F–9377

Sub. Code
7MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics

ALGEBRA – II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a homomorphism of two vector spaces over F.

2. If U and W are subspaces of a vector space V, prove that $U+W=\{v \in V / v = u+w, u \in V, w \in W\}$ is a subspace of V.

3. Prove that $A(W)$ is a subspace of $\hat{V}$.

4. If $\varepsilon(u,v)=\|u-v\|$, then prove that $\varepsilon(u,v) \le \varepsilon(u,w)+\varepsilon(w+v)$.

5. When we say that an element $a \in K$ is algebraic of degree n over F?

6. Define the derivative of $f(x)$ in $F(x)$.

7. What is the fixed field of $G(k,F)$?

8. Write down the elementary symmetric functions in x_1, x_2, x_3, x_4.

9. If $T_1, T_2 \in Hom(v,v)$ then prove that $T_1T_2 \in Hom(v,v)$.

10. If $T \in A(v)$ then prove that $T^* \in A(v)$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If V is the internal direct sum of $u_1,...u_n$ then prove that V is isomorphic to the external direct sum of $u_1,...u_n$.

Or

(b) If $v_1,...,v_n$ are in V then prove that either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1,...,v_{k-1}$.

12. (a) In $F^{(n)}$ define, for $u = (\alpha_1 \ldots, \alpha_n)$ and $v = (\beta_1, \ldots, \beta_n)$, $< u,v >= \alpha_1\overline{\beta}_1 + \alpha_2\overline{\beta}_2 + \cdots + \alpha_n\overline{\beta}_n$. Verify that this defines an inner product on $F^{(n)}$.

Or

(b) If V is a finite – dimensional inner product space and if W is a subspace of v, then prove that $V = W + W^{\perp}$.

13. (a) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

Or

(b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

14. (a) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

Or

(b) Prove that $G(k,F)$ is a subgroup of the group of all automorphisms of K.

15. (a) If V is n-dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1,...,v_n$ and the matrix $m_2(T)$ in the basis $w_1,...,w_n$ of v over F, then prove that there is an element $C \in F_n$ such that $m_2(T) = Cm_1(T)c^{-1}$.

Or

(b) If $T \in A(V)$ is such that $\langle vT,v \rangle = 0$ for all $v \in V$, then prove that $T = 0$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) If $v_1,...,v_n$ is a basis of v over F and if $w_1,...,w_m$ in V are linearly independent over F then prove that $m \le n$.

(b) If V is finite-dimensional and if W is a subspace of V, then prove that W is finite-dimensional, $\dim W \le \dim V$ and $\dim V/W = \dim V - \dim W$.

17. If V and W are of dimensions m and n, respectively, over F, then prove that $Hom(V,W)$ is of dimension mn over F.

18. If F is of characteristic o and if a,b, are algebraic over F, then prove that there exists an element $c \in F(a,b)$ such that $F(a,b) = F(C)$.

19. State and prove the fundamental theorem of Galois theory.

20. If V is finite-dimensional over F then prove that for $S, T \in A(V)$.

(a) $r(ST) \leq r(T)$

(b) $r(TS) \leq r(T)$

(c) $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$.

F–9378

Sub. Code
7MMA2C2

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics

ANALYSIS – II

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. If $f \in \mathscr{R}$, then prove that $\underline{\int_{a}^{b}} f\,d\alpha \leq \overline{\int_{a}^{b}} f\,d\alpha$.

2. Define the unit step function.

3. Define an equicontinuous function on a set E in a metric space x.

4. State the stone – Weierstars theorem.

5. Prove that the function E is periodic with period 2π.

6. Define gamma function.

7. Define a measurable set.

8. Define a Lebesgue measurable function.

9. Define characteristic function.

10. State bounded convergence theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If f is monotoni on (a,b) and if α is continuous on (a,b), then prove that $f \in \mathscr{R}(\alpha)$.

Or

(b) State and prove fundamental theorem of calculus.

12. (a) State and prove the Cauchy criterion for uniform convergence.

Or

(b) If K is a compact metric, if $f_n \in \mathscr{C}$(K) for $n = 1,2,3,\ldots$, and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

13. (a) Suppose $\sum C_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} C_n x^n$, $-1 < x < 1$. Then prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} C_n$.

Or

(b) Suppose $a_0, \ldots a_n$ are complex numbers, $n \geq 1,\ a_n \neq 0.\ p(z) = \sum_{k=0}^{n} ak z^k$. Then prove that $P(z) = 0$ for some complex number z.

14. (a) If E_1 and E_2 are measurable, then prove that $E_1 \cup E_2$ is measurable.

Or

(b) If f and g are two measurable real valued functions defined on the same domain. Then prove that the functions $f+g$ and fg are also measurable.

15. (a) Let φ and ψ be simple functions which vanish outside a set of finite measure. Then prove that $\int(a\varphi+b\psi)=a\int\varphi+b\int\psi$ and if $\varphi\geq\psi$ a.e. then prove that $\int\varphi\geq\int\psi$.

Or

(b) State and prove monotone convergence theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If γ' is continuous on (a,b), then prove that γ is rectifiable and $\wedge(\gamma)=\int_a^b|\gamma'(t)|\,dt$.

17. Suppose $\{f_n\}$ is a sequence of functions differentiable on (a,b) and such that $\{f_n(x_0)\}$ converges for some point x_0 on (a,b) If $\{f_n'\}$ converges uniformly on (a,b) then prove that $\{f_n\}$ converges uniformly on (a,b) to a function f and $f'(x)=\lim_{n\to\infty}f_n'(x)$ $a\leq x\leq b$.

18. State and prove Parseral's theorem.

19. Let $< Ei >$ be a sequence of measurable sets. Then prove that $m(UEi) \leq \sum mEi$. Also if the sets E_n are pairwise disjoint, then prove that $m(UEi) = \sum mEi$.

20. Let f be defined and bounded on a measurable set E with mE finite. Prove that the necessary and sufficient foundation for f to be measurable is $\inf_{f \leq \psi} \int_E \psi(x)dx = \sup_{f \geq \varphi} \int_E \varphi(x)dx$ for all simple functions φ and ψ.

F–9379

Sub. Code
7MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the parametric equations of the spherical surface $x^2 + y^2 + z^2 = a^2$.

2. Define orthogonal trajectories of a system.

3. Form a partial differential equation by eliminate the arbitrary constants from $ax^2 + by^2 + z^2 = 1$.

4. Define singular integral of the equation $F(x, y, z, p, q) = 0$.

5. Write down the general form of Clairout equation.

6. Find the complete integral of the equation $p^2 z + q^2 = 1$.

7. If $u = f(x+iy) + g(x-iy)$, where the functions f and g are arbitrary, Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

8. Solve $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.

9. Define exterior Dirichlet problem.

10. Write down the Helmholtz equation.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Integrate the equation

$(y+z)dx+(z+x)\,dy+(x+y)\,dz = 0$.

Or

(b) Find the integral curves of the equations

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}.$$

12. (a) Find the surface which intersects the surfaces of the system $z(x+y) = C(3z+1)$.

Orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$.

Or

(b) Form a partial differential equation by eliminate the arbitrary function from $f(x^2+y^2+z^2, z^2-2xy) = 0$.

13. (a) Show that the equations $x\,p = yq, z(xp+yq) = 2xy$ are compatible and solve them.

Or

(b) Find the integral surface of the differential equation $(y+zq)^2 = z^2(1+p^2+q^2)$ circumscribed about the surface $x^2-z^2=2y$.

14. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2}+x^2\frac{\partial^2 z}{\partial y^2}=0$ to canonical form.

Or

(b) Solve $r-s+2q-z=x^2y^2$.

15. (a) Prove that the solutions of a certain Neumann problem can differ from one another by a constant only.

Or

(b) Prove that $r\cos\theta$ and $r^{-2}\cos\theta$ satisfy Laplace's equation, where r, θ, ϕ are spherical polar coordinates.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Find the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersections with the family of planes parallel to $z=0$.

(b) Verify that the differential equation $yz\,dx+2xz\,dy-3xy\,dz=0$ is integrable.

17. If u is a function of x, y and z which satisfies the partial differential equation $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$. Show that u contains x, y and z only in combinations $x+y+z$ and $x^2+y^2+z^2$.

18. Find the complete integral of the equations

(a) $p^2 x + q^2 y = z$

(b) $zpq = p + q$.

19. By separating the variables, show that the one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ has solutions of the form A $\exp(\pm inx, \pm inct)$, where A and n are constants. Hence show that functions of the form $z(r, t) = \sum_r \left\{A_r \cos\frac{r\pi ct}{a} + B_r \sin\frac{r\pi ct}{a}\right\} \sin\frac{r\pi x}{a}$ where the A_r's and B_r's are constants, satisfy the wave equation and the boundary conditions $z(0, t) = 0$, $z(a, t) = 0$ for all t.

20. The faces $x=0, x=a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slap is described by the equation $\theta = f(x)$ $(0 \le x \le a)$. Determine the temperature at a subsequent time t.

F–9380

Sub. Code
7MMA2C4

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics

MECHANICS

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define configuration space.

2. What are ignorable coordinates?

3. Write down the Lagrange's equations for a holonomic system.

4. Define a liouville's system.

5. State the multiplier rule for the analysis of the stationary values of $\int_{t_0}^{t_1} L dt$.

6. State the Jacobi form of the principle of least action.

7. Write a short notes on Hamiton's principle function.

8. State the stackel's theorem.

9. What is meant by canonical transformation?

10. Define Lagrange bracket.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the D' Alembert's principle.

Or

(b) In the position of the particle is given by the spherical coordinates (r, θ, ϕ), find the components of generalised momentum.

12. (a) Derive Lagravge's equations from D'Alembert's principle.

Or

(b) A double pendulum consists of two particles suspended by massless rods (each of length l). Assuming that all motion takes place in a vertical plane, find the differential equations of motion.

13. (a) Obtain the solution of Brachistochrone problem by the method of calculus of variations.

Or

(b) A particle of mass m is attracted to a fixed point 0 by an inverse square force $F_r = -\frac{\mu m}{r^2}$ where μ is the gravitational coefficient. Using the plane polar coordinates (r, θ) to describe the position of the particle. Find the equation of motion.

14. (a) Explain Pfaffian differential forms. Also derive the Hamilton's canonical equations.

Or

(b) Derive the Hamilton – Jacobi equations, in the form $\frac{\partial s}{\partial t} + H\left(q, \frac{\partial s}{\partial q}, t\right) = 0$.

15. (a) Consider the transformation $Q = q - tp + \frac{1}{2}gt^2$, $p = p - gt$. Find k – H and the generating functions.

Or

(b) State and prove the Poisson's theorem

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Find the angular momentum of a system of particles about a fixed point.

(b) A practical of mass m can slide without friction on a fixed circular wire of radius r which lies in a vertical plane. Using D' Alembert's principle and the equation of constraint, show that $y\ddot{x} - x\ddot{y} - gx = 0$.

17. Explain Routhian function. Bring out its utility.

18. Use the Jacobi form of the principle of least action and obtain the orbit for the kepler problem.

19. Show that the kepler problem described by the spherical coordinates (r, θ, ϕ) is separable in accordance with the stackel and check Liouville criteria.

20. Consider the transformation $Q = \sqrt{2q}e^{t} \cos p$, $P = \sqrt{2q}e^{-t} \sin p$. Show that this transformation is canonical and final the generating function $\phi(q, Q, t)$.

F–9381

Sub. Code
7MMA2E1

M.Sc. DEGREE EXAMINATION, APRIL 2023

Second Semester

Mathematics

***Elective* — GRAPH THEORY**

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a complete graph. Give an example.

2. Draw all the trees with 6 vertices.

3. Define a block of a graph with an example.

4. When will you say that a graph is said to be an Eulerian? Give an example.

5. Find the number of different perfect matching in $K_{n,n}$.

6. Draw 4-edge chromatic graph.

7. Define an independent set and maximum independent set of a graph. Give an example for each.

8. State the Brook's theorem.

9. Embed K_5 on the torus.

10. If G is a simple planar graph, then prove that $\delta \leq 5$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that if $G \cong H$, then $\gamma(G) = \gamma(H)$ and $\varepsilon(G) = \varepsilon(H)$. Is the converse true?

Or

(b) Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

12. (a) If G is a block with $\gamma \geq 3$, then prove that any two edges of G lie on a common cycle.

Or

(b) If G is a simple graph with $\gamma \geq 3$ and $\delta \geq \frac{v}{2}$, then prove that G is Hamiltonian.

13. (a) State and prove the Berge theorem.

Or

(b) Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

14. (a) With the usual notations, prove that $\alpha' + \beta' = \gamma$ if $\delta > 0$.

Or

(b) State and prove the Dirac theorem.

15. (a) Define a planar graph. Also prove that K_5 is nonplanar.

Or

(b) State and prove Euler's formula for a connected plane graph.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. With the usual notations, prove that

$$\tau(K_5) = n^{n-2}$$

17. Prove that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

18. State and prove the Vizing's theorem.

19. (a) Show that, for all k and l, $r(k, l) = r(l, k)$.

(b) State and prove the Erdos theorem.

20. Show that every planar graph is 5-vertex-colourable.

F–9382

Sub. Code
7MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Find the radius of convergence of the power series $\Sigma n^p z^n$.

2. Prove that the reflection $z \to \bar{z}$ is not a linear transformation.

3. Define the winding number of γ with respect to a.

4. Write down the Cauchy's representation formula.

5. Define meromorphic function with an example.

6. Show that the function $\cos z$ have essential singularities at ∞.

7. Determine the residue of $\frac{1}{\sin^2 z}$.

8. State the Rouche's theorem.

9. State the Laurent series.

10. Find the genus of $\cos\sqrt{z}$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that a harmonic function satisfies the formal differential equation $\frac{\partial^2 4}{\partial z \partial \bar{z}} = 0$

Or

(b) Define the cross ratio. Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$.

12. (a) With the usual notations, prove that $\left| \int_a^b f(t) dt \right| \leq \int_a^b |f(t) dt|$.

Or

(b) State and prove Liouville's theorem. Also state the fundamental theorem of algebra.

13. (a) State and prove the Wierestrass theorem for essential singularity.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the argument principle

Or

(b) Evaluate $\int_0^\infty \frac{\cos x}{x^2 + a^2} dx$, a real.

15. (a) State and prove the Hurwitz theorem.

Or

(b) Prove that for $|z| < 1$.

$$(1+z)(1+z^2)(1+z^4)(1+z^8)\ldots\ldots = \frac{1}{1-z}.$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Abel's theorem.

17. State and prove Cauchy's theorem for a rectangle.

18. State and prove the Schwarz lemma.

19. State and prove Cauchy's residue theorem.

20. Derive the Jensen's formula.

F–9383

Sub. Code
7MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2023

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2017 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Let Y be a subspace of X. If U is open in Y and Y is open in X, then prove that U is open in X.

2. Define Hausdorff space.

3. Give and example for quotient topology.

4. Define box topology.

5. If the sets C and D form a separation of X, and if Y is a connected subset of X, then prove that Y lies entirely within either C or D.

6. What are the components and path components of R_l?

7. State the tube lemma.

8. Show that every closed interval in R is compact.

9. State Urysohn lemma.

10. Is R^w normal in the product topology? In the uniform topology?

Part B $(5 \times 5 = 25)$

Answer **all** the questions, choosing either (a) or (b).

11. (a) Let $\mathfrak{B}$ and $\mathfrak{B}'$ be bases for the topologies τ and τ' respectively on X. Then prove that the following are equivalent :

    (i) τ' is finer than τ

    (ii) For each $x \in X$ and each basis element $\mathfrak{B} \in \mathfrak{B}$ containing x, there is a basis element $\mathfrak{B}' \in \mathfrak{B}'$ such that $x \in \mathfrak{B}' \subset \mathfrak{B}$.

Or

(b) Let A be a subset of the topological space X; let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$.

12. (a) Let $f : A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$ then prove that f is continuous if and only if the functions $f_1 : A \to X$ and $F_2 : A \to Y$ are continuous.

Or

(b) Show that the uniform topology on R^{τ} is finer than the product topology; they are different if τ is finite.

13. (a) Show that if $\prod X_{\alpha}$ is connected and nonempty, then each X_{α} is connected.

Or

(b) Prove that the components of X are connected disjoint subsets of X whose union is X, such that each connected subset of X, intersects only one of then.

14. (a) Show that finite union of compact set is compact.

Or

(b) Show that compactness implies limit point compactness, but not conversely.

15. (a) Suppose that X has a countable basis. Then prove that every open covering of X contains a countable subcollection covering X.

Or

(b) Prove that every compact Hausdorff space is normal.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let X and X' denote a single set in the topologies τ and τ' respectively; let Y and Y' denote a single set in the topologies U and U' respectively.

(a) Show that if $\tau' \supset \tau$ and $u' \supset u'$ then the product topology on $X' \times Y'$ is finer than the product topology on $X \times Y$.

(b) Does the converse of (a) hold? Justify your answer.

(c) What can you say if $\tau' \supset \tau$ and $u' \supset u'$ and $u' \neq u$?

17. Show that the topologies on R^n included by the Euclidean metric d and the square metric P are the same as the product topology on R^n?

18. Prove that the Cartesian product of connected spaces is connected.

19. State and prove Lebesgue number lemma.

20. State and prove Tietze extension theorem.

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F–9384

Sub. Code
7MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Let $\psi(t) = \ln \mu(t)$, where $\mu(t)$ is the m.g.f of a distribution. Prove that $\psi'(0) = \mu$.

2. If C_1 and C_2 are subsets of the sample space S, show that $P(C_1 \cap C_2) \le P(C_1) \le P(C_1 \cup C_2) \le P(C_1) + P(C_2)$.

3. Let X and Y have the p.d.f $f(x, y) = 1$, $0 < x < 1$, $0 < y < 1$ zero elsewhere. Find the p.d.f. of the product $Z = XY$.

4. If the correlation coefficient ρ of X and Y exists, show that $-1 \le \rho \le 1$.

5. If the m,g,f, of a random variable X is $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^5$, find $\Pr(X = 2 \text{ or } 3)$.

6. If X is $N(\mu, \sigma^2)$, show that $E(1X - \mu) = \sigma \sqrt{2}/\pi$.

7. If $f(x)=\frac{1}{2}, -1<x<1$, zero elsewhere, is the p.d.f. of the random variable X, find the p.d.f. of $Y=X^2$.

8. Find the mean of $S^2=\sum_{1}^{n}\left(X_i-\overline{X}\right)^2 \Big/ n$, where $X_1, X_2, \ldots X_n$ is a random sample from $N(\mu, \sigma^2)$.

9. Define convergence in probability with an example.

10. If Y is $b\left(100, \frac{1}{2}\right)$, approximate the value of $\Pr(Y=50)$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) A drawer contains eight pairs of socks. If size socks are taken at random and without replacement, Compute the probability that there is atleast one matching pair among these six socks.

Or

(b) State and prove Chebyshev's inequality.

12. (a) Let the random variable X_1 and X_2 have the joint p.d.f. $f(x_1, x_2)=1/\pi$, $(x_1-1)^2+(x_2+2)^2<1$, zero elsewhere. Find $f_1(x_1)$ and $f_2(x_2)$. Are X_1 and X_2 independent?

Or

(b) Let X_1, X_2, X_3 and X_4 be four independent random variables, each with p.d.f. $f(x)=3(1-x)^2$, $0<x<1$, zero elsewhere. If Y is the minimum of these four variables, find the distribution function and the p.d.f. of Y.

13. (a) Compute the measures of skewness and kurtosis of a gamma distribution with parameters α and β.

Or

(b) If X is $N(1, 4)$, compute the probability $\Pr(1 < X^2 < 9)$.

14. (a) Let X_1 and X_2 denote a random sample of sized from a distribution that $N(\mu, \sigma^2)$. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 + 2X_2$. Show that the joint p.d.f. of Y_1 and Y_2 is bivariate normal with correlation coefficient $3/\sqrt{10}$.

Or

(b) Show that the t-distribution with $r = 1$ degree of freedom and the Cauchy distribution are the same.

15. (a) Let X_n have a gamma distribution with parameter $\alpha = n$ and β, where β is not a function of n. Let $Y_n = X_n/n$. Find the limiting distribution of Y_n.

Or

(b) A part is produced with a mean of 6.2 ounces and a standard deviation of 0.2 ounce. What is the probability that the weight of 100 such items is between 616 and 624 ounces?

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let $f(x) = (4 - x)/16$, $-2 < x < 2$, zero elsewhere, be the p.d.f. of X.

(a) Sketch the distribution function and the p.d.f of X on the same set of axes

(b) If $Y = |X|$, compute $\Pr(Y \leq 1)$

(c) If $Z = X^2$ compute $\Pr\left(Z \leq \frac{1}{4}\right)$.

17. Let X_1 and X_2 denote random variables that have the joint p.d.f. $f(x_1, x_2)$ and the marginal probability density functions $f_1(x_1)$ and $f_2(x_2)$ respectively. Furthermore, let $\mu(t_1,t_2)$ denote the m.g.f of the distribution. Prove that X_1 and X_2 are independent if and only if $M(t_1,t_2) = M(t, 0)\, M(0, t_2)$.

18. (a) If X has a gamma distribution with $\alpha = 3$ and $\beta = 4$, find $\Pr(3.8 < X < 25.2)$.

(b) Let X and Y have a bivariate normal distribution with parameters $\mu_1 = 5$, $\mu_2 = 10$, $\sigma_1^2 = 1$, $\sigma_2^2 = 25$ and $\rho > 0$. If $\Pr\left(4 < Y < 16 / X = 5\right) = 0.954$, determine ρ.

19. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size $n = 4$ from a distribution with p.d.f. $f(x) = 2x,\ 0 < x < 1$.

(a) Find the joint p.d.f. of Y_3 and Y_4

(b) Find the conditional p.d.f. of Y_3 given $Y_4 = y_4$

(c) Evaluate $E(Y_3/y_4)$.

20. State and prove central limit theorem.

F–9385

Sub. Code
7MMA3E1

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Third Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Let $A = \{a, b\}$. How many binary operations can be defined on A?

2. Define monoid.

3. Write $p(x) = x^5 + 3x^4 - 15x^3 + x - 10$ in telescopic form and find the number of multiplications and additions/substractions involved in them.

4. Let $D(K) = 2k + 9$. Find the recurrence relation on D.

5. Solve $S(K) - S(K-1) - 6S(K-2) = 0$.

6. Show that $f(x) = \frac{x}{2}$ is partial recursive.

7. Define totally ordered set.

8. Give an example of an infinite lattica without a zero and a one.

9. Show that lattica D(18) is not a Boolean algebra.

10. Write down the minterm normal form of $f(x_1, x_2) = \overline{x}_1, v\overline{x}_2$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that $f:(N,+) \rightarrow (z_4,+_4)$ is a homomorphism. Also, check that Is it an isomorphism.

Or

(b) Prove that the inverse of the product of two elements of a group G is the product of the inverse taken in the reverse order.

12. (a) Prove that $\sqrt{2}$ is irrational.

Or

(b) Write the recurrance relation for Fibonacci numbers and solve it.

13. (a) If $P(K) - 6P(K-1) + 5P(K-2) = 0$, $P(0) = 2$, $P(1) = 2$. What is the generating function of P?

Or

(b) Show that $f_1(x, y) = x + y$, $x, y \in N$ is primitive recurrsive.

14. (a) Let $(L, \leq)$ be Lattice. For any $a, b \in L$, prove the following are equivalent:

(i) $a \leq b$; (ii) $a \vee b = b$; (iii) $a \wedge b = a$.

Or

(b) Show that the direct product of any two distributive lattices is a distributive lattice.

15. (a) Express the polynomial $P(X_1, X_2, X_3) = X_1 \vee X_2$ in an equivalent sum-of-products canonical form in the three variables X_1, X_2 and X_3.

Or

(b) Consider the Boolean function $f(x_1, x_2, x_3) = ((x_1 + x_2) + (x_1 + x_3)). \quad x_1.\bar{x}_2$ simplify this function and draw the circute gate diagram for it.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Show that (z_n, t_n) is an abelian group.

(b) Let $S = \{(a, b) : a, b \in Q \text{ (rational number)}\}$ and defined on the operation $*$ by $(a, b) * (c, d) = (ac, ad + b)$. Is S a semi group?

17. (a) Prove that, for all $n \geq 1, 3 + 11 + \ldots.. + (8n - 5) = 4n^2 - n$ using induction.

(b) Solve $D(K) - 8D(K-1) + 16D(K-2) = 0$ where $D(2) = 16$, $D(3) = 80$.

18. Solve $S(K) - 4S(K-1) + 4S(K-2) = 3K + 2^k$, $S(0) = 1, S(1) = 1$.

19. Show that $(L \times M, \wedge, \vee)$ is a lattice.

20. Simplify the following using Karnaugh diagram $f(x_1, x_2, x_3, x_4) = x^1{}_1 x^1{}_2 + x_1 x_3 x_4 + x_1 x_2 x^1{}_4 + x^1{}_2 x_3$.

F–9386

Sub. Code
7MMA3E4

M.Sc. DEGREE EXAMINATION, APRIL 2023

Third Semester

Mathematics

Elective — FUZZY MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours　　　　　　　　Maximum : 75 Marks

Part A　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Define union of two fuzzy sets with an example.

2. Compute the scalar cardinality and the fuzzy cardinality of the fuzzy set $A = \frac{\cdot 4}{v} + \frac{\cdot 2}{w} + \frac{\cdot 5}{x} + \frac{\cdot 4}{y} + \frac{1}{z}$.

3. Prove that, for all $a, b \in [0, 1]$, $u(a, b) \le u_{\max}(a, b)$.

4. Does the function $C(a) = (1-a)^w$ qualify for each $w > 0$ as a fuzzy complement.

5. Let $R(X, Y)$ be a fuzzy relation on $X = \{x, y, z\}$ and $Y = \{a, b\}$ such that $M_R = \begin{matrix} & a & b \\ x & \cdot 3 & \cdot 2 \\ y & 0 & 1 \\ z & \cdot 6 & \cdot 4 \end{matrix}$. Then find $M_{R^{-1}}$.

6. Define tolerance relation.

7. What is meant by plausibility measure?

8. Write down the Dempster's rule of combination.

9. What is meant by Boltzman entropy.

10. State the Gibbs' theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Propose an extension of the standard fuzzy set operations (min, max, 1–a) to interval-valued fuzzy sets.

Or

(b) Show that all α-cuts of any fuzzy set A defined on $\mathbb{R}^n$ $(n \geq 1)$ are convex if and only if

$\mu_A[\lambda r + (1-\lambda)s] \geq \min[\mu_A(r), \mu_A(s)]$ for all $r_1 s \in \mathbb{R}^n$ and all $\lambda \in [0, 1]$.

12. (a) Prove that every fuzzy complement has at most one equilibrium.

Or

(b) Show that $\lim_{w \to \infty} \min[1, (a^w + b^w)^{1/w}] = \max(a, b)$.

13. (a) Prove that the max-min composition and min join are associative operations on binary fuzzy relations.

Or

(b) Show that for every fuzzy partial ordering on X, the sets of undominated and undominating elements of X are nonempty.

14. (a) Show that a belief measure Bel on a finite power set $\mathcal{P}(x)$ is a probability measure if and only if its basic assignment m is given by $m(\{x\}) = Bel(\{x\})$ and $m(A) = 0$ for all subsets of X that are not single tons.

Or

(b) Show that a plausibility measure that statistics equation $\pi(A \cup B) = \max[\pi(A), \pi(B)]$ is based on nested focal elements.

15. (a) Prove that $H(X/Y) = H(X, Y) - H(Y)$.

Or

(b) Show that the maximum of the measure of fuzziness defined by

$$f(A) = \sum_{x \in X} (\mu_A(x) \log_2 \mu_A(x) + |[1 - \mu_A(x)] \log_2 [1 - \mu_A(x)])$$

is $|x|$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let the membership grade functions of sets A, B and C

$$\mu_A(x) = \frac{x}{x+2}, \ \mu_B(x) = 2^{-x}, \ \mu_C(x) = \frac{1}{1 + 10(x-2)^2}$$ be

defined on the set $X = \{0, 1, \ldots. 10\}$ and $f(x) = x^2$ for all $x \in X$. Use the extension principle to derive $f(A), f(B)$ and $f(C)$.

17. Prove that the following properties are satisfied by all fuzzy intersections in the yaser class.

(a) $i_w(a, 0) = 0$

(b) $i_w(a, i) = a$

(c) $i_w(a, a) \leq a$

(d) If $w \leq w^1$, then $i_w(a, b) \sum i_w(a, b)$

(e) $\lim\limits_{w \to 0} i_w(a, b) = i_{\min}(a, b)$.

18. Solve the following fuzzy relation equations

$$p \cdot \begin{bmatrix} \cdot 5 & 0 & \cdot 3 & 0 \\ \cdot 4 & 1 & \cdot 3 & 0 \\ 0 & \cdot 1 & 1 & \cdot 1 \\ \cdot 4 & \cdot 3 & \cdot 3 & \cdot 5 \end{bmatrix} = \begin{bmatrix} \cdot 5 & \cdot 3 & \cdot 3 & \cdot 1 \\ \cdot 5 & \cdot 4 & \cdot 4 & \cdot 2 \end{bmatrix}$$

19. Given two noninteractive marginal possibility distributions $r_X = (1, \cdot 8, \cdot 5)$ and $r_Y = (1, \cdot 7)$ on sets $X = \{a, b, c\}$ and $Y = \{\alpha, \beta\}$, respectively, determine the corresponding basic distributions. Then, calculate the joint basic distribution in two different ways.

(a) by the min operator

(b) by Dempster's rule

Show visually the focal elements of the marginal and joint distributions.

20. Prove that the function $I(N) = \log_2 N$ is the only function that satisfies $I(N \cdot M) = I(N) + I(M)$ for all $N, M \in N$ through $I(2) = 1$.

F–9387

Sub. Code
7MMA3E5

M.Sc. DEGREE EXAMINATION, APRIL 2023

Third Semester

Mathematics

Elective : STOCHASTIC PROCESSES

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Covariance Stationary.

2. Show that martingale is a constant mean.

3. Define transition probability matrix.

4. Draw the transitive diagram for the transition matrix.

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix} \end{array}$$

5. State the first entrance theorem.

6. Define an intree Tj to a specific point j in a directed graph G.

7. Write down the probability for the Birth Process.

8. What do you mean by a point process?

9. State the Blackwell's theorem.

10. Define directly Riemann integrable.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the mean and variance of Poisson Process.

Or

(b) Let {Zi ; i = 1, 2, 3, ...} to be sequence of independent and identically distributed random variable with $E(Z_i)=1$ and Let $X_n = \prod_{i=1}^{n} Z_i$. Prove that $\{X_n; n \geq 1\}$ is a martingale.

12. (a) Let $\{X_n; n \geq 0\}$ be a Markov chain with three states 0, 1, 2 with transition matrix $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution $P_r\ \{X_0 = i\} = \frac{1}{3}, i = 0,1,2.$

Find (i) $P_r\ \{X_2 = 2,\ X_1 = 1,\ X_0 = 2\}$;

(ii) $P_r\ \{X_3 = 1, X_2 = 1,\ X_1 = 1,\ X_0 = 2\}$;

(iii) $P_r\ \{X_2 = 2,\ X_1 = 1 / X_0 = 2\}$

Or

(b) If $\{X_n;\ n \geq 0\}$ is a Markov dependents trails, then prove that $P_n = \frac{a}{a+b} + \left(P_1 - \frac{a}{a+b}\right)(1-1-b)^n,\ n \geq 1.$

13. (a) If the State j is persistent, then prove that for every state k that can be reached from state j, $F_{kj} = 1$.

Or

(b) Consider three states Markov Chain 1, 2, 3 with transition probability matrix

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \end{array}$$

Find the stationary distribution if it exists.

14. (a) Enumerate Yule – Furry Process.

Or

(b) Find whether difference of two independent poisson processes is poisson or not.

15. (a) State and prove Wald's equation.

Or

(b) Consider the distribution for W(t) with $P_r(t) = P_r\{N(t) = n\} = F_n(t) - F_{n+1}(t)$. Find the renewal function.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let $X(t) = A_1 + A_2 t$ where A_1 and A_2 are uncorrelated random variables with mean a_i; variance σ_i^2, $i = 1, 2$. Find the mean value function, variance function and covariance function of $\{X(t); t \in T\}$. Is the process evolutionary?

(b) Define state space with an example.

17. Consider the Markov chain with transition probability matrix

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{2} \end{pmatrix} \end{array}$$

(a) Is a chain irreducible.

(b) Is the State 1 and 4 Ergodic.

18. State and prove Ergodic theorem.

19. In Poisson process, if M(t) is the number of occurances recorded in an interval of length t, then prove that M(t) is also a Poisson process with parameter λP.

20. State and prove the Elementary Renewal theorem.

F–9388

Sub. Code
7MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2023

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the sequence space.

2. Define the operator norm.

3. Define a support hyperplane for a set.

4. What is meant by a Banach space?

5. State the Banach-steinhaus theorem.

6. Define the graph of F.

7. Define the normed dual.

8. State the Riesz representation theorem for $\mathcal{C}([a,b])$.

9. Define a Hilbert space. Give an example.

10. When will you say that a set is said to be orthogonal?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove the Riesz lemma.

Or

(b) Let X and Y be normed spaces. If X is finite dimensional, then prove that every linear map from X to Y is continuous.

12. (a) Let X be a normed space over K, and f be a non zero linear functional on X. If E is an open subset of X, then prove that $f(E)$ is an open subset of K.

Or

(b) Let X be a normed space and Y be a banach space. Let X_0 be a dense subspace of X and $f_0 \in BL(X_0, Y)$. Prove that there is a unique $f \in BL(x, y)$ such that $F_{|x_0} = F_0$. Also prove $\|F\| = \|F_0\|$.

13. (a) State and prove Resonance theorem.

Or

(b) Let X and Y be Banach spaces and $F : x \to y$ be a linear map which is closed and surjective. Prove that F is continuous and open.

14. (a) State and prove closed range theorem of Banach.

Or

(b) Let X and Y be normed spaces. Let $F \in BL(X, Y)$. Prove that $\|F'\| = \|F\| = \|F''\|$ and $F'' J_x = J_y F$, where J_x and J_y are the canonical embeddings of x and y into x' and y'', respectively.

15. (a) State and prove polarization identity.

Or

(b) State and prove the projection theorem.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let y be a closed subspace of a normed space x for $x + y$ in the quotient space x / y and let $\|\|x + y\|\| - \inf\{\|\|x + y\|\| : y \in y\}$. prove that $\|\| \ \|\|$ is a norm on x / y.

17. State and prove the Hahn-Banach extension theorem.

18. State and prove uniform boundedness principle theorem.

19. State and prove the Riesz representation theorem for $\underline{|P}$.

20. Discuss the Gram-Schmidt orthonormalization process.

F–9389

Sub. Code
7MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Construct a network for the project whose activities and their precedence relationship are given below.

Activity	A	B	C	D	E	F	G	H	I
Precedence	–	A	A	–	D	B,C,E	F	D	G,H

2. Write the formula for purchasing cost per unit time in EOQ with price breaks.

3. What is the difference between PERT and CPM?

4. Define the effective lead time.

5. Define the lack of memory in queuing system.

6. Define poisson distribution.

7. Draw the transition – rate diagram.

8. Find L_S of the model $(M / M / R) : (GD / K / K), R < K$.

9. Define the general constrained NLPP.

10. Write a short note on separable convex programming.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain the equipment replacement with an illustration.

Or

(b) The following table gives optimistic time (i), the most likely (m), and the pessimistic time (ii). Draw the network of the project and calculate the slack for each event. Find the critical path.

Activity :	1-2	2-3	2-4	3-4	4-5	5-6
a :	0.8	3.7	6.2	2.1	0.8	0.9
m :	1.0	5.6	6.6	2.7	3.4	3.4
b:	1.2	9.9	15.4	6.1	3.6	2.7

12. (a) Find the optimal order quantity for a product for which the price breaks are as follows:

Quantity	Unit Cost (Rs)
$0 \le y_1 \le 500$	25
$500 \le y_2 \le 1500$	24.80
$1500 \le y_3 \le 3000$	24.60
$3000 \le y_4$	24.40

Or

(b) Narrate the no-setup model.

13. (a) What is a queue? Give an illustration. Also describe the basic elements of a queuing model.

Or

(b) An art collector travels to art auctions once a month on the average. Each trip is guaranteed to produce one purchase. The time between trips is exponentially distributed. Determine the following.

(i) The probability that no purchase is made in a 3-month period.

(ii) The probability that the time between successive trips will exceed one month.

(iii) The probability that no more than eight purchases are made per year.

14. (a) Derive L_q, L_s, W_q, W_s for $(M/M/C):(GD/\infty/\infty)$ queuing model.

Or

(b) Show that the P-K formula reduces to L_S of the $(M/M/1):(GD/\infty/\infty)$ when the service time is exponential with a mean of $\frac{1}{\mu}$ time units.

15. (a) Solve the following problem, using Golden section method,

Maximize $f(x) = \begin{cases} 4x; 0 \le x \le 2 \\ 4-x; 2 \le x \le 4 \end{cases}$

Assume that $\Delta = 0.05$.

Or

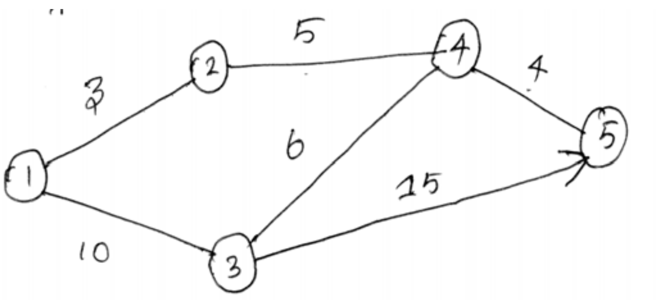
(b) Solve the following problem, using Dichotomous method maximize $f(x) = \begin{cases} 3x; 0 \le x \le 2 \\ \frac{1}{3}(-x+20); 2 \le x \le 3 \end{cases}$

Assume that the maximum value of $f(x)$ occurs at $x = 2$ and $\Delta = 0.10$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Apply Floyd's algorithm to find the shortest routes between every two nodes for the following network



17. Using Dynamic programming Algorithm with constant, to find the optimal inventory policy for the following six-period inventory situation: The unit production cost is \$2 for all the periods.

Period i	Di (units)	ki (\$)	hi (\$)
1	10	20	1
2	15	17	1
3	7	17	1
4	20	18	3
5	13	5	1
6	25	50	1

18. The springdale High School band is performing a benefit jazz concert in its new 400-seat auditorium. Local businesses buy the tickets in blocks of 10 and donate them to youth organizations. Tickets go on sale to business entities for four hours only the day before the concert. The process of placing orders for tickets is sold at a discount as "rush tickets" one hour before the concert starts. Determine the following

    (a) The probability that it will be possible to buy rush tickets.

    (b) The average number of rush tickets available.

19. Visitor's parking at ozark college is limited to five spaces only. Cars making use of this space arrive according to a poisson distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors who cannot find an empty space on arrival may temporarily wait inside the lot unit a parked waiting space must go elsewhere.

    Determine the following:

    (a) The probability, Pn of n cars in the system.

    (b) The effective arrival rate for cars that actually use the lot.

    (c) The average number of cars in the lot.

    (d) The average time a car waits for a parking space inside the lot.

    (e) The average utilization of the parking lot.

20. Solve the following problem using restricted basis method

Maximize $z = e^{-x_1} + x_1 + (x_2 + 1)^2$

Subject to

$x^2{}_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

F–9390

Sub. Code
7MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics

TOPOLOGY – II

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a locally compact space.

2. State the countable inter section property.

3. Give an example of a completely regular space.

4. When will you say that two compactification is said to be equivalent?

5. Define countably locally finite.

6. Prove that in a metric space, every closed set is a G_δ set.

7. Give an example of a space which is both complete and totally bounded.

8. Define an equicontinuous.

9. What is meant by evaluation map?

10. Define a Baire space with an example.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Define the one-point compactification. Prove that the rational Q are not locally compact.

Or

(b) Let x be a space. Let $\mathscr{D}$ be a collection of subsets of x that is maximal with respect to the finite intersection property. Let $D \in \mathscr{D}$. If $A \supset D$ then prove that $A \in \mathscr{D}$.

12. (a) Prove that a product of completely regular spaces is completely regular.

Or

(b) Let $A \subset X$ and let $f : A \to z$ be continuous map of A into the Hausdorff space z. Prove that there is at most one extension of f to a continuous function $g : \overline{A} \to z$.

13. (a) Show that if x has a countable basis, a collection $\mathcal{A}$ of subsets of x is countably locally finite if and only if it is countable.

Or

(b) Let x be normal and let A be a closed G_δ set in x. Prove that there is a continuous function $f : x \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) If the space y is complete in the metrical, then prove that the space y^J is complete in the uniform metric $\overline{\rho}$ corresponding to d.

Or

(b) Let x be a compactly generated space and let (y,d) be a metric space. Prove that $\mathcal{C}(x,y)$ is closed in y^x in the topology of compact convergence.

15. (a) Prove that the sets $B_c(f,\in)$ form a basis for a topology on y^x.

Or

(b) Let $C_1 \supset C_2 \supset \ldots$ be a nested sequence of non empty closed sets in the complete metric space X. If diam $C_n \to 0$, then prove that $nC_n \neq \phi$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that an arbitrary product of compact spaces is compact in the product topology.

17. Let x be a completely regular space. Prove that there exists a compactification Y of X having the property that every bounded coutinuous map $f : x \to \mathbb{R}$ extends uniquely to a continuous map of Y int $\mathbb{R}$.

18. State and prove the Nagata-Smirnov metrization theorem.

19. Prove that a metric space (x,d) is compact if and only if it is complete and totally bounded.

20. State and prove the Ascoli's theorem.

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F–9391

Sub. Code
7MMA4E1

M.Sc. DEGREE EXAMINATION, APRIL 2023.

Fourth Semester

Mathematics

Elective – ADVANCED STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an unbiased estimator.

2. What is meant by critical region?

3. Define minimax decision function.

4. What do you mean by complete family of probability density function?

5. Write a short notes on Bayes confidence interval.

6. What is meant by efficiency of statistic?

7. What do you mean by Likelihood ratio test?

8. Define uniformly most powerful critical region of size α.

9. Define contrasts.

10. What is meant by real quadratic form?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let X have a p.d.f of the form $f(x;\theta) = \theta x^{\theta-1}, 0 < c < 1$, zero elsewhere, where $\theta \in \{\theta : \theta = 1,2\}$. To test the simple Hypothesis $H_0 : \theta = 1$ against the alternate simple hypothesis $H_1 : \theta = 2$, use a random sample x_1, x_2 of size $n = 2$ and define the critical region to be $C = \left\{(x_1 x_2); \frac{3}{4} \leq x_1 x_2\right\}$. Find the power function of the test.

Or

(b) Let $x_1, x_2 \ldots x_n$ denote a random sample from a distribution having the following probability density function $f(x;\theta) = \begin{cases} 1/\theta e^{-x/\theta}, & 0 < x < \infty, \quad 0 < \theta < \infty \\ 0 & elsewhere \end{cases}$

Find the maximum likelihood estimator $\hat{\theta}$ of θ.

12. (a) Let $y_1 < y_2 < \cdots y_n$ denote the order statistics of a random sample of size n from the distribution with p.d.f. $f(x,\theta) = e^{-(x-\theta)} I_{(\theta,\infty)}^{(x)}$. Show that $y_1 = \min x_i$ is the sufficient statistic for θ.

Or

(b) Let $x_1, x_2, \ldots x_n$ denote a random sample from a distribution that has p.d.f. $f(x;\theta), \theta \in \Omega$. If a sufficient statistic $y_1 = u_1(x_1, x_2, \ldots x_n)$ for θ exists and if a maximum likelihood estimator $\hat{\theta}$ of θ also exists uniquely. Prove that $\hat{\theta}$ is a function of $y_1 = u_1(x_1, x_2, \ldots x_n)$.

13. (a) Suppose that $Y = \overline{X}$, the sufficient statistic is the mean of a random sample of size n that arises form the normal distribution $N(\theta, \sigma^2)$, where σ^2 is known. Prove that $g(y/\theta)$ is $N(\theta, \sigma^2/n)$

Or

(b) Let s^2 denote the variance of a random sample of size $n > 1$ from a distribution that is $N(\mu, \theta)$, $0 < \theta < \infty$, where μ is known. We know that $E[nS^2/(n-1)] = \theta$. What is the efficiencies of the estimator $nS^2/(n-1)$?

14. (a) Let $X_1, X_2, \ldots X_n$ denote the random sample from a distribution that is $N(\theta, 1)$, where the mean θ is unknown. The simple hypothesis $H_o : \theta = \theta'$ where θ' is a fixed number, the alterative composite hypothesis $H_1 : \theta \neq \theta'$. Show that there is no uniformly most powerful test for testing H_o against H_1

Or

(b) Let $X_1, X_2, \ldots X_{25}$ be a random sample of size 25 from a normal distribution $n(\theta, 100)$. Find a uniformly most powerful critical region of size $\alpha = 0.10$ for testing $H_o : \theta = 75$ against $H_1 : \theta > 75$.

15. (a) With the usual notations, prove that $Q = Q_3 + Q_4$.

Or

(b) State and prove Boole's inequality.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Derive the confidence intervals for difference of means.

(b) Let two independent random samples, each of size 10, from two normal distribution $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\bar{x} = 4.8, s_1^2 = 8.64$, $\bar{y} = 5.6$, $s_2^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$.

17. State and prove the Neyman factorization theorem.

18. (a) Derive the Fisher information.

(b) Enumerate the limiting distribution of maximum likelihood estimator.

19. State and prove the Neyman-Pearson theorem.

20. Describe the analysis of variance for two-way classification.

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F–9393

Sub. Code
7MMA4E3

M.Sc. DEGREE EXAMINATION, APRIL 2023

Fourth Semester

Mathematics

***Elective* — NUMERICAL METHODS**

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write a short notes on the asymptotic error constant.

2. State the Sturm theorem.

3. Write down the Matrix norm.

4. State the Brauer theorem.

5. Define the Hermite interpolating polynomial.

6. Write down the Lagrange bivariate interpolating polynomial.

7. What is meant by the Richardson's extrapolation?

8. Write down the two-point formula for Gauss-Legendre integration methods.

9. Define absolutely stable.

10. State the Backward Euler method.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Obtain the number of real roots between 0 and 3 of the equation $P(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$ using Sturm sequence.

Or

(b) Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $P_3(x) = x^3 + x^2 - x + 2 = 0$. Use the initial approximations $p_0 = -0.9, q_0 = 0.9$.

12. (a) Find the condition number of the system $\begin{bmatrix} 2.1 & 1.8 \\ 6.2 & 5.3 \end{bmatrix}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}$, using the spectral norm.

Or

(b) Let A be a square matrix. Prove that $\lim_{m \to \infty} A^m = 0$ if $\|A\| = 1$, or $\Leftrightarrow P(A) < 1$.

13. (a) The following data for a function $f(x, y)$ is given

y / x	0	1	3
0	1	2	10
1	2	4	14
3	10	14	28

Construct the bivariate interpolating polynomial and hence find $f(0.5, 0.5)$.

Or

(b) Obtain a linear polynomial approximation to the function $f(x) = x^3$ on the interval [0, 1] using the least squares approximation with $W(x) = 1$.

14. (a) The following data for the function $f(x) = x^4$ is given

x :	0.4	0.6	0.8
$f(x)$:	0.0256	0.1296	0.4096

Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation compare with the exact solution. Obtain the bound on the truncation errors.

Or

(b) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$, using Gauss-Legendre three point formula.

15. (a) Explain the Euler method for solving ordinary differential equation with initial value problem.

Or

(b) Given the initial value problem $u' = t^2 + u^2$, $u(0) = 0$. Determine the first three non-zero terms in the Taylor series for $u(t)$ and hence obtain the value for $u(1)$. Also determine t when the error in $u(t)$ obtained from the first two non-zero terms is to be less than 10^{-6} after rounding.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find all the roots of the polynomial $x^3 - 6x^2 + 11x - 6 = 0$ using the Graeffe's root squaring method.

17. Using the Jacobi method find all the eigenvalues and the corresponding eigenvectors of the matrix.

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

18. Obtain the cubic spline approximation for the function defined by the data :

x :	0	1	2	3
$f(x)$:	1	2	33	244

with $M(0)=0, M(3)=0$. Hence find an estimate of $f(2.5)$.

19. Find the quadrature formula $\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$ which is exact for polynomials of highest possible degree. Then use the formula on $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ and compare with the exact value.

20. Solve the initial value problem $u' = -2tu^2, u(0)=1$ using the mid-point method, with $h=0.2$, over the interval [0, 1]. Use the Taylor series method of second order to compute $u(0.2)$. Determine the percentage relative error at $t = 1$.