ converges uniformly on $[a,b]$, to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x)$ where $a \leq x \leq b$.

18. Given a double sequence $\{a_{ij}\}$, $i = 1,2,\ldots$, $j = 1,2\ldots$ suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i$, $i = 1,2,3,\ldots$ and $\sum b_i$ converges. Then prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.

19. Prove that the outer measure of an interval is its length.

20. Let f be defined and bounded on a measurable set E with mE finite. In order that $\inf_{f \le \psi} \int_E \psi(x)dx = \sup_{f \ge \phi} \int_E \phi(x)dx$.

For all simple functions ϕ and ψ, prove that it is necessary and sufficient that f be measurable.

F–6321

Sub. Code
7MMA2C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define orthogonal trajectories of a system of curves on a surface.

2. Solve the equation $a^2y^2z^2dx + b^2z^2x^2dy + c^2x^2y^2dz = 0$.

3. Form a partial differential equation, by eliminating the constants a and b from $ax^2 + by^2 + z^2 = 1$.

4. Define the singular integral of the equation.

5. Find the complete integral of the equation $p + q = pq$.

6. Write a short notes on Jacobi's method.

7. If u is the complementary function and z_1 a particular integral of a linear partial differential equation, then prove that $u + z_1$ is a general solution of the equation.

8. Classify the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.

9. Show that $r\cos\theta$ satisfying the Laplace equation when r, θ and ϕ are spherical polar coordinates.

10. Write down the D'Alembert's solution of the one dimensional wave equation.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the integral curves of the equation $\frac{dx}{xz - y} = \frac{dy}{yz - x} = \frac{dz}{1 - z^2}$.

Or

(b) If $\overline{X}$ is a vector such that $\overline{X} \cdot curl\overline{X} = 0$ and μ is an arbitrary function of x, y, z then prove that $(\mu\overline{X}) \cdot curl(\mu\overline{X}) = 0$.

12. (a) Find the integral surface of the linear partial differential equation

$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0$, $z = 1$.

Or

(b) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis.

13. (a) Find the condition that two partial differential equations are compatible.

Or

(b) Solve the equation $p^2x + q^2y = z$ using Charpit's method.

14. (a) Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{\partial z}{x}$ is satisfied by $z = \frac{1}{x}\varphi(y-x) + \varphi'(y-x)$, where φ is an arbitrary function.

Or

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

15. (a) Prove that the solutions of a certain Neumann problem can differ from one another by a constant only.

Or

(b) Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r,\theta)$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Verify that the equation $z(z+y^2)dx + z(z+x^2)dy - xy(x+y)dz = 0$ is integrable and find its primitive.

17. Prove that the general solution of the linear partial differential equation $P_p + Q_q = R$ is $F(u,v) = 0$, where F is an arbitrary function and $u(x,y,z) = c_1$ and $v(x,y,z) = c_2$ form a solution of the equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

18. (a) Find a complete integral of the partial differential equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0$, $z^2 = 4y$.

(b) Solve the equation $z^2 = pqxy$ using Jacobi's method.

19. Determine the solution of the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^2 z}{\partial y^2} = 0$ $(-\infty < x < \infty,\ y \geq 0)$ satisfying the conditions:

(a) z and its partial derivatives tend to zero as $x \to \pm\infty$;

(b) $z = f(x)$, $\partial z / \partial y = 0$ on $y = 0$.

20. Determine the temperature $\theta(\rho, t)$ in the infinite cylinder $0 \leq \rho \leq a$ when the initial temperature is $\theta(\rho, 0) = f(\rho)$ and the surface $\rho = a$ is maintained at zero temperature.

F–6323

Sub. Code
7MMA2E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021.

Second Semester

Mathematics

Elective : GRAPH THEORY

(CBCS – 2017 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part A　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Define the incidence matrix of a graph. Give an example.

2. Define a spanning tree of G. Give an example.

3. What is meant by edge connectivity? Give an example.

4. Give an example of a graph which is Eulerian but not Hamiltonian.

5. What is meant by matching in graph G? Give an example.

6. Find the edge chromatic number of K_4

7. Define the Ramsey numbers.

8. State the Brook's theorem.

9. Draw a graph on embedding of K_5 on the torus.

10. Is the Petersen graph nonplanar? Justify your answer.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If a k-regular bipartite graph with $k > 0$ has bipartition (X, Y) then prove that $|X| = |Y|$.

Or

(b) If e is a link of G, then prove that $\tau(G) = \tau(G - e) + \tau(G.e)$.

12. (a) State and prove the Whitney theorem.

Or

(b) Prove that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.

13. (a) Prove that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

Or

(b) (i) State the Vizing's theorem.

(ii) If G is bipartite, then prove that $\chi' = \Delta$.

14. (a) If $\delta > 0$, then prove that $\alpha' + \beta' = \gamma$.

Or

(b) Prove that every critical graph is a block.

15. (a) (i) What is meant by dual graphs?

(ii) Let v be a vertex of a planar graph G. Prove that G can be embedded in the plane in such a way that v is on the exterior face of the embedding.

Or

(b) If G is a connected plane graph, then prove that $\gamma - \varepsilon + \phi = 2$. Also deduce that all planar embeddings of a given connected planar graph have the same number of faces.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Show that every simple graph on n vertices is isomorphic to a subgraph of K_n.

(b) Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

17. (a) If G is a simple graph with $\gamma \geq 3$ and $\delta \geq \gamma/2$, then prove that G is Hamiltonian.

(b) Prove that $C(G)$ is well defined.

18. (a) State and prove Hall's theorem.

(b) State and prove the marriage theorem.

19. With the usual notations, prove the following :

(a) $r(k,l) \leq \binom{k+l-2}{k-1}$.

(b) $r(k,k) \geq 2^{k/2}$.

20. Show that every planar graph is 5-vertex- colourable.

F–6324

Sub. Code
7MMA3C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the Hadamard's formula.

2. What is meant by conformal mapping?

3. Define the winding number of γ with respect to a.

4. Compute $\int_{\gamma} x dz$ where γ is the directed line segment from 0 to $1 + i$.

5. Define : zero and pole.

6. Show that the function $\cos z$ have essential singularity at ∞.

7. Find the residue for $\frac{1}{\sin z}$ at $z = 0$.

8. How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2?

9. Write down the formula for the series expansion of arc $\tan z$.

10. Find the genus of $\cos\sqrt{z}$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Define an analytic function. If $g(w)$ and $f(z)$ are analytic functions, then prove that $g(f(z))$ is also analytic.

Or

(b) State and prove the symmetry principle.

12. (a) Show that the line integral $\int_\gamma pdx + qdy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p$, $\frac{\partial U}{\partial y} = q$.

Or

(b) State and prove the Liouville's theorem. Also deduce that fundamental theorem of algebra.

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

(b) State and prove the Schwarz lemma.

14. (a) State and prove the argument principle theorem.

Or

(b) Evaluate $\int_0^\pi \frac{d\theta}{a + \cos\theta}$, $a > 1$.

15. (a) State and prove the Weierstrass's theorem for power series.

Or

(b) With the usual notations, prove that $\frac{\pi^2}{\sin^2 \pi z} = \sum_{n=-\infty}^{\infty} \frac{1}{(z-n)^2}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the Lucas's theorem.

(b) Find the linear transformation which carries (0, i, –i) into $(1,-1,0)$.

17. State and prove the Cauchy's integral formula. Also deduce that $f^{(n)}(z) = \frac{n!}{2\pi i} \int_c \frac{f(r)dr}{(r-z)^{n+1}}$.

18. State and prove the local mapping theorem.

19. State and prove the residue theorem. Also find $\int_C \frac{e^z dz}{(z-a)(z-b)}$ at its poles.

20. State and prove the Jensen's formula.

F–6325

Sub. Code
7MMA3C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021.

Third Semester

Mathematics

TOPOLOGY–I

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define topological space.

2. Is the set $\{x \times y / x \geq 0 \,\&\, y \geq 0\}$ in the plane R^2 closed? Justify your answer.

3. Define continuous function.

4. Prove that the function $f : R \to R$ defined by $f(x) = 3x + 1$ is a homomorphism.

5. Define connected space.

6. Define linear continuum.

7. Is the real line R compact or not? Justify your answer.

8. State the Lebesgue number lemma.

9. Define normal space.

10. State Urysohn lemma

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11\. (a) Let B and B' be bases for the topologies J and J' respectively on X. Prove the following are equivalent:

(i) J' is finer than J ;

(ii) For each $x \in X$ and each basis element $B \in B$ containing x, there is a basis element $B' \in B'$ such that $x \in B' \subset B$.

Or

(b) Let A be a subset of the topological space X; let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$

12\. (a) Let X and Y be topological spaces; let $f : X \to Y$. Then prove that the following are equivalent:

(i) f is continuous

(ii) For every subset A of X, one has $f(\overline{A}) \subset f(\overline{A})$

(iii) For every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

Or

(b) State and prove uniform limit theorem.

13\. (a) Prove that the union of a collection of connected sets that have a point in common is connected.

Or

(b) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

14. (a) State and prove maximum and minimum value theorem.

Or

(b) State and prove uniform continuity theorem.

15. (a) Suppose that X has a countable basis. Then prove that

(i) Every open covering of X contains a countable sub collection covering X.

(ii) There exists a countable subset of X which is dense is X.

Or

(b) Prove that every metrizable space is normal

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

(b) Let A be a subset of the topological space X. Then prove that (i) $x \in \overline{A}$ if and only if every open set U containing x intersects A. (ii) $x \in \overline{A}$ if and only if every basis elements B containing x intersects A.

17. Prove that the topologies on R^n induced by the euclidean metric d and the square metric p are the same as the product topology on R^n.

18. (a) State and prove intermediate value theorem.

(b) Let X be locally path connected. Show that every connected open set in X is path connected.

19. (a) Prove that every cloud subset of a compact space is closed.

(b) Prove that every compact subset of an Hausdorff space is closed.

20. (a) Prove that a subspace of a Hausdorff space is Hausdorff and product of Hausdorff spaces is Hausdorff.

(b) Prove that a subspace of a regular space is regular and product of regular spaces is regular.

F–6326

Sub. Code
7MMA3C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021.

Third Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. A person has purchased 10 of 1000 tickets sold in a certain raffle. To determine the five prize winners, 5 tickets are to be drawn at random and without replacement. Compute the probability that this person will win atleast one prize.

2. Define moment generating function.

3. Let x_1 and x_2 have the joint p.d.f as

$$f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$$

Find the marginal p.d.f of x_1 and x_2

4. Let $f(x_1, x_2) = \begin{cases} 4x_1 x_2 & 0 < x_1 < 1, \quad 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$

be the joint p.d.f of x_1 and x_2 find $\Pr\left(0 < x_1 < \frac{1}{2}, \frac{1}{4} < x_2 < 1\right)$

5. Define Bernouli distribution.

6. Let Y be the number of successes in n independent repetitions of a random experiment having the probability of success $p = 2/3$ if $n=3,$ then find $\Pr(2 \le Y)$

7. If the sample size is $n=2$ find the constant c so that $s^2 = c(X_1 - X_2)^2$.

8. Let X have the p.d.f $f(x) = \begin{cases} x^2/9, & 0 < x < 3 \\ 0 & elsewhere \end{cases}$ find the p.d.f of $Y = X^3$

9. Define convergence in distribution

10. Let Y be $b\left(72, \frac{1}{3}\right)$ find Pr $(22 \le y \le 28)$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let X have the p.d.f $f(x) = \begin{cases} x/6 & x = 1,2,3 \\ 0 & elsewhere \end{cases}$ find $E(X), E(X^2), E(X^3)$ and $E(6X + X^3)$

Or

(b) Let X be a random variable such that $E\left[(x-b)^2\right]$ exists for all real b show that $E\left[(x-b)^2\right]$ is a minimum when $b = E(x)$

12. (a) Let X and Y have the p.d.f

$$f(x,y) = \begin{cases} 1 & 0 < x < 1, \quad 0 < y < 1 \\ 0 & elsewhere \end{cases}$$

find the p.d.f of the product $Z = XY$.

Or

(b) Show that the random variables x_1 and x_2 with joint p.d.f

$$f(x_1, x_2) = \begin{cases} 12x_1\ x_2(1 - x_2), & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$$

are independent.

13. (a) Show that the m.g.f of the negative binomial distribution is $M(t) = P^r\left[1-(1-P)e^t\right]^{-r}$

Or

(b) Derive the moment generating function of the gamma distribution.

14. (a) Find the probability that exactly four observations of a random sample of size 5 from the distribution having p.d.f $f(x) = \begin{cases} \frac{x+1}{2}, & -1 < x < 1 \\ 0 & elsewhere \end{cases}$ exceed zero.

Or

(b) Derive the mean and variance of beta distribution.

15. (a) Let Y_1 denote the first order statistic of a random sample of size n from a distribution that has the p.d.f $f(x) = \begin{cases} e^{-(x-\theta)}, & \theta < x < \infty \\ 0 & elsewhere \end{cases}$ let $z_n = n(Y_1 -)$ find the limiting distribution of z_n

Or

(b) Let Z_n be $\chi^2(n)$ and let $W_n = Z_n / n^2$ find the limiting distribution of W_n.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the chebyshev's inequality

(b) If X is a random variable such that $E(X)=3$ and $E(X^2)=13,$ use chebyshev's inequality to determine a lower bound for the probability $\Pr(-2 < X < 8)$

17. Let $f(x_1 / x_2) = \begin{cases} c_1 \; x_1/x_2^2, & 0 < x_1 < x_2,\ 0 < x_2 < 1 \\ 0 & elswhere \end{cases}$ and

$f_2(x_2) = \begin{cases} c_2 \, x_2^4, & 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$ denote the conditional p.d.f of X_1 given $X_2 = x_2$ and the marginal p.d.f of X_2. determine

(a) The constants c_1 and c_2

(b) The joint p.d.f of X_1 and X_2

(c) $\Pr\left(1/4 < X_1 < \frac{1}{2} / X_2 = \frac{5}{8}\right)$

(d) $\Pr(1/4 < X_1 < 1/2)$

18. Let $f(x_1, x_2) = \begin{cases} \binom{x_1}{x_2}(1/2)^{x_1}\left(\frac{x_1}{15}\right) & x_2 = 0, 1, ...x_1, \\ & x_1 = 1, 2, 3, 4, 5 \\ 0 & elsewhere \end{cases}$

be the joint p.d.f. of X_1 and X_2. Determine

(a) $E(X_2)$

(b) $U(x_1) = E(X_2 / x_1)$

(c) $E[u(X_1)]$

19. Let the independent random variables X_1 and X_2 have the same p.d.f. $f(x) = \begin{cases} \frac{x}{6} & x = 1,\ 2,\ 3 \\ 0 & elsewhere \end{cases}$. Determine

(a) Joint p.d.f of X_1 and X_2 w

(b) $Pr(X_1 = 2,\ X_2 = 3)$

(c) $\Pr(X_1 + X_2 = 3)$

(d) m.g.f of $Y = X_1 + X_2$

20. State and prove the central limit theorem.

F–6327

Sub. Code
7MMA3E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Third Semester

Mathematics

Elective : DISCRETE MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define a semigroup with an example.

2. Define a semigroup isomorphism.

3. Find the characteristics equation of

   $J(k)-4J(k-1)+4J(k-2)=0$

4. Write down $p(x)=x^5+3x^4-15x^3+x-10$ in telescopic form.

5. When we say that a function is recursive?

6. Find the homogeneous solution for

   $S(k)-3S(k-1)-4S(k-2)=4^k$.

7. Prove that every lattice with 0 and 1 is not complemented.

8. Define a meet-homomorphism and a join-homomorphism.

9. Give any two examples for Boolean polynomials in the variables x_1, x_2 and x_3.

10. Define :

(a) identity gate and

(b) AND-gate.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let A be a set with n elements. How many commutative binary operations can be defined on A?

Or

(b) Let $(S, *)$ and (T, Δ) be monoids with identities e and e' respectively. Let $g : S \to T$ be an onto homomorphism. Then prove that $g(e) = e'$.

12. (a) Prove that for all $n \geq 1, 1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$ using induction.

Or

(b) For the sequence defined by $A(k) = k^2 - k, k \geq 0$, obtain the recurrence relation if A is a sequence of integers.

13. (a) Find T(27) where $T(n)$ denotes the worst time for binary search of a file with n records.

Or

(b) Show that $f(x, y) = x^y$ is primitive recursive.

14. (a) Let $(L, \leq)$ be a lattice. For any $a, b \in L$, Prove that the following are equivalent.

(i) $a \leq b$

(ii) $a \vee b = b$

(iii) $a \wedge b = a$

Or

(b) State and prove cancellation rule for distribution lattices.

15. (a) In a Boolean algebra, prove that the De Morgan's laws given by $(a \wedge b)' = a' \vee b'$ and $(a \vee b)' = a' \wedge b'$ holds for all $a, b \in L$.

Or

(b) Write down an algorithm applied to the K-map of an Boolean function to find a minimal expression for the function.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let N be the set of positive integers and $*$ the operation of L.C.M on N. Find whether $(N, *)$ is a commutative semigroup. Is it a monoid? Specify the identity element. Which elements in N have inverse and what are they?

17. Write down the recurrence relation for Fibonacci numbers and solve it.

18. Show that the following functions over N are primitive recursive.

(a) Constant function over N

(b) Predecessor function

(c) Proper subtraction function

(d) Zero test function

(e) Odd and even parity function.

19. If L and M are lattices then prove that $(L \times M, \wedge, \vee)$ is a lattice.

20. Describe the addition of two one-digit binary numbers.

F–6328

Sub. Code
7MMA3E4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Third Semester

Mathematics

Elective – FUZZY MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define Normalized fuzzy set.

2. If an element has an membership grade of 0.8 in fuzzy set A. Find $\mu_A -(x)$.

3. Define Sugeno class of fuzzy complement.

4. What do you mean by Yager's class of fuzzy union?

5. Define projection of the Relation.

6. Write down the formula for μ_{P*Q}?

7. Define Belief Measure.

8. Show that $Pl(A)+Pl(\overline{A}) \geq 1$.

9. Prove that $H(X|Y) = H(X,Y) - H(Y)$

10. Write down the formula for $I(X|Y)$ and $I(Y|X)$.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Find the scalar and fuzzy cardinality of the fuzzy set $A = \frac{0.4}{v} + \frac{0.2}{w} + \frac{0.5}{x} + \frac{0.4}{y} + \frac{1}{z}$.

Or

(b) Write the truth values and truth tables of $(\overline{a} \wedge b) \Rightarrow c$ in L_3.

12. (a) Prove that every fuzzy complement has atmost one equilibrium point.

Or

(b) Prove that for all $a, b \in [0,1]$ $i(a,b) \leq \min(a,b)$.

13. (a) Let R be a binary relation defined by the following membership matrix $M_R = \begin{bmatrix} 0.7 & 0.4 & 0 \\ 0.9 & 1 & 0.4 \\ 0 & 0.7 & 1 \\ 0.7 & 0.9 & 0 \end{bmatrix}$ Obtain its resolution form.

Or

(b) Let $M_p = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 0.7 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.4 & 0.5 \end{pmatrix} \end{array}$ and $M_Q = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cc} \alpha & \beta \end{array} \\ \begin{pmatrix} 0.6 & 0.8 \\ 0 & 1 \\ 0 & 0.9 \end{pmatrix} \end{array}$

Find max-min composition.

14. (a) Determine the possibility distribution, possibility measure and necessity measure. Given the basic assignment $m = (0.1, 0.1, 0.1, 0, 0.1, 0.2, 0.2, 0.2)$

Or

(b) Show that every possibility measure can be uniquely determined by a possibility distribution function.

15. (a) Explain in detail about measures of fuzziness.

Or

(b) Consider two fuzzy sets, A and B defined on the set of real numbers $X = [0, 4]$ by the membership grade functions $\mu_A(x) = \dfrac{1}{1+x}$ and $\mu_B(x) = \dfrac{1}{1+x^2}$. To calculate the actual degree of fuzzy sets of these sets. Let us use the Hamming distance $w = 1$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that all α-cuts of any fuzzy set defined on $\mathbb{R}^n$ $(n \geq 1)$ are convex if and only if $\mu_A[\lambda r + (1-\lambda)s] \geq 1$ $\forall\ r, s \in \mathbb{R}^n$, $\lambda \in [0,1]$.

17. Prove that

$$\lim_{w \to \infty} (a,b) = \lim_{w \to \infty} \min(1, (a^w + b^w)^{1/w}) = \max(a,b).$$

18. Solve the following fuzzy relation equation

$$p_0 \begin{bmatrix} 0.9 & 0.6 & 1 \\ 0.8 & 0.8 & 0.5 \\ 0.6 & 0.4 & 0.6 \end{bmatrix} = [0.6 \quad 0.6 \quad 0.5].$$

19. Prove : Given a consonant body of evidence ($\mathcal{F}$, m), the associated consonant belief and plausibility measures possess the following properties :

(a) $Bel(A \cap B) = \min[Bel(A), Bel(B)]$ for all $A, B, \in \mathcal{P}(X)$.

(b) $Pl(A \cup B) = \max[Pl(A), Pl(B)]$ for all

$A, B, \in \mathcal{P}(X)$.

20. Let m_x and m_y be the marginal basic assignments on set x and y respectively, and let m be a joint basic assignment on $x \times y$ such that $m(A \times B) = M_x(A), m_y(B)$ for all $A \in \mathcal{P}(x)$ and $B \in \mathcal{P}(y)$. Prove that $E(m) = E(m_x) + E(m_y)$.

F–6329

Sub. Code
7MMA3E6

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021.

Third Semester

Mathematics

Elective : COMBINATORIAL MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an exponential generating function of the sequence $(a_0, a_1, a_2, ..., a_r, ...)$.

2. Write down all partitions of the integer 6.

3. Define a linear recurrence relation with an example.

4. Define a recurrence relation with an example.

5. How many ways to place one non taking rook, two non taking rooks and three or more non taking rooks in a staircase chessboard?

6. What is mean by permutations with forbidden positions?

7. Give an example for an equivalence relation.

8. Define a store enumerator of the set R.

9. Define a balanced incomplete block design of a set.

10. Define a distance between two code words with an example.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that the ordinary generating function of the sequence $\binom{0}{0}, \binom{2}{1}, \binom{4}{2}, \binom{6}{3}, \ldots, \binom{2r}{r} \ldots\ldots$ is $(1-4x)^{-1/2}$.

Or

(b) Find the number of r-digit quaternary sequences that contain an even number of 0's.

12. (a) Solve the recurrence relation $a_n = a_{n-1} - a_{n-2}$ with boundary conditions $a_1 = 1, a_2 = 0$.

Or

(b) Solve the difference equation $a_n + 2a_{n-1} = n + 3$ with boundary condition $a_0 = 3$.

13. (a) Find the number of integers between 1 and 250 that are not divisible by any of the integers 2, 3, 5 and 7.

Or

(b) Find the number of derangements of n objects.

14. (a) Find the number of distinct strings of length 2 that are made up of blue beads and yellow beads.

Or

(b) In how many ways can five books, two of which are the same, be distributed to four children, if among them there is a set of identical twins?

15. (a) Prove that, in a balanced incomplete block design, $bk = vr$ and $r(k-1) = \lambda(v-1)$.

Or

(b) Prove that there are atmost $n-1$ Latin squares in a set of orthogonal Latin squares of order n.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Evaluate the sum $\sum_{i=0}^{r} \frac{r!}{(r-i+1)!(i+1)!}$

17. Find the number of n-digit binary sequences that have the pattern 010 occurring for the first time at the n^{th} digit.

18. Find the number of permutations of the letters $\alpha, \alpha, \beta, \beta, \gamma$ and γ so that no α appears in the first and second positions, no β appears in the third position, and no γ appears in the fifth and sixth position.

19. State and prove Burside's theorem.

20. Prove that, for $n \geq 3$ and $n = p^{\alpha}$, where p is a prime number and α is a positive integer, there exists a set of $n-1$ orthogonal Latin squares of order n.

F–6331

Sub. Code
7MMA4C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Network.

2. What is the difference between PERT and CPM?

3. Define Periodic review.

4. Define effective lead time.

5. Define the lack of memory in queuing system.

6. Define Poisson distribution.

7. State Little's formula.

8. Define aspiration level.

9. Write a short notes on dichotomous search method.

10. Define separable.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Write the Minimal spanning tree algorithm.

Or

(b) Construct the network diagram comprising activities $B, C \ldots Q$ and N such that the following constraints are satisfied.

$B < E, F$; $C < G, L$; $E, G < H$; $L, H < I$; $L < M$; $H < N$, $H < J, I, J < P$; $P < Q$. The notation $X < Y$ means that the activity X must be finished before Y can begin.

12. (a) Enumerate the no-setup model.

Or

(b) Explain in detail about general dynamic programming algorithm.

13. (a) Describe the basis elements of a queuing model.

Or

(b) Explain in detail about Pure Death model.

14. (a) Explain in detail about multiple server model with real life application.

Or

(b) Explain in detail about cost model.

15. (a) Solve maximize $f(x) = \begin{cases} 3x & 0 \le x \le 2 \\ \frac{1}{3}(-x + 20) & 2 \le x \le 3 \end{cases}$

using golden section method. Given the maximum value of $f(x)$ occurs at $x = 2$ and $\Delta = 0.10$.

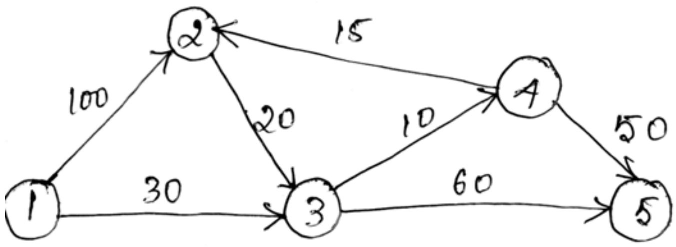
Or

(b) Explain in detail about Gradient method.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Determine the shortest routes between city 1 and each of the remaining four cities for the following network.



17. Neon lights on the U of A campus are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs \$ 100 to initiate a purchase order. A neon light kept in storage is estimated to cost about \$ .02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

18. Babies are born in a large city at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following

(a) The average number of births per year.

(b) The probability that no births will occur during 1 day.

(c) The probability of issuing 50 birth certificates in 3 hours, given that 40 certificates were issued during the first 2 hours of the 3 hour period.

19. Visitors parking at Ozark College is limited to five spaces only. Cars making use of this space arrive according to a Poisson distribution at the rate of 6 cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors who cannot find an empty space on arrival may temporarily wait inside the lot until a parked car leaves. The temporary space can hold only three cars other cars that cannot park or find a temporary waiting space must go elsewhere. Determine the following

(a) The probability, P_n of n cars in the system.

(b) The effective arrival rate for cars that actually are the lot.

(c) The average number of cars in the lot.

(d) The average time a car waits for a parking space inside the lot.

(e) The average utilization of the parking lot.

20. Solve the following problem using restricted basis method

Maximize $z = x_1 + x_2^4$

Subject to $3x_1 + 2x_2^2 \leq 9$

$x_1, x_2 \geq 0$.

F–6332

Sub. Code
7MMA4C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Fourth Semester

Mathematics

TOPOLOGY – II

(CBCS – 2017 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A — $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Riemann sphere.

2. Show that every locally compact Hausdroff space is completely regular.

3. Show that the rationals Q are not locally compact.

4. Define completely regular space.

5. Show that the irrationals are a G_δ set in R.

6. Define open refinement.

7. Give an example of metric for the product topology R^w.

8. Define equicontinuous function.

9. Show that in the compact-open topology $\mathcal{C}(x, y)$ is Hausdroff if y is Hausdroff.

10. Define Baire space with an example.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Show that if G is a locally compact topological group and H is a subgroup, then G/H is locally compact.

Or

(b) Prove that the quasi components of X equal the components of X if X is compact Hausdroff.

12. (a) Show that every Hausdroff topological group is completely regular.

Or

(b) Let X be completely regular; let $\beta(X)$ be its stone-Čech compactification. Prove that every bounded continuous real-valued function on X can be uniquely extended to a continuous real-valued function on $\beta(X)$.

13. (a) Let X be a metrizable space, then show that X has a basis that is countably locally finite.

Or

(b) Let G be a locally finite collection of sunsets of X. Then prove that $\overline{U_{A \in G} A} = U_{A \in G} \overline{A}$.

14. (a) Let (X,d) be a metric space. Show that there is an isometric imbedding of X into a complete metric space.

Or

(b) Let X be a compactly generated space; let (Y,d) be a metric space. Prove that $\mathcal{C}(X,Y)$ is closed in Y^X in the topology of compact convergence.

15. (a) If X is a compact Hausdroff space, or a complete metric space, then prove that X is a Baire space.

Or

(b) Let X be a space and let (Y,D) be a metric space. Prove that for the space $\mathcal{C}(X,Y)$, the compact-open topology and the topology of compact convergence coincide.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. State and prove Tychonoff theorem.

17. (a) Show that every continuous real-valued function defined on s_Ω is eventually constant.

(b) Show that one point compactification of s_Ω and the stone- Čech compactification are equivalent.

(c) Conclude that every compactification of s_Ω is equivalent to the one-point compactification.

18. Let X be a metrizable space. If G is an open covering of X then prove that there is a collection D of subsets of X such that:

(a) D is an open covering of X

(b) D is a refinement of G.

(c) D is countably locally finite

19. Let $I = [0,1]$. Show that there exists a continuous map $f : I \to I^2$ whose image fills up the entire square I^2.

20. State and prove Ascoli's theorem.

| F–6333 |

Sub. Code
7MMA4E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021.

Fourth Semester

Mathematics

Elective : ADVANCED STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a maximum likelihood estimator of θ.

2. Define an alternative hypothesis.

3. Define an unbiased minimum variance estimator of the parameter.

4. What is meant by an absolute-error loss function?

5. What is meant by Bayesian statistics?

6. Define an efficient estimator of the parameter.

7. Define a best critical region of size α for testing the simple hypothesis against the alternative simple hypothesis.

8. What is meant by likelihood ratio tests?

9. Give an example for not a quadratic form.

10. When we say that a random variable is a central chi-square variable?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $X_1, X_2, \ldots\ldots, X_n$ represent a random sample of the distribution having the p.d.f. $f(x;\theta) = \left(\frac{1}{\theta}\right) e^{-x/\theta}$, $0 < x < \infty,\ 0 < \theta < \infty,$ zero elsewhere. Find the *m.l.e.* $\hat{\theta}$ of θ.

Or

(b) A die was cast $n = 120$ independent times and the following data resulted.

Spots up	1	2	3	4	5	6
Frequency	b	20	20	20	20	40-b

If we use a chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at the 0.025 significance level?

12. (a) Let $X_1, X_2, \ldots. X_n$ denote a random sample from a normal distribution with mean zero and variance θ, $0 < \theta < \infty$. Show that $\sum_1^n X_i^2 / n$ is an unbiased estimator of θ and has variance $2\theta^2 / n$.

Or

(b) If $az^2 + bz + c = 0$ for more than two values of z, then $a = b = c = 0$. Use this result to show that the family $\{b\,(2, \theta) : 0 < \theta < 1\}$ is complete.

13. (a) Suppose that the random sample arises from a distribution with p.d.f. $f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1$, $\theta \in \Omega = \{0 : 0 < \theta < \infty\}$, zero elsewhere.

Show that $\hat{\theta}$ is asymptotically efficient.

Or

(b) Prove that $I_n(\theta) = nI(\theta)$.

14. (a) Let $X_1, X_2, ...X_{10}$ be a random sample of size 10 from a normal distribution $N(0, \sigma^2)$. Find a best critical region of size $\alpha = 0.05$ for testing $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 = 2$.

Or

(b) Let X have a Poisson distribution with mean θ. Find the sequential probability ratio test for testing $H_0 : \theta = 0.02$ against $H_1 : \theta = 0.07$.

15. (a) Compute the mean and the variance of a random variable that is $\chi^2(r, \theta)$.

Or

(b) Let $X_1, X_2, ..., X_n$ be a random sample from a normal distribution $N(\mu, \sigma^2)$. Show that $\sum_{i=1}^{n}(X_i - \overline{X})^2 = \sum_{i=2}^{n}(X_i - \overline{X'})^2 + \frac{n-1}{n}(X_i - \overline{X'})^2$, where $\overline{X} = \sum_{i=1}^{n} X_i / n$ and $\overline{X'} = \sum_{i=2}^{n} X_i / (n-2)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X have a p.d.f. of the form $f(x;\theta)=\theta x^{\theta-1}, 0<x<1$. Zero elsewhere, where $\theta \in \{\theta : \theta = 1,2\}$. T_0 test the simple hypothesis $H_0 : \theta = 1$ against the alternative simple hypothesis $H_1 : \theta = 2$, use a random sample X_1, X_2 of size $n = 2$ and define the critical region to be $C = \left\{(x_1, x_2) : \frac{3}{4} \le x_1 x_2\right\}$. Find the power function of the test.

17. State and prove the factorization theorem of Neyman.

18. State and prove the Rao-Cramer inequality.

19. State and prove Neyman-Pearson theorem.

20. State and prove Boole's inequality.

F–6516

Sub. Code
7MMA3E5

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Third Semester

Mathematics

Elective – STOCHASTIC PROCESSES

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   (10 × 2 = 20)

Answer **all** questions.

1. Define Martingale process.

2. Give an example of Gaussian process $x(t)$ is strictly stationary if $x(t)$ is covariance stationary.

3. State strong Markov property.

4. State first entrance theorem.

5. Define stationary distribution.

6. Find the eigen values of the two state Markov chain $P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$, $0 < a, b < 1$.

7. Define time dependent Poisson process.

8. What do you mean by Immigration-Death process.

9. State Blackwell's theorem.

10. Write any two applications of Renewal theorem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Consider the process $\{x(t), t \in T\}$ whose probability distribution under a certain condition, is given by

$$\Pr\{x(t)=n\}=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,\ldots \\ \dfrac{at}{1+at} & n=0\end{cases}$$

Show that the process $\{x(t), t \in T\}$ is not stationary.

Or

(b) Find the covariance function of $\{Y_n, n \geq 1\}$ given by $Y_n = a_0 x_n + a_1 x_{n-1} + \cdots + a_k x_{n-k}, n = 1,2,\ldots$

Where a's are constants and x_n's are uncorrelated random variables.

12. (a) Suppose that a ball is thrown at random to one of the 3 cells, Let $x_n (n \geq 1)$ be said to be in state $k (= 1,2,3)$ if after n throws k cells are occupied. Find p and p^n.

Or

(b) Let $\{y_n, n \geq 1\}$ be a sequence of independent random variables with $\Pr(y_n = 1) = p = 1 - \Pr(y_n = -1)$. Let x_n be defined by $x_0 = 0, x_{n+1} = x_n + y_{n+1}$. Examine whether $\{x_n, n \geq 1\}$ is a Markov chain. Find $\Pr\{x_n = k\}, k = 0,1,2,\ldots$

13. (a) If state j is persistent, then prove that for every state k that can be reached from $j, F_{kj} = 1$.

Or

(b) Classify the Markov chain whose transition matrix is

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{2} \end{pmatrix} \end{array}$$

14. (a) If $N_1(t), N_2(t)$ are two independent Poisson processes with parameters λ_1, λ_2 respectively, then show that $\Pr\{N_1(t) = k / N_1(t) + N_2(t) = n\} = \binom{n}{k} p^k q^{n-k}$ where $p = \frac{\lambda_1}{(\lambda_1 + \lambda_2)}, q = \frac{\lambda_2}{(\lambda_1 + \lambda_2)}$.

Or

(b) Show that the p.g.f. of a non-homogeneous process $\{N(t), t \geq 0\}$ is $Q(s,t) = \exp\{m(t)(s-1)\}$, where $M(t) = \int_0^t \lambda(x) dx$ is the expectation of $N(t)$.

15. (a) Show that the renewal function M satisfies the equation.

$M(t) = F(t) + \int_0^t M(t - x) dF(x)$.

Or

(b) Show that $\Pr\{Y(t) \leq x\} = F(t + x) - \int_0^t [1 - F(t + x - y)] d\, M(y)$ if $Y(t)$ is the distribution of residual life time.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let $X(t) = \sum_{r=1}^{k} (A_r \cos\theta_r t + B_r \sin\theta_r t)$, where A_r, B_r are uncorrelated random variables with mean θ, variance σ^2 and θ_r are constants. Show that $\{x(t), t \geq 0\}$ is covariance stationary.

(b) What is meant by a Martingale process?

17. Show that in a Markov chain with a finite state space S, there is at least one non-null persistent state and therefore atleast one invariant distribution.

18. State and prove Ergodic theorem.

19. Find the differential equation of pure death process. If the process starts with i individuals, find the mean and variance of the number $N(t)$ present at time t.

20. (a) State and prove central limit theorem for renewals.

(b) Derive Wald's equation.

F–5403

Sub. Code
7MMA3E3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

Third Semester

Mathematics

Elective — AUTOMATA THEORY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Draw a block diagram of a finite automation.
2. Write any one property of transition function.
3. Is two grammars of different types can generate the same language? Justify your answer.
4. What is a type 1 production?
5. Is a recursive set recursively enumerable? Justify your answer.
6. What is meant by concatenation? Give an example.
7. Define a regular set. Give an example.
8. Write any two application of pumping lemma.
9. Define a leftmost derivation.
10. Let G be a grammar $S \to sbs|a$. Is G ambiguous? Justify.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that for any transition functions δ and for any two input strings x and y, $\delta(q, xy) = \delta(\delta(q, x), y)$.

Or

(b) Construct a deterministic automation equivalent to $M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$, where δ is defined by its state table

State/ε	a	b
$\rightarrow q_0$	q_0, q_1	q_2
q_1	q_0	q_1
(q2)		q_0, q_1

12. (a) If $G = (\{S\}, \{0,1\}, \{S \rightarrow OS1, S \rightarrow \wedge\}, S)$, Find $L(G)$.

Or

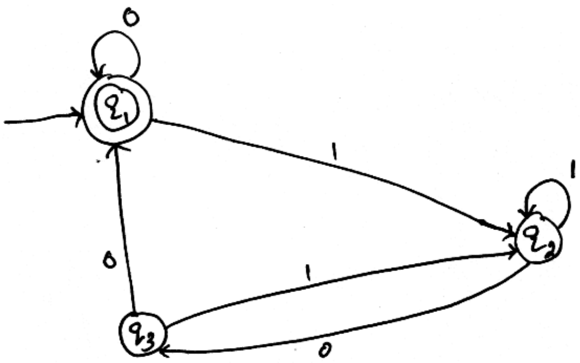
(b) Prove that every monotonic grammar G is equivalent to a type 1 grammar.

13. (a) Consider the grammar G given by $S \rightarrow OSA_1 2$, $S \rightarrow 012$, $2A_1 \rightarrow A_1 2$, $1A_1 \rightarrow 11$. Examine whether $001122 \in L(G)$. Justify your answer.

Or

(b) Show that L_0 is closed under the transpose operation.

14. (a) Construct a regular expression corresponding to the state diagram described by the following figure :



Or

(b) If x and y are regular sets over Σ, then prove that $x \cap y$ is also regular over Σ.

15. (a) If $A \overset{*}{\Rightarrow} w$ in G, then prove that there is a left most derivation of w.

Or

(b) Let G be $S \rightarrow AB, A \rightarrow a, B \rightarrow c/b$, $C \rightarrow D$, $D \rightarrow E$ and $E \rightarrow a$. Eliminate unit productions and get an equivalent grammar.

Part C (3 × 10 = 30)

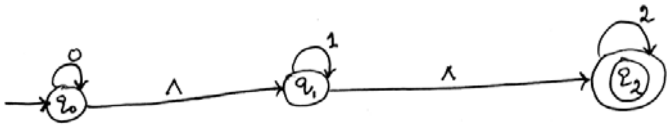
Answer any **three** questions.

16. If L is the set accepted by NDFA, then prove that there exists a DFA which also accepts L.

17. Construct a grammar G generating $\{a^n b^n c^n / n \geq 1\}$.

18. Prove that a context-sensitive language is recursive.

19. (a) Consider a finite automation, with $\wedge$-moves, given in the following figure. Obtain an equivalent automation without $\wedge$-moves.



(b) State and prove the pumping lemma for regular.

20. Let $G=(V_N, \Sigma, P, S)$ be a CFG. Prove that $S \overset{*}{\Rightarrow} \alpha$ if and only if there is a derivation tree for G with yield α.