| F–7344 |
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| Sub. Code |
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| 7MMA1C1 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**First Semester**

**Mathematics**

**ALGEBRA – I**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** (10 × 2 = 20)

Answer **all** questions.

1. Is $(Z,-)$ a group? Give reason.

2. Define a permutation group.

3. Define an external direct product of groups.

4. Prove that $Z(G)$ is a subgroup of $G$.

5. Define a division ring. Give an example.

6. If $\varphi : R \to R'$ define $\varphi(x) = x$ is a homomorphism find the kernel of $\varphi$.

7. Define two-sided ideal of a ring.

8. Define a maximal ideal of $R$.

9. When will you say that the polynomial is said to be integer monic?

10. Define an unique factorization domain.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) If $G$ is a finite group and $a \in G$, then prove that $O(a) / O(G)$.

Or

(b) State and prove the Cauchy's theorem for Abelian groups.

12. (a) If $p$ is a prime number and $p / O(G)$, then prove that $G$ has an element of order $p$.

Or

(b) Suppose that $G$ is the internal direct product of $N_1, N_2, \ldots N_n$. Prove that for $i \neq j$, $N_i \cap N_j = (e)$, and if $a \in N_i$, $b \in N_j$ then $ab = ba$.

13. (a) If $R$ is a ring, then prove that for all $a, b \in R$

(i) $a0 = 0a = 0$

(ii) $a(-b) = (-a)b = -(ab)$

(iii) $(-a)(-b) = ab$

Or

(b) If $\varphi$ is a homomorphism of $R$ into $R'$, then prove the following:

(i) $\varphi(0) = 0$

(ii) $\varphi(-a) = -\varphi(a)$ for every $a \in R$.

14. (a) If $U$ is an ideal of the ring $R$, then prove that $R/U$ is a ring and is a homomorphic image of $R$.

Or

(b) If $[a,b]=[a',b']$ and $[c,d]=[c',d']$, then prove that $[a,b][c,d]=[a',b'][c',d']$.

15. (a) Define a Euclidean ring. prove that a Euclidean ring possesses a unit element.

Or

(b) If $R$ is an integral domain, then prove that $R[x]$ is also integral domain.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Cayley's theorem.

17. State and prove second part of Sylow's theorem.

18. (a) If $p$ is a prime number then prove that $J_p$, the ring of integers mod $p$, is a field.

(b) Show that any field is an intogral domain.

19. Let $R$ be a commutative ring with unit element whose only ideals are (o) and R itself. Prove that R is a field.

20. State and prove Gauss' lemma.

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| F–7345 |
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| 7MMA1C2 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**First Semester**

**Mathematics**

**ANALYSIS – I**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** (10 × 2 = 20)

Answer **all** the questions.

1. Define compact.

2. State Weierstrass theorem.

3. If $P > 0$ prove that $\lim\limits_{n\to\infty}\frac{1}{n^P} = 0$.

4. State Root Test.

5. Define absolutely converges.

6. Define product series.

7. Suppose $E \subset X$, a metric space, $P$ is a limit point of $E$, $f$ and $g$ are complex functions on $E$ and $\lim\limits_{x\to P} f(x) = A$, $\lim\limits_{x\to P} g(x) = B$. Prove that $\lim\limits_{x\to P}(f+g)(x) = A + B$.

8. Define bounded function.

9. Suppose $f$ is differentiable in $(a,b)$ if $f'(x) \geq 0 \, \forall \, x \in (a,b)$, prove that $f$ is monotonically increasing.

10. Define differentiable function.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that every infinite subset of a countable set $A$ is countable.

Or

(b) Prove that compact subsets of metric spaces are closed.

12. (a) Suppose $\{s_n\}$ is monotonic. Prove that $\{s_n\}$ converges if and only if it is bounded.

Or

(b) Prove that $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

13. (a) Suppose

(i) $|C_1| \geq |C_2| \geq |C_3| \geq \ldots$

(ii) $C_{2m-1} \geq 0, \, C_{2m} \leq 0$ ; $m = 1,2,3 \ldots$

(iii) $\lim_{n \to \infty} C_n = 0$

Prove that $\sum C_n$ converges.

Or

(b) Given the power series $\sum C_n z^n$, put $\alpha = \lim_{n\to\infty}\sup \sqrt[n]{|C_n|}$, $R = \frac{1}{\alpha}$ if $\alpha = 0$, $R = +\infty$; if $\alpha = +\infty$, $R = 0$. Prove that $\sum C_n z^n$ converges if $|z| < R$ and diverges if $|z| > R$.

14. (a) Suppose $f$ is a continuous 1 – 1 mapping of a compact metric space $X$ onto a metric space $Y$. Prove that the inverse mapping $f^{-1}$ defined on $Y$ by $f^{-1}(f(x)) = x$, $x \in Y$ is a continuous mapping of $Y$ onto $X$.

Or

(b) Prove that a continuous function of a continuous function is continuous.

15. (a) Suppose $f$ is a continuous mapping of $[a,b]$ into $R^k$ and $f$ is differentiable in $(a,b)$. Prove that there exists $x \in (a,b)$ such that $|f(b) - f(a)| \le (b-a)|f'(x)|$.

Or

(b) State and prove intermediate value theorem.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Suppose $k \subset Y \subset X$. Prove that $k$ is compact relative to $X$ if and only if $k$ is compact relative to $Y$.

17. State and prove Ratio test.

18. Suppose (a) $\sum_{n=0}^{\infty} a_n$ converges absolutely (b) $\sum_{n=0}^{\infty} a_n = A$ (c) $\sum_{n=0}^{\infty} b_n = B$ (d) $C_n = \sum_{k=0}^{n} ak\, b_{n-k}$ , $n = 0,1,2\ldots$ Prove that $\sum_{n=0}^{\infty} C_n = AB$.

19. Prove that inverse image of an open set is open.

20. State and prove Taylor's theorem.

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| F–7346 |
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| 7MMA1C3 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**First Semester**

**Mathematics**

**DIFFERENTIAL GEOMETRY**

**(CBCS – 2017 onwards)**

Time : 3 Hours                Maximum : 75 Marks

**Part A**                $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the principle normal at $p$.

2. Define a cylindrical helix.

3. What is meant by right helicoid?

4. Define direction coefficients.

5. Write down the canonical equations for geodesics.

6. State the Whitehead theorem.

7. Define the geodesic curvature vector.

8. What is meant by geodesic?

9. Write down the second fundamental form a surface.

10. Define the characteristic line.

## Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that $\left[\dot{\vec{r}}, \ddot{\vec{r}}, \dddot{\vec{r}}\right] = 0$ is a necessary and sufficient condition that the curve be plane.

Or

(b) Show that the torsion of an involute of a curve is equal to $\dfrac{\rho(\sigma\rho' - \sigma'\rho)}{(\rho^2 + \sigma^2)(c - s)}$.

12. (a) Find the unit normal on the surface of revolution $\vec{r} = (g(u)\cos v, g(u)\sin v, f(u))$

Or

(b) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the places $z =$ constant.

13. (a) Prove that the curves of the family $v^3/u^2 =$ constant are geodesics on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2$ $(u > 0, v > 0)$.

Or

(b) Narrate the following terms:

(i) Geodesic parallels;

(ii) Geodesic polar.

14. (a) Prove that the components $\lambda, \mu$ of the geodesic curvature vector are given by the following formulae with $s$ as parameter.

$$\lambda = \frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial v'} = -\frac{1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial v'};$$

$$\mu = \frac{1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial u'} = -\frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial u'}.$$

Or

(b) Find the Gaussian curvature at the point $(u,v)$ of the anchor ring and show that the total curvature of the whole surface is zero.

15. (a) State and prove the Euler's theorem.

Or

(b) Show that developable consists of two sheets which are tangent to the edge of regression along a sharp edge which are tangent to the edge of regression along a sharp edge.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces

$$ax^2 + by^2 + cz^2 = 1,\ a^1x^2 + b^1y^2 + c^1z^2 = 1.$$

17. A helicoids is generated by the screw motion of a straight line which meets the axis at an angle $\alpha$. Find the orthogonal trajectories of the generators. Find also the metric of the surface referred to the generators and their orthogonal trajectories as parametric curves.

18. State and prove the Christoffel symbols of the second kind.

19. Derive the Liouville's formula for Kg.

20. Show that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

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| F–7348 |
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| 7MMA1E1 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**First Semester**

**Mathematics**

**Elective – NUMBER THEORY**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define composite number. Give an example.

2. State the Euclidean algorithm.

3. Define the Mangoldt function $\lambda(n)$.

4. State the Selberg identity.

5. If $n$ is a positive integer and $x$ is real number, then prove that $[x+n]=[x]+n$.

6. When will you say that two lattice points are mutually visible?

7. Is the linear congruence $2x \equiv 3 (\bmod 4)$ has no solution? Justify your answer.

8. State the Wolstenwholme's theorem.

9. Find the quadratic residues modulo 11.

10. What is meant by Legendre symbol?

**Part B** (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If $2^n + 1$ is prime, then prove that $n$ is a power of 2.

Or

(b) State and prove the fundamental theorem of arithmetic.

12. (a) If $n \geq 1$, then prove that $\phi(n) = n \prod_{p|n}\left(1 - \frac{1}{p}\right)$.

Or

(b) If $f$ and $g$ are arithmetical functions, then prove that

(i) $(f + g)' = f' + g'$

(ii) $(f * g)' = f' * g + f * g'$.

13. (a) For all $x \geq 1$, prove that $\left|\sum_{n \leq x} \frac{\mu(n)}{n}\right| \leq 1$, with equality holding only if $x < 2$.

Or

(b) If $a$ and $b$ are positive real numbers such that $ab = x$, then prove that

$$\sum_{\substack{q,d \\ qd \leq x}} f(d)g(q) = \sum_{n \leq a} f(n)G\left(\frac{x}{n}\right) + \sum_{n \leq b} g(n)F\left(\frac{x}{n}\right) - F(a)G(b)$$

14. (a) Solve the congruence $5x \equiv 3 \pmod{24}$.

Or

(b) State and prove the Chinese Remainder theorem.

15. (a) State and prove Euler's criterion.

Or

(b) Determine whether 888 is a quadratic residue or non-residue of the prime 1999.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. (a) Given any two integers $a$ and $b$, prove that there is a common divisor $d$ of $a$ and $b$ of the form $d = ax + by$, where $x$ and $y$ are integers. Moreover, every common divisor of $a$ and $b$ divides this $d$.

(b) If $(a,b)=1$ and if $d|(a+b)$, then prove that $(a,d)=(b,d)=1$.

17. (a) Define the Liouville's function $\lambda(n)$. For every $n \geq 1$, prove that $\sum_{d|n} \lambda(d) = \begin{cases} 1 & \textit{if } n \textit{ is a square,} \\ 0 & \textit{otherwise} \end{cases}$. Also prove $\lambda^{-1}(n) = |\mu(n)|$ for all $n$.

(b) If $f$ is multiplicative then prove that $f(1)=1$.

18. Derive the Dirichlet's asymptotic formula for the partial sums of the divisor function $d(n)$.

19. State and prove the Lagrange theorem.

20. If $p$ and $q$ are distinct odd primes, then prove that $(p|q)(q/p) = (-1)^{(p-1)(q-1)/4}$.

**F–7349**

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| 7MMA2C1 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Second Semester**

**Mathematics**

**ALGEBRA – II**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a vector space. Give an example.

2. Define linearly independent.

3. What is meant by dual space?

4. Define an orthonormal set.

5. Define algebraic of degree $n$ over $f$.

6. What is meant by splitting field over $F$?

7. Define an automorphism of the field $k$.

8. Define normal extension of $F$.

9. Define the rank of $T$.

10. Define skew-Hermitian. give an example.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that the intersection of two subspaces of $v$ is a subspace of $v$.

Or

(b) If $v_1, v_2, ... v_n \in v$ are linearly independent, then prove that every element in their linear span has a unique representation in the form $\lambda_1 v_1 + ... + \lambda_n v_n$ with the $\lambda_i \in F$.

12. (a) Prove that $A(W)$ is a subspace of $\hat{v}$.

Or

(b) If $u, v \in v$ then prove that $|\langle u, v\rangle| \leq \|u\| \cdot \|v\|$.

13. (a) Prove that a polynomial of degree $n$ over a field can have at most $n$ roots in any extension field.

Or

(b) For any $f(x), g(x) \in F[x]$ and any $\alpha \in F$, prove the following:

(i) $(f(x) + g(x))' = f'(x) + g'(x)$

(ii) $(\alpha f(x))' = \alpha f'(x)$

(iii) $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

14. (a) If $K$ is a field and if $\sigma_1, \sigma_2, ..., \sigma_n$ are district automorphisms of $k$, then prove that it is impossible to find elements $a_1, a_2, ... a_n$, not all 0, in $K$ such that $a_1\sigma_1(u) + a_2\sigma_2(u) + ... a_n\sigma_n(u) = 0$ for all $u \in k$.

Or

(b) Prove that $k$ is a normal extension of $F$ if and only if $k$ is the splitting field of some polynomial over $F$.

15. (a) If $V$ is finite-dimensional over $F$, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for $T$ is not 0.

Or

(b) If $T \in A(V)$ is such that $(vT, v) = 0$ for all $u \in v$ then prove that $T = 0$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. If $V$ is finite dimensional and if $W$ is a subspace of $V$, then prove that $W$ is finite dimensional. $\dim W \leq \dim V$ and $\dim \frac{V}{W} = \dim V - \dim W$.

17. If $V$ is finite dimensional and $W$ is a subspace of $V$, then prove that $\hat{W}$ is isomorphic to $\hat{V} / A(W)$ and $\dim A(W) = \dim V - \dim W$.

18. If $L$ is a finite extension of $k$ and if $k$ is a finite extension of $F$, then prove that $L$ is a finite extension of $F$. Also prove that $[L:F]=[L:k][k:F]$.

19. If $K$ is a finite extension of $F$, then prove that $G(K, F)$ is finite group and its order $O(G(K, F))$ satisfies $O(G(K, F)) \leq [K:F]$.

20. If $T \in A(V)$ has all its characteristic roots in $F$, then prove that there is a basis of $V$ in which the matrix of $T$ is triangular.

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| F–7350 |
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| 7MMA2C2 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Second Semester**

**Mathematics**

**ANALYSIS — II**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define Riemann integral.

2. State theorem of integration by parts.

3. Define pointwise convergence of a function.

4. Define equicontinuous function.

5. Define Trignometric polynomial.

6. Define Dirichlet kernel.

7. Define Lebesgue measurable.

8. Define characteristic function.

9. Define step function.

10. Let $f$ be a non negative measurable function. Show that $\int f = 0$ implies $f = 0$ almost everywhere.

## Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If $f$ is continuous on $[a, b]$ then prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$.

Or

(b) State and prove theorem on change of variable.

12. (a) If $k$ is a compact metric space and if $f_n \in \mathscr{C}(k)$ for $n = 1, 2, 3, ..,$ if $\{f_n\}$ converges uniformly on $k$ then prove that $\{f_n\}$ is equi continuous on $k$.

Or

(b) Prove that there exist a real continuous function on the real line which is nowhere differentiable.

13. (a) State and prove Taylor's theorem.

Or

(b) If $\Sigma C_n$ converges and let $f(x) = \sum_{n=0}^{\infty} C_n x^n \ (-1 < x < 1)$

then prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} C_n$ .

14. (a) Prove that the interval $(a, \infty)$ is measurable.

Or

(b) Let $\{A_n\}$ be a countable collection of sets of real numbers then prove that $m^*(\cup A_n) \leq \Sigma m^* A_n$ .

15. (a) State and prove Fatou's lemma.

Or

(b) State and prove Lebesgue convergence theorem.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Assume $\alpha$ increases monotonically and $\alpha' \in \mathscr{R}$ on $[a,b]$. Let $f$ be a bounded real function on $[a,b]$. Then prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha' \in \mathscr{R}$. Also Prove that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)\,dx$.

17. State and prove the stone-Weierstrass theorem.

18. State and prove the Parseval's theorem.

19. Prove that the outer measure of an interval is its length.

20. State and prove Bounded convergence theorem.

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| F–7351 |
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| 7MMA2C3 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Second Semester**

**Mathematics**

**PARTIAL DIFFERENTIAL EQUATIONS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Find the orthogonal trajectories on the surface $x^2 + y^2 + 2fyz + d = 0$ of its curves of intersection with planes parallel to the plane $x \circ y$.

2. Write down a pfaffian differential equation.

3. Eliminate the arbitrary function $f$ form the equation $z = x + y + f(xy)$.

4. Define a complete integral of the partial differential equation.

5. Find the complete integral of the equation $z^2 = pq\,xy$.

6. Write down the Clairaut equation.

7. Find the particular integral of the equation $(D^2 - D')z = 2y - x^2$.

8. Classify the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.

9. Show that $r\cos\theta$ satisfying the Laplace equation, when $r, \theta$ and $\varphi$ are spherical polar coordinates.

10. Write a short notes on exterior Neumann problem.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the integral curves of the equation $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

Or

(b) Verify that the differential equation $\left(y^2 + yz\right)dx + \left(xz + z^2\right)dy + \left(y^2 - xy\right)dz = 0$ is integrable and find its primitive.

12. (a) If $u$ is a function of $x, y$ and $z$ which satisfies the partial differential equation

$(y-z)\frac{\partial y}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial y}{\partial z} = 0$. Show that $u$ contains $x, y$ and $z$ only in combinations $x+y+z$ and $x^2 + y^2 + z^2$.

Or

(b) Find the general integral of the partial differential equation $(2xy-1)p + \left(z - 2x^2\right)q = 2(x - yz)$ and also the particular integral which passes through the line $x = 1,\ y = 0$.

13. (a) Find the condition that two partial differential equations are compatible.

Or

(b) Solve the equation $p^2x + q^2y = z$ using Jacobi's method.

14. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

Or

(b) Derive the solution of the equation :

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} = 0 \text{ for the region } r \geq 0,\ z \geq 0,$$

satisfying the conditions :

(i) $v \to 0$ as $z \to \infty$ and as $r \to \infty$

(ii) $v = f(r)$ on $z = 0, r \geq 0$.

15. (a) (i) What is meant by a boundary value problem for Laplace's equation?

(ii) Explain about the types of Dirichlet problem.

Or

(b) Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r,\theta)$.

## Part C (3 × 10 = 30)

Answer any **three** questions.

16. Verify that the equation $z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$ is integrable and find its primitive.

17. Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the $x$-axis.

18. (a) Solve the equation $(p^2 + q^2)y = qz$ using Charpit's method.

(b) Find a complete integral of the equation $p^2x + qy = z$, and hence derive the equation of an integral surface of which the line $y = 1$, $x + 2 = 0$ is a generator.

19. Solve the one-dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

20. The paints of trisection of a string are pulled side through a distance on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest.

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| F–7352 |
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| Sub. Code |
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| 7MMA2C4 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Second Semester**

**Mathematics**

**MECHANICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by holonomic system? Give an example.

2. Define the potential energy.

3. Write down the Christoffel symbol of the first kind.

4. Write down the standard form Lagrange's equation.

5. State the Hamilton's principle.

6. State the Jacobi's form of the principle of least action.

7. What is meant by Hamilton's principal function?

8. Write down the modified Hamilton-Jacobi equation.

9. Define a Canonical transformation.

10. When will you say that the transformation is said to be Mathieu transformation?

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Enumerate the following term:

(i) Virtual displacement;

(ii) Virtual work

Or

(b) With the normal notations, prove that

$$\vec{H} = \vec{r}_c + m\dot{\vec{r}}_c + \sum_{i=1}^{N} \vec{P}_i \times m_i \dot{\vec{P}}_i .$$

12. (a) A particle of mass $m$ can slide without friction on the inside of a small tube which is bent in the form of a circle of radius $r$. the tube rotates about a vertical diameter with a constant angular velocity $w$. Find the differential equation of motion.

Or

(b) Suppose a mass-spring system is attached to a frame which is translating with a uniform velocity $v_0$. Let $l_0$ be the unstressed spring length and use the elongation $x$ as the generalized coordinate. Find the Jacobi integral for the system.

13. (a) Derive the Euler-Lagrange equation in the form

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 .$$

Or

(b) A particle of mass $m$ is attracted to a fixed point $o$ by an inverse square force, that is $F_r = \frac{-\mu m}{r^2}$, where $\mu$ is the gravitational coefficient. The plane polar co-ordinates $(r, \theta)$ are used to described the position of the particle. Find the equations of motion.

14. (a) Discuss the Pfaffian differential forms. Also deduce that the Hamilton's canonical equations.

Or

(b) State the prove the Stackel's theorem.

15. (a) Define a homogeneous canonical transformation and analyse the generating functions associated with such a transformation.

Or

(b) Derive the Jacobi's identify.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the Konig's therem.

(b) Find the Kinetic energy of a system in terms of the motion with respect to an arbitrary reference point P.

17. Derive Lagrange's equations in the standard form for a nonholonomic system.

18. Using Jacobi's form of the principle of least action, obtain the orbit for the Kepler problem in the form $r = \dfrac{c^2 / \mu m^2}{1 + \sqrt{1 + 2hc^2 / \mu^2 m^3 \cos\theta}}$, with an eccentricity $e = \sqrt{1 + 2hc^2 / \mu^2 m^3}$ .

19. State and prove the Jacobi's theorem.

20. Show that the transformation $Q = \log\left(\dfrac{\sin p}{p}\right)$, $P = q \cot p$ is canonical. Obtain the four major types of generating functions associated with this canonical transformation.

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**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Second Semester**

**Mathematics**

**Elective – GRAPH THEORY**

**(CBCS – 2017 onwards)**

Time : 3 Hours — Maximum : 75 Marks

**Part A** — $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Vertex - transitive of a graph G. Give an example.

2. What is meant by edge cut of graph G? Give an example.

3. Define a block of a graph. Give an example.

4. Draw a graph which is Eulerian but not Hamiltonian.

5. Find the number of different perfect matching in $K_{n,\,n}$.

6. Find the edge chromatic number of a Petersen graph.

7. Determine $r(2, l)$.

8. Define a critical graph with an example.

9. State the Jordan Curve theorem for planar graph.

10. Define dual graphs. Give an example.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Define a degree of vertex of graph G. Prove that in any graph, the number of vertices of odd degree is even. Also prove $\delta \leq 2E/\nu \leq \Delta$.

Or

(b) Define a tree of graph G. If G is a tree then prove that $\varepsilon = \nu - 1$.

12. (a) Prove that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.

Or

(b) Let G be a simple graph and let $u$ and $\nu$ be nonadjacent vertices in G such that $d(u) + d(\nu) \geq \nu$. Prove that G is Hamiltonian if and only if $G + \upsilon\nu$ is Hamiltonian.

13. (a) State and prove the Marriage theorem.

Or

(b) If G is bipartite graph, then prove that $x' = \Delta$.

14. (a) With the usual notations, prove that $r(k,l) \leq \binom{k+l-2}{k-1}$.

Or

(b) If G is k-critical, then prove that $\delta \geq k - 1$.

15. (a) Define a planar graph with an example. Prove that $k_5$ is nonplanar.

Or

(b) If two bridges overlap, then prove that either they are skew or else they are equivalent 3-bridges.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. With the usual notations, prove that $\tau(k_n) = n^{n-2}$.

17. Prove that a graph G with $\nu \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths.

18. State and prove the Berge theorem.

19. If G is a connected simple graph and is neither an odd cycle nor a complete graph, then prove that $\chi \leq \Delta$.

20. State and prove the five-colour theorem.

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**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Third Semester**

**Mathematics**

**COMPLEX ANALYSIS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** (10 × 2 = 20)

Answer **all** questions.

1. Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} Z^n/n!$.

2. Prove that the reflection $z \to \bar{z}$ is not a linear transformation.

3. Compute $\int_{|z|=2} z^n (1-z)^m dz$.

4. State the Liouville's theorem.

5. Define a removable singularity. Give an example.

6. State the Weierstrass theorem for an essential singularity.

7. Find the residues of $\frac{1}{z^2+5z+6}$ and its poles.

8. State the argument principle theorem.

9. Write down the power series expansion of arc sin $z$.

10. Define the canonical product with an example.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that a harmonic function satisfies the formal differential equation $\frac{\partial^2 u}{\partial z \partial \overline{z}} = 0$.

Or

(b) If $T_1(z) = \frac{z+2}{z+3}$, $T_2(z) = \frac{z}{z+1}$, find $T_1T_2(z)$, $T_2T_1(2)$ and $T_1^{-1}T_2(z)$.

12. (a) Write the usual notations, prove that $\left|\int_a^b f(t)dt\right| \leq \int_a^b |f(t)|dt$. Also find $\int_{|z|=1} |z-1||dz|$.

Or

(b) State and prove the Morera's theorem.

13. (a) State and prove the Taylor's theorem.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the residue theorem.

Or

(b) Evaluate $\int_0^\infty \frac{x^2 dx}{x^4 + 5x^2 + 6}$.

15. (a) State and prove the Hurwitz theorem.

Or

(b) Define an entire function with an example. Also prove that every function which is meromorphic in the whole plane is the quotient of two entire functions.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. (a) Define the cross ratio. If $z_1, z_2, z_3, x_4$ are distinct points in the extended plane and T any linear transformation, then prove that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1, z_2, z_3, z_4)$.

(b) State and prove the Abel's limit theorem.

17. State and prove the Cauchy's theorem for a rectangle.

18. State and prove the Schwarz lemma.

19. Evaluate $\int_0^{\pi} \log \sin\theta \, d\theta$.

20. State and prove the Laurent series.

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**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Third Semester**

**Mathematics**

**TOPOLOGY – I**

**(CBCS – 2017 onwards)**

Time : 3 Hours — Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define basis for a topology.
2. Define Hausdorff space.
3. State pasting lemma.
4. Define metric topology.
5. Is the subset $X = \{x \times y / y = 0\} \cup \{x \times y / x > 0$ and $y = \frac{1}{x}\}$ connected or not? Justify your answer.
6. Define locally path connected space.
7. Prove that continuous image of compact space is compact.
8. State uniform continuity theorem.
9. Define Lindelof's space.
10. State the Urysohn lemma.

## Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $X$ be a topological space. Suppose that @ is a collection of open sets of $X$ such that for each $x$ in $X$ and each open set $U$ of $X$ contains $x$ there is an element $C$ of @ such that $x \in C \subset U$. Then prove that @ is a basis for the topology of $X$.

Or

(b) Let $A$ be a subset of topological space X, let $A'$ be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$.

12. (a) Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each $\alpha$. Let $\prod X_\alpha$ have the product topology. Then prove that the function $f$ is continuous if and only if each function $f_x$ is continuous.

Or

(b) Let $X$ be a metric space with matrix $d$; $d : X \times X \to R$ defined as $d(x, y) = \min\{d(x, y), 1\}$. Then prove that d is a metric that induces the topology of $X$.

13. (a) State and prove intermediate value theorem.

Or

(b) Prove that a space $X$ is locally connected if and only if for every open set $U$ of $X$ each component of $U$ is open in $X$.

14. (a) Let $Y$ be a subspace of $X$. Then prove that $Y$ is compact if and only if every covering of $Y$ by sets open in $X$ contains a finite subcollection covering $Y$.

Or

(b) Prove that compactness implies limit point compactness but not conversely.

15. (a) Suppose that $X$ has a countable basis. Then prove that

(i) Every open covering of $X$ contains a countable subcollection covering $X$.

(ii) There exist a countable subset of $X$ which is dense in $X$.

Or

(b) Prove that every compact Hausdorff space is normal.

**Part C** $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Let $X$ be a topological space. Then prove that the following conditions hold :

(i) $\phi$ and $X$ are closed

(ii) Arbitrary intersections of closed sets are closed

(iii) Finite unions of closed sets are closed.

(b) Let $Y$ be a subspace of $X$, let $A$ be a subset of $Y$ and $\overline{A}$ denote the closure of $A$ in $X$. Then prove that the closure of A in Y equals $\overline{A} \cap Y$ .

17. Let X and Y be topological spaces and let $f : X \to Y$. Prove that the following are equivalent :

(a) $f$ is continuous

(b) for every subset $A$ of $X$, one has $f(\overline{A}) \subset \overline{f(A)}$

(c) for every closed set $B$ of $Y$, the set $f^{-1}(B)$ is closed in x

(d) for each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of X such that $f(U) \subset V$.

18. If $L$ is a linear continuum in the order topology, then prove that $L$ is connected and so is every interval and ray in $L$.

19. Prove that the product of finitely many compact spaces is compact.

20. State and prove Urysohn metrization theorem.

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**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Third Semester**

**Mathematics**

**PROBABILITY AND STATISTICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. If $c_1$ and $c_2$ are subsets of @ such that $c_1 \subset c_2$, then prove that $P(c_1) \leq P(c_2)$.

2. Let X be a random variable such that $\Pr(X \leq 0) = 0$ and let $\mu = E(x)$. Show that $\Pr(X \geq 2\mu) \leq 1/2$.

3. Let $f(x_1, x_2) = \begin{cases} 4x_1 x_2 & 0 < x_1 < 1, \quad 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$ be the joint p.d.f. of $x_1$ and $x_2$. Find $\Pr(0 < x_1 < 1/2, 1/4 < x_2 < 1)$.

4. Let the random variables $x_1$ and $x_2$ have the joint p.d.f. described as follows

| $(x_1, x_2)$ | (0, 0) | (0, 1) | (0, 2) | (1, 0) | (1, 1) | (1, 2) |
|---|---|---|---|---|---|---|
| $f(x_1, x_2)$ | $\frac{2}{12}$ | $\frac{3}{12}$ | $\frac{2}{12}$ | $\frac{2}{12}$ | $\frac{2}{12}$ | $\frac{1}{12}$ |

and $f(x_1, x_2)$ is equal to zero elsewhere. Write these probabilities in a rectangular array recording each marginal p.d.f. in the margins.

5. Define Binomial distribution.

6. If the moment generating function of a random variable X is $\left(\frac{1}{3}+\frac{2}{3}e^t\right)^5$, find $\Pr(X=2 \text{ or } 3)$.

7. Let $X$ have the p.d.f. $f(x)=\begin{cases} x^2/9, & 0<x<3 \\ 0 & \text{elsewhere} \end{cases}$ find the p.d.f. of $Y=X^3$.

8. Let $\overline{X}$ be the mean of a random sample of size 25 from a distribution $N(75, 100)$ find $pr(71 < \overline{x} < 79)$.

9. Define convergence in distribution.

10. Let $y$ be $b\left(72, \frac{1}{3}\right)$. Find $\Pr(22 \le y \le 28)$.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $X$ have the p.d.f. $f(x)=\begin{cases} 2(1-x), & 0<x<1 \\ 0 & \text{elesewhere} \end{cases}$. Find $E(X)$, $E(X^2)$ and $E(6X+3X^2)$.

Or

(b) Let X be a random variable such that $E\left[(x-b)^2\right]$ exists for all real b. Show that $E\left[(x-b)^2\right]$ is a minimum when $b=E(x)$.

12. (a) Let $f(x_1, x_2) = \begin{cases} 21 x_1^2 \; x_2^3, & 0 < x_1 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$ be the joint p.d.f. of $X_1$ and $X_2$. Find the conditional mean and variance of $X_1$ given $X_2 = X_2$, $0 < x_2 < 1$.

Or

(b) State and prove the theorem for the random variable $X_1$ and $X_2$ to be independent.

13. (a) The m.g.f of a random variable $X$ is $\left(\frac{2}{3} + \frac{1}{3}e^t\right)^9$.

Show that

$$\Pr(\mu - 2\sigma < x < \mu + 2\sigma) = \sum_{x=1}^{5} \binom{9}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{9-x}.$$

Or

(b) Let X have a gamma distribution with $\alpha = r/2$ where $r$ is a positive integer and $\beta > 0$. Find the distribution function for the random variable $y = 2x/\beta$.

14. (a) If $x_i = i$, $i = 1, 2, ..., n$, compute the values of $\bar{x} = \Sigma x_i / n$ and $s^2 = \Sigma (x_i - \bar{x})^2 / n$.

Or

(b) Derive the mean and variance of beta distribution.

15. (a) Let $y_n$ denote the $n^{th}$ order statistic of a random sample from distribution

$f(x) = \begin{cases} \frac{1}{\theta} \; 0 < x < \theta, \; 0 < \theta < \infty \\ 0 \quad \text{elsewhere} \end{cases}$. Find the limiting distribution of $Z_n = n(\theta - y_n)$.

Or

(b) Let $z_n$ be $\chi^2(n)$ and let $y_n = (z_n - n)/\sqrt{2n}$. Find the limiting distribution of $y_n$.

## Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) State and prove the chebyshev's inequality.

(b) If X is a random variable such that $E(x)=3$ and $E(x^2)=13$, use Chebyshev's inequality to determine a lower bound for the probability $\Pr(-2<x<8)$.

17. Let $f(x, y)=\begin{cases} 2 & 0<x<y,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$ be the joint p.d.f. of x and y. Show that the conditional means are respectively $(1+x)/2$, $0<x<1$ and $y/2, 0<y<1$. Show that the correlation coefficient of $X$ and $Y$ is $\rho=1/2$.

18. (a) Derive the m.g.f. of normal distribution.

(b) Let x and y have a bivariate normal distribution with $\mu_x=2.8, \mu_y=110, \sigma_x{}^2=0.16$, $\sigma_y{}^2=100$ and $\rho=0.6$. Compute.

(i) $\Pr(106<y<124)$

(ii) $\Pr(106<y<124/x=3.2)$.

19. Let the independent random variables $x_1$ and $x_2$ have the same p.d.f.

$f(x)=\begin{cases} \frac{x}{6}, & x=1,2,3 \\ 0 & \text{elsewhere} \end{cases}$. Determine

(a) The joint p.d.f. of $x_1$ and $x_2$.

(b) $\Pr(x_1=2, x_2=3)$

(c) $\Pr(x_1+x_2=3)$

(d) m.g.f. of $y=x_1+x_2$.

20. State and prove the central limit theorem.

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**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Third Semester**

**Mathematics**

**Elective – DISCRETE MATHEMATICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Give an example of a subgroup which is not a monoid.

2. Define a sub semigroup with an example.

3. Calculate $F_4$ of the Fibonacci number using recursion.

4. Find the Characteristics equation of

   $J(k) - 4J(k-1) + 4J(k-2) = 0$.

5. What is meant by projection function?

6. Define a partial recursive of a set.

7. Define a bounded lattice.

8. Determine all the partial orders and draw their Harse diagrams on the set $L\{a,b,c\}$. Which of them are chains?

9. Define a Boolean polynomial.

10. Draw a 2-variable Karaugh map.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Show that there exists a homomorphism from the algebraic system $(N,+)$ to the system $(Z_4,+_4)$ where

(i) $N$ is the set of natural numbers and

(ii) $Z_4$ is the set of integers modulo 4. Is it an isomorphism?

Or

(b) For any commutative monoid $(M,*)$, prove that the set of idempotent elements of $M$ forms a submonoid.

12. (a) Prove by induction method, for $n \geq 1$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

Or

(b) Find the recurrence relation satisfying

$$y_n = A(3)^n + B(-4)^n$$

13. (a) Solve $T(K) - 7T(K-1) + 10T(K-2) = 6 + 8K$ with $T(0) = 1$ and $T(1) = 2$

Or

(b) Prove that $f(x,y) = x^y$ is primitive recursive.

14. (a) Prove that every chain is a lattice.

Or

(b) Show that every distributive lattice is modular. Is the converse true? Justify your answer.

15. (a) Find the principal disjunctive normal form of

$$p(x_1, x_2, x_3) = (x_2 + x_1x_3)(x_1 + x_3)x_2)$$

Or

(b) Consider the Boolean function

$$f(x_1, x_2, x_3) = ((x_1 + x_2) + (x_1 + x_3)).x_1.\bar{x}_2.$$

Simplify this function and draw the circuit gate diagram for it.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let $T$ be the set of all even integers, show that the semigroups $(z,+)$ and $(T,+)$ are isomorphic.

(b) Let $(S,*)$ and $(T,\Delta)$ be monoids with identities e and e' respectively. Let $g: S \to T$ be an onto (semigroup) homomorphism. Prove that $g(e) = e'$.

17. Solve the recurrence relation

$S(K) - 4S(K-1) - 11S(K-2) + 30S(K-3) = 0$, $S(0) = 0$, $S(1) = -35$, $S(2) = -85$.

18. Find the generating function of Fibonacci sequence.

19. Show that the direct product of any two distributive lattices is a distributive lattice.

20. Simplify

$$f(a,b,c,d,e) = \Sigma(0,1,3,8,9,13,14,15,16,17,19,24,25,27,31).$$

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**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Third Semester**

**Mathematics**

**Elective – FUZZY MATHEMATICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define $\alpha-$cut of a fuzzy set.

2. Write down the formula for LUKASTIEUICZ truth values $T_n$.

3. Define Yager's class of fuzzy complement.

4. What do you mean by general aggregation operation?

5. Define Cylindrical extension.

6. Write down the formula for max product composition.

7. Define Plausibility measure.

8. If $A \subseteq B$ then prove that $Bel(A) \leq Bel(B)$.

9. Prove that $H(X) \geq H(X|Y)$.

10. Write down the formula for $H(X|Y)$ and $H(Y|X)$.

**Part B** (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Find the union of fuzzy set.

| X | 5 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| Adult | 0 | 0 | 0.8 | 1 | 1 | 1 | 1 | 1 | 1 |
| Old | 0 | 0 | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1 | 1 |

Or

(b) If $L_3$ prove that $a \Leftrightarrow b = (a \Rightarrow b) \wedge (b \Rightarrow a)$.

12. (a) If $C$ is a continuous fuzzy complement, prove that '$c$' has unique equilibrium.

Or

(b) Prove that for all $a, b \in [a, b], u(a, b) \geq \max(a, b)$.

13. (a) Let $M_p = \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix}\begin{bmatrix} a & b & c \\ 0.7 & 0.5 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.4 & 0.5 \end{bmatrix}$ and $M_Q = \begin{matrix} \\ a \\ b \\ c \end{matrix}\begin{bmatrix} \alpha & \beta \\ 0.6 & 0.8 \\ 0 & 1 \\ 0 & 0.9 \end{bmatrix}$.

Find max-min composition.

Or

(b) Let $R$ be binary relation defined by the following membership matrix $M_R = \begin{bmatrix} 1 & 0 & 0.7 \\ 0.3 & 0.2 & 0 \\ 0 & 0.5 & 1 \end{bmatrix}$. Obtained its resolution form.

14. (a) Calculate the joint basic assignment $m_{1,2}$ for the focal elements C, RUD and DUC.

| Focal elements | C | RUD | DUC |
|---|---|---|---|
| $m_1$ | 0.05 | 0.15 | 0.05 |
| $m_2$ | 0.05 | 0.05 | 0.05 |

Also determine $Bel_{1,2}, Pl_{1,2}$ for these focal elements.

Or

(b) Prove: Given a consonant of evidence ($\mathcal{F}$, $m$) the associated *Bel* and *Pl* measures possess the following properties:

(i) $Bel(A \cup B) = \min\{Bel(A), Bel(B)\} \forall A, B \in \mathcal{P}(X)$

(ii) $Pl(A \cup B) = \max\{Pl(A), Pl(B)\} \forall A, B \in \mathcal{P}(X)$

15. (a) Using the measures of fuzzyness defined by $f_{c,w}(A) = (b-a)^{1/w} - \left( \int_a^b \delta_{c,\,A}\, w\,(x)\, dx \right)^{1/w}$ for $w = 1$ and the standard fuzzy complements. Calculate the degree of fuzziness of the fuzzy set defined on the interval [0,10] of real numbers by the following membership functions

(i) $\mu_A(x) = \frac{x}{10}$

(ii) $\mu_B(x) = 2^{-x}$

(iii) $\mu_c(x) = \begin{cases} \frac{x}{5} \; if \; \leq 5 \\ 2 - 0.2x \;\; if \;\; x \geq 5 \end{cases}$.

Or

(b) Show that the measure of fuzziness defined by the equations $f(A) = -\sum_{x \in X} \{\mu_A(x) \log_2 \mu_A(x) + [1 - \mu_A(x)]$

$\log_2 [1 - \mu_A(x)]$ and $f_C(A) = |x| - \sum_{x \in x} |\mu_A(x) - c\,\mu_A(x)|$.

Express fuzziness in bits (or) nots $(\log c)$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Explain the following terms in fuzzy logic

(a) Conditional

(b) Biconditional

(c) Tautology

(d) Negation

(e) Contradiction.

17. Prove that the fuzzy set operations union, intersection, continuous complement that satisfies the law of excluded middle and the law of contradiction. but not idempotent.

18. Solve the following fuzzy relation

$$P_0 \begin{bmatrix} 0.1 & 0.4 & 0.5 & 0.1 \\ 0.9 & 0.7 & 0.2 & 0 \\ 0.8 & 1 & 0.5 & 0 \\ 0.1 & 0.3 & 0.6 & 0 \end{bmatrix} = [0.8 \quad 0.7 \quad 0.5 \quad 0]$$

19. Prove that a belief measure Bel on a finite power set $\mathcal{P}(X)$ is a probability measure if and only if its basic assignment '$m$' is given by $m(\{x\}) = Bel[\{x\}]$ and $m(A) = 0$ for all subsets of $x$ that are not single tons.

20. Prove that $H(x, y) \le H(x) + H(y)$.

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**M.Sc. DEGREE EXAMINATION, APRIL 2022.**

**Fourth Semester**

**Mathematics**

**FUNCTIONAL ANALYSIS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by sequence space?

2. Define bounded linear map.

3. Define a convex body.

4. What is meant by a Banach limit?

5. State the uniform boundedness principle.

6. Define the graph of $F$.

7. Define the normed dual.

8. State the Riesz representation theorem for $c[a,b]$.

9. Define an orthonormal set.

10. Define the orthogonal projection.

**Part B** $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the Riesz lemma.

Or

(b) Let $X$ and $Y$ be normed spaces. If $X$ is finite dimensional, then prove that every linear map from $X$ to $Y$ is continuous. Also, prove if $X$ is infinite dimensional and $Y \neq \{0\}$, then there is a discontinuous linear map from $X$ to $Y$.

12. (a) Let $X$ be a normed space over $k$, and f be a nonzero linear functional on $X$. If $E$ is an open subset of $X$, then prove that $f(E)$ is an open subset of $k$.

Or

(b) Show that a Banach space cannot have a denumerable basis.

13. (a) State and prove the Banach-Steinhaus theorem.

Or

(b) Let $X$ and $Y$ be Banach spaces and $F : X \to Y$ be a linear map which is closed and subjective. Prove that $F$ is continuous and open.

14. (a) Let $X$ and $Y$ be normed spaces and $F \in BL(x, y)$. Prove that $R(F) \subset \{y \in Y : y'(y) = 0 \, for\, all \;\, y' \in z(F')\}$ where equality holds if and only if $R(F)$ is closed in $Y$.

Or

(b) Let $1 \leq p \leq \infty$ and $\frac{1}{p} + \frac{1}{\varepsilon} = 1$. Prove that the dual of $C_{00}$ with the norm $\| \; \|_p$ is linearly isometric to $l^q$.

15. (a) (i) With the usual notations, prove the following:

For all

$x, y \in X, \|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2).$

(ii) Let $X$ be an inner product space and let $\{x_1, .......x_n\}$ be an orthogonal set in $X$. Prove that $\|x_1 + ..... + x_n\|^2 = \|x_1\|^2 + ....... + \|x_n\|^2$.

Or

(b) Let $X$ be an inner product space and $f \in X'$. Let $\{u_1, u_2, ...\}$ be an orthonormal set in $X$. Prove that $\sum_n |f(u_n|^2 \leq \|f\|^2$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Let $X$ be a normed space. Prove the following are equivalent:

(a) Every closed and bounded subset of $X$ is compact.

(b) The subset $\{x \in X : |x| \leq 1\}$ of $X$ is compact.

(c) $X$ is finite dimensional.

17. Let $X$ be a normed space and $Y$ be a closed subspace of $X$. Prove that $X$ is a Banach space if and only if $Y$ and $X/Y$ are Banach spaces in the induced norm and the quotient norm, respectively.

18. State and prove the closed graph theorem.

19. State and prove the Riesz representation theorem for $\lfloor p$ .

20. Discuss the Gram-Schmidt orthonormalization process.

| F–7362 |
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| 7MMA4C2 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Fourth Semester**

**Mathematics**

**OPERATIONS RESEARCH**

**(CBCS – 2017 onwards)**

Time : 3 Hours — Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a spanning tree. Give an example.

2. Draw the network defined by $N = \{1, 2, 3, 4, 5, 6\}$ $A\{(1,2), (1,5), (2,3), (2, 4), (3,5), (3,4), (4,3), (4,6)(5,2), (5,6)\}$.

3. What is meant by shortage cost?

4. Write down the setup model.

5. Define Queue discipline.

6. Write down the formula truncated Poisson distribution.

7. Write down the Little's formula.

8. Write the formula for finding w(I) in waiting time distribution for (M/M/1) : (FCFS/$\infty$/$\infty$).

9. Classify the solution methods of nonlinear programming with example.

10. Define the general constrained nonlinear programming problem.

## Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain Dijkstra's Algorithm.

Or

(b) A publisher has a contract with an author to publish a textbook. The activities associated with the production of the text book are given below. Develop the associated network for the project

| | Activity | Predecessors | Duration (Weeks) |
|---|---|---|---|
| A : | Manuscript proof reading by editor | – | 3 |
| B : | Sample pages prepared by typesetter | – | 2 |
| C : | Book cover design | – | 4 |
| D : | Preparation of art work for book figures | – | 3 |
| E : | Author's approval of edited manuscript and sample pages | A, B | 2 |
| F : | Book typesetting | E | 2 |
| G : | Author checks typeset pages | F | 2 |
| H : | Author checks artwork | D | 1 |
| I : | Production of printing plates | G, H | 2 |
| J : | Book production and binding | C, I | 4 |

12. (a) Explain multi-Item EOQ with storage limitation.

Or

(b) Find the optimal inventory policy for the following 6-period inventory situation

| Period i | Di (units) | Ki ($) | hi ($) |
|---|---|---|---|
| 1 | 10 | 20 | 1 |
| 2 | 15 | 17 | 1 |
| 3 | 7 | 10 | 1 |
| 4 | 20 | 18 | 3 |
| 5 | 13 | 5 | 1 |
| 6 | 25 | 50 | 1 |

The unit production cost is $ 2 for all the periods.

13. (a) Explain forgetfulness of the exponential.

Or

(b) Derive pure death model.

14. (a) Explain (M/M/1) : (GD/N/$\infty$) queuing model.

Or

(b) An investor invests $ 1000 a month in one type of stock market security. Because the investor must wait for a good "buy" opportunity, the actual time of purchase is totally random. The investor usually keeps the securities for about 3 years on the average but will sell them at random times when a "sell" opportunity presents itself. Although the investor is generally recognized as a shrewd stock market player, past experience indicates that about 25% of the securities decline at about 20% a year. The remaining 75% appreciate at the rate of about 12% a year. Estimate the investor's average equity in the stock market.

15. (a) Solve the following problem using gradient method.

Maximize $f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1 x_2 - 2x_2^2$.

Or

(b) Explain separable programming.

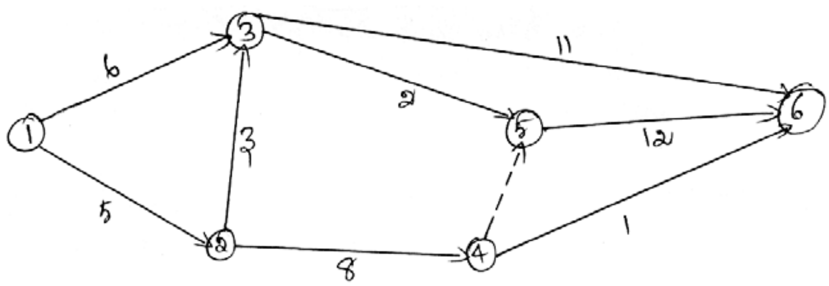
**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. (a) Determine the critical path for the following project network.

(b) Determine the time schedule for the following project network

(c) Compute the floats for the noncritical activities of the following project network.



17. A four-period inventory model operates with the following data

| Period i | Demand Di (units) | Setup cost ki ($) |
|---|---|---|
| 1 | 76 | 98 |
| 2 | 26 | 114 |
| 3 | 90 | 185 |
| 4 | 67 | 70 |

The initial inventory $x_1 = 15$ units. The unit production cost is $2, and the unit holding cost per period is $ 1 for all the periods. Determined by the forward algorithm.

18. Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following

(a) The average number of births per year

(b) The probability that no births will occur in any one day

(c) The probability of issuing 50 birth certificates in 3 hours given that 40 certificates were issued during the first 2 hours of the 3-hour period.

19. Visitor's parking at Ozark college is limited to five spaces only. Cars making use of this space arrive according to a Poisson distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked cars leaves.

That temporary space can hold only three cars. Other cars that cannot park or find a temporary waiting space must go elsewhere. Determine the following:

(a) The probability, $p_n$ of having n cars in the system.

(b) The effective arrival rate for cars that actually use the lot.

(c) The average number of cars in the lot.

(d) The average time a car waits for a parking space inside the lot.

20. Solve the following nonlinear programming problem using separable programming method

Maximize $Z = x_1 + x_2^4$

Subject to

$$3x_1 + 2x_2^2 \leq 9$$
$$x_1, x_2 \geq 0$$

**F–7363**

| Sub. Code |
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| 7MMA4C3 |

**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Fourth Semester**

**Mathematics**

**TOPOLOGY – II**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** $(10 \times 2 = 20)$

Answer **all** questions.

1. Define one-point compactification.

2. State the countable intersection property.

3. Define completely regular space.

4. If $X$ is completely regular, then prove that $X$ can be imbedded in $[0,1]^J$ for some $J$.

5. Prove that the collection $A = \{(n, n+2) / n \in z\}$ is locally finite.

6. Define locally discrete set.

7. Define uniform metric.

8. Define equicontinuous function.

9. State Ascoli's theorem.

10. Check whether the Set $Q$ of rationals is a Baire space or not ? Justify your answer.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $X$ be a locally compact Hausdroff space and $Y$ be a subspace of $X$. If $Y$ is closed in $X$ or open in $X$, then prove that $Y$ is locally compact.

Or

(b) Let $X$ be a space. Let $\mathfrak{D}$ be a collection of subsets of $X$. That is maximal with respect to the finite intersection property. Let $D \in \mathfrak{D}$. If $A \supset \mathfrak{D}$ then prove that $A \in \mathfrak{D}$.

12. (a) Prove that a subspace of a completely regular space is completely regular. Also prove that a product of completely regular spaces is completely regular.

Or

(b) Let $A \subset X$ and let $f : A \to Z$ be a continuous map of $A$ into the Hausdroff space $Z$. Prove that there is almost one extension of $f$ to a continuous function $g : \overline{A} \to Z$.

13. (a) Let $ea$ be a locally finite collection of subsets of $X$. Then prove that

(i) Any subcollection of $ea$ is locally finite

(ii) The collection $\mathcal{B} = \{\overline{A}\}_{A \in ea}$ of the closures of the elements of $ea$ is locally finite.

(iii) $\overline{\bigcup_{A \in ea} A} = \bigcup_{A \in ea} \overline{A}$

Or

(b) Let $X$ be normal and $A$ be a closed $G_S$ set in $X$. Prove that there is a continuous function $f : x \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) If the space $Y$ is complete in the metric $d$, then prove that the space $Y^J$ is complete in the uniform metric $\overline{\rho}$ corresponding to $d$.

Or

(b) Let $X$ be a compactly generated space and let $(Y,d)$ be a metric space. Then prove that $\mathcal{C}(X,Y)$ is closed in $Y^x$ in the topology of compact convergence.

15. (a) Let $X$ be a locally compact Hausdroff, let $\mathcal{C}(X,Y)$ have the compact-open topology. Then prove that map $e: X \times \mathcal{C}(X,Y) \to Y$ defined by the equation $\mathcal{C}(x,f) = f(x)$ is continuous.

Or

(b) Let $C_1 \supset C_2 \supset ......$ be a nested sequence of nonempty closed sets in the complete metric space $X$. If diam $c_n \to 0$, then prove that $\cap C_n \neq \phi$.

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Let $X$ be a locally compact Hausdroff space which is not compact and $Y$ be the one-point compactification of $X$. Then prove that $Y$ is a compact Hausdroff space, $X$ is a subspace of $Y$, the set $Y - X$ consists of a single point and $\overline{X} = Y$.

17. Let $X$ be a completely regular space. If $Y_1$ and $Y_2$ are two compactifications of $X$ satisfying the extension property. Prove that $Y_1$ and $Y_2$ are equivalent.

18. Prove that a space $X$ is metrizable if and only if $X$ is regular and has a countably locally finite basis.

19. Prove that a metric space $(X,d)$ is compact if and only if it is complete and totally bounded.

20. If $X$ is a compact Hausdoff space or a complete metric space then prove that $X$ is a Baire space.

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**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Fourth Semester**

**Mathematics**

**Elective : ADVANCED STATISTICS**

**(CBCS – 2017 onwards)**

Time : 3 Hours Maximum : 75 Marks

**Part A** ($10 \times 2 = 20$)

Answer **all** the questions.

1. Define a statistical hypothesis.

2. Define a significance level of the test.

3. What is meant by an exponential class of p.d.f. of the continuous type?

4. Define an absolute – error loss function.

5. Define the Fisher information in the random sample.

6. What is an efficiency of a statistic?

7. When we say that a critical region is a uniformly most powerful critical region?

8. State the likelihood ratio test principle.

9. What is meant by noncentral $\chi^2$ ?

10. Define quadratic and real quadratic forms.

**Part B** (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) Let $x_1, x_2, ...., x_n$ represent a random sample from the distribution has the following p.d.f.
$f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \infty < \infty$, zero elsewhere
Find the m. l. e. $\hat{\theta}$ of $\theta$.

Or

(b) Let $\overline{X}$ be the mean of a random sample of size $n$ from a distribution that is $N(\mu, q)$. Find $n$ such that $\Pr(\overline{X} - 1 < \mu < \overline{X} + 1) = 0.90$, approximately.

12. (a) Let $X_1, X_2, ...., X_n$ be a random sample from the normal distribution $N(0, \theta), 0 < \theta < \infty$. Show that $\sum_{1}^{n} X_i^2$ is a sufficient statistic for $\theta$.

Or

(b) Let $X_1, X_2, ...., X_n$ denote a random sample from a distribution that is $N(\theta, 1), -\infty < \theta < \infty$. Find the unbiased minimum variance estimation of $\theta^2$.

13. (a) Prove that $I_n(\theta) = nI(\theta)$.

Or

(b) Let $X$ be $N(0, \theta)$, $0 < \theta < \infty$. If $X_1, X_2, ...., X_n$ is a random sample from this distribution, show that the m. l. e of $\theta$ is an efficient estimator of $\theta$.

14. (a) Consider the one random variable $X$ that has a binomial distribution with $n = 5$ and $p = \theta$. Let $f(x;\theta)$ denote the p.d.f of $X$ and let $H_0 : \theta = \frac{1}{2}$ and $H_1 : \theta = \frac{3}{4}$ find the values of $f\left(x;\frac{1}{2}\right)$, $f\left(x;\frac{3}{4}\right)$ and $f\left(x;\frac{1}{2}\right) \Big/ f\left(x;\frac{3}{4}\right)$ at points of positive probability density function.

Or

(b) Let $X$ have a Poisson distribution with mean $\theta$. Find the sequential probability ratio test for testing $H_0 : \theta = 0.02$ against $H_1 : \theta = 0.07$.

15. (a) Let $X_1, X_2, ...., X_n$ be a random sample from a normal distribution $N(\mu, \sigma^2)$. Show that $\sum_{i=1}^{n}(X_i - \overline{X})^2 = \sum_{i=2}^{n}\left(X_i - \overline{X^1}\right)^2 + \frac{n-1}{n}\left(X_i - \overline{X^1}\right)^2$, where $\overline{X} = \sum X_i / n$ and $\overline{X^1} = \sum_{i=2}^{n} X_i / (n-1)$.

Or

(b) Show that $R = \frac{\sum_1^n (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_1^n (X_i - \overline{X})^2 \sum_1^n (Y_i - \overline{Y})^2}}$

$$= \frac{\sum_1^n X_i Y_i - n\overline{X}\,\overline{Y}}{\sqrt{\left(\sum_1^n X_i^2 - n\overline{X}^2\right)\left(\sum_1^n Y_i^2 - n\,\overline{Y}^2\right)}}$$

**Part C** (3 × 10 = 30)

Answer any **three** questions.

16. Let $X$ have a p.d.f of the form $f(x;\theta) = \theta x^{\theta - 1}$, $0 < x < 1$, zero elsewhere, where $\theta \in \{\theta : \theta = 1, 2\}$. To test the simple hypothesis $H_0 : \theta = 1$ against the alternative simple hypothesis $H_1 : \theta = 2$ use a random sample $X_1, X_2$ of size $n = 2$ and define the critical region to be $C = \left\{(x_1, x_2) : \frac{3}{4} \le x_1, x_2\right\}$. Find the power function of the test.

17. State and prove the factorization theorem of Neyman.

18. Derive the Rao-Cramer inequality.

19. State and prove Neyman-Pearson theorem.

20. Students scores on the mathematics portion of the ACT examination, $x$ and on the final examination in first-semester calculus (200 points possible), $y$, are given.

    (a) Calculate the least squares regression line for these data.

    (b) Plot the points and the least squares regression line on the same graph.

    (c) Find point estimates for $\alpha, \beta$, and $\sigma^2$.

| $x$ | $y$ |
|---|---|
| 25 | 138 |
| 20 | 84 |
| 26 | 104 |
| 26 | 112 |
| 28 | 88 |
| 29 | 90 |
| 32 | 183 |
| 20 | 100 |
| 25 | 143 |
| 26 | 141 |
| 28 | 161 |
| 25 | 124 |
| 31 | 118 |
| 30 | 168 |

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**M.Sc. DEGREE EXAMINATION, APRIL 2022**

**Fourth Semester**

**Mathematics**

**Elective – NUMERICAL METHODS**

**(CBCS – 2017 onwards)**

Time : 3 Hours                Maximum : 75 Marks

**Part A**                $(10 \times 2 = 20)$

Answer **all** questions.

1. What is transcendental equation? Give an example.

2. What is meant by order of convergence?

3. Define the term Pivot element.

4. Under what condition power method is suitable to find the Eigen value of the matrix?

5. Define a cubic spline.

6. State the properties of Qudratic Spline Interpolation.

7. What do you mean by Numerical differentiation?

8. Define extrapolation.

9. Give the multistep methods available for solving ordinary differential equation.

10. State the special advantage of Runge-Kutta method over Taylor Series method.

**Part B** (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the number of real and complex roots of the polynomial equation $p_4(x) = 4x^4 + 2x^2 - 1 = 0$.

Or

(b) Find all the roots of the polynomial $x^3 - 6x^2 + 11x - 6 = 0$ using the Graeff's root squaring method.

12. (a) Find the condition number of the system $\begin{bmatrix} 2.1 & 1.8 \\ 6.2 & 5.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}$.

Or

(b) Find all the eigen values and the corresponding eigen vectors for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

13. (a) Using the following values of $f(x)$ and $f'(x)$.

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| –1 | 1 | –5 |
| 0 | 1 | 1 |
| 1 | 3 | 7 |

Estimate the values of $f(-0.5)$ and $f(0.5)$ using piecewise cubic Hermite interpolation.

Or

(b) Obtain the least squares straight line to fit the following data :

| $x$ | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|
| $f(x)$ | 0.447 | 0.632 | 0.775 | 0.894 | 1 |

14. (a) The following tables of values is given :

| $x$ | –1 | 1 | 2 | 3 | 4 | 5 | 7 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 1 | 16 | 81 | 256 | 625 | 2401 |

Using the formula $f'(x_1) = (f(x_2) - f(x))/(2h)$ and the Richardson extrapolation, find $f'(3)$.

Or

(b) Define $s(h) = \dfrac{-y(x+2h) + 4y(x+h) - 3y(x)}{2h}$ show that $y'(x) - s(h) = c_1h^2 + c_2h^3 + c_3h^4 + ....$ and state $c_1$.

15. (a) Find the single step methods for the differential equation $y' = f(t, y)$ which produce exact results for $y(t) = a + be^{-t}$.

Or

(b) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$, with $h = 0.2$ on the interval [0, 0.4], using the backward Euler method.

## Part C (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the complex roots of the equation $f(z) = z^3 + 1 = 0$ correct to eight decimal places. Use the initial approximation to a root as $(x_0, y_0) = (0.25, 0.25)$ compare with the exact values of the roots $(1 + i\sqrt{3})/2$.

17. Find all the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ using the Jacobi method. Iterate till the off-diagonal elements, in magnitude, are less than 0.0005.

18. Obtain the piecewise quadratic interpolating polynomials for the function $f(x)$ defined by the data.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | –3 | –2 | –1 | 1 | 3 | 6 | 7 |
| $f(x)$ | 369 | 222 | 171 | 165 | 207 | 990 | 1779 |

Hence find an approximate value of $f(-2.5)$ and $f(6.5)$.

19. Using the method of undetermined coefficients find the nodes and weights of the quadrature formula $\int_0^\infty e^{-x} f(x)dx = \lambda_0 f(x_0) + \lambda_1 f(x_1)$.

20. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval [0, 0.4]. Using the second order implicity Runge-Kutta method.