F–0619

Sub. Code
7MMA2C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Second Semester

Mathematics

ALGEBRA — II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. If V is a vector space over F Prove that $Ov = 0$ for $v \in V$.

2. Define linearly dependent and linearly independent over F.

3. Define an inner product space.

4. Prove that $W^{\perp}$ is a subspace of V.

5. What is the degree of $\sqrt{2} + \sqrt{3}$ over Q?

6. For any $f(x), g(x) \in F(x)$, prove that $(f(x) + g(x))' = f'(x) + g'(x)$.

7. Define a fixed field of a group of automorphisms of K.

8. Let K be the field of complex numbers and let F be the field of real numbers, then compute $G(K, F)$.

9. When we say that two linear transformations are similar?

10. If $T, S \in A(V)$ then prove that $(S+T)^* = S^* + T^*$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Define a linear span of a non empty subset of the vector space. Also prove that linear span is a subspace of V.

Or

(b) If $v_1, \ldots.., v_n$ is a basis of V over F and if $w_1, \ldots.., w_n$ in V are linearly independent over F, then prove that $m \le n$.

12. (a) Prove that $A(A(W)) = W$.

Or

(b) State and prove the Schwarz inequality.

13. (a) State and prove the Remainder Theorem.

Or

(b) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

14. (a) If K is a finite extension of F, then prove that $G(K, F)$ is a finite group and its order, $O(G(K,F))$ satisfied $O(G(K,F)) \le [K:F]$.

Or

(b) Show that the fixed field of G is a subfield of K.

15. (a) If V is finite-dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not O.

Or

(b) If N is normal and $AN = NA$, then prove that $AN^* = N^*A$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. If V is a vector space over F and if W is a subspace of V, then prove that V/W is a vector space over F, where for $v_1 + W, v_2 + W \in V/W$ and $\alpha \in F$,

(a) $(v_1 + W) = (v_2 + W) = (v_1 + v_2) + W$

(b) $\alpha(v_1 + W) = \alpha v_1 + W$.

17. Let V be a finite-dimensional inner product space, then prove that V has an orthonormal set as a basis.

18. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover $[L : F] = [L : K][K : F]$.

19. State and prove the fundamental theorem of Galois theory.

20. (a) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

(b) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that λ is a root of the minimal polynomial of T. In particular, T only has a finite number of characteristic roots in F.

F–0620

Sub. Code
7MMA2C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Second Semester

Mathematics

ANALYSIS — II

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the Reimann-Stieltjes integral of f with respect to α, over $[a, b]$.

2. Define the unit step function.

3. Define a uniformly bounded function on a set E.

4. Define an algebra of a family A of complex functions defined on a set E.

5. Let e^x be defined on R^1. Prove that $(e^x)' = e^x$.

6. When will you say that a function is said to be orthonormal?

7. Define the outer measure with an example.

8. State the Egoroff's theorem.

9. With the usual notations, prove that $\chi_{A \cap B} = \chi_A \cdot \chi_B$.

10. Define the following terms :

(a) $f^+(x)$

(b) $f^-(x)$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If $f_1, f_2 \in \mathscr{R}(\alpha)$ on $[a, b]$, then prove that $f_1 + f_2 \in \mathscr{R}(\alpha)$, $cf \in \mathscr{R}(\alpha)$, $\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$ and $\int_a^b cf d\alpha = c \int_a^b f d\alpha$.

Or

(b) If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)| \, dt$.

12. (a) Let $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ (x real, n = 1, 2, 3,.....). Show that $\{f_n'\}$ does not converge to f' where $f(x) = \lim_{n \to \infty} f_n(x)$.

Or

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

13. (a) State and prove Parseval's theorem.

Or

(b) If $x > 0$ and $y > 0$, then prove that

$$\int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}.$$

14. (a) If E_1 and E_2 are measurable, then prove that $E_1 \cup E_2$ is measurable.

Or

(b) State and prove the Lusin's theorem.

15. (a) Let ϕ and ψ be simple functions which vanish outside a set of finite measure. Prove that $\int(a\phi + b\psi) = a\int\phi + b\int\psi$, and if $\phi \geq \psi$ a.e., then $\int\phi \geq \int\psi$.

Or

(b) State and prove Fatou's lemma.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the fundamental theorem of calculus.

(b) If $f \in \mathcal{R}(\alpha)$ on $[a, b]$, then prove that $|f| \in \mathcal{R}(\alpha)$ and $\left|\int_a^b f\,d\alpha\right| \leq \int_a^b |f|\,d\alpha$.

17. State and prove the Stone-Weierstrass theorem.

18. Define the Gamma function and prove the following :

(a) $\Gamma(x+1) = x\,\Gamma(x)$;

(b) $\Gamma(1) = 1$;

(c) $\log \Gamma(x)$ is convex on $(0, \infty)$;

(d) $\Gamma(n+1) = n!$ for $n = 1, 2, 3, \ldots.$

19. Show that the outer measure of an interval is its length.

20. State and prove the Lebesgue convergence theorem.

F–0621

Sub. Code
7MMA2C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Show that the direction cosinas of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

2. Verify that the differential equation $\left(y^2 + yz\right)dz + \left(xz + z^2\right)$ $dy + \left(y^2 - xy\right)dz = 0$ is integrable.

3. Form a partial differential equation by eliminating a, b from $2z = (ax + y)^2 + b$.

4. Show that every characteristic strip of the partial differential equation $f(x, y, z, p, q)$ is a constant.

5. Find a complete integral of the equation $p^2 z^2 + q^2 = 1$.

6. Write down the fundamental idea of Jacobi's method.

7. Solve $r = a^2 t$.

8. Eliminate the arbitrary functions f and g from the equation $z = f(ax + y) + g(-ax - y)$.

9. Write down the exterior Dirchlet boundary value problem for Laplace's equation.

10. Define interior Churchill problem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Solve the Pfaffian differential equation $(y+z)\,dx + (z+x)\,dy + (x+y)\,dz = 0$.

Or

(b) Find the integral curves of the set of equation $\dfrac{a\,dx}{(b-c)\,yz} = \dfrac{bdy}{(c-a)\,zx} = \dfrac{cdz}{(a-b)xy}$.

12. (a) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$.

Or

(b) Find the general integrals of the linear partial differential equations $px(x+y) - qy(x+y) + (x-y)(2x+2y+z) = 0$.

13. (a) Show that the equations $xp - yq = x$, $x^2 p + q = xz$ are compatible and find their solutions.

Or

(b) Find a complete integral of the equation $p^2 x + qy = z$, and hence derive the equation of an integral surface of which the line $y = 1$, $x + z = 0$ is a generator.

14. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

Or

(b) Determine the solution of the equation $\frac{\partial^2 z}{\partial x^4} + \frac{\partial^2 z}{\partial y^2} = 0$ $(-\infty < x < \infty, y > 0)$ satisfying the conditions.

(i) z and its partial derivatives tend to zero as $x \to \pm\infty$;

(ii) $z = f(x), \frac{\partial z}{\partial y} = 0$ on $y = 0$.

15. (a) If $p > 0$ and $\psi(t)$ is given by $\psi = \int_V \frac{\rho(r')d\tau'}{|r - r'|}$ where the volume v is bounded. Prove that $\lim_{r\to\infty} r\psi(r) = M$, where $M = \int_V \rho(r')dr'$.

Or

(b) Prove that the solution of a certain Neumann problem can differ from one another by a constant only.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that a necessary and sufficient condition that the Pfaffian differential equation $X \cdot dr = 0$ should be integrable is that $X \cdot curl X = 0$.

17. Verify that the equations (a) $z=\sqrt{2x+a}+\sqrt{2y+b}$ (b) $z^2+\mu=2\left(1+\lambda^{-1}\right)(x+\lambda y)$ are both complete integrals of the partial differential $z=\frac{1}{p}+\frac{1}{q}$. Show that the complete integral (b) is the envelope of the one parameter subsystem obtained by taking $b=\frac{a}{\lambda}-\frac{\mu}{1+\lambda}$ in the solution (a).

18. Find a complete, singular and general integrals of $\left(p^2+q^2\right)y=qz$.

19. Solve $\frac{\partial^2 z}{\partial x^2}=\frac{1}{C^2}\frac{\partial^2 z}{\partial t^2}$, by separable variable method.

20. The points of trisection of a string are pulled aside through a distance ε on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest.

F–0622

Sub. Code
7MMA2C4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Second Semester

Mathematics

MECHANICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What are nonholonomic constraints?

2. State the principle of virtual work.

3. Write down the Lagrange's equations for nonholonomic system.

4. Define the Routhian function.

5. Show that for a conservative holonomic system, the Hamiltonian has a constant value.

6. State modified Hamilton principle.

7. Write down the Hamilton's canonical equations.

8. Give the modified Hamilton-Jacobi equation.

9. Define Momentum transformation.

10. State the Poisson bracket.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Narrate the following terms :

(i) Configuration space;

(ii) Generalized force.

Or

(b) State and prove the Konig's theorem.

12. (a) Show that the Lagrange's equation for a holonomic system can be put in the form $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$, $i = 1,2,\ldots n$.

Or

(b) Define a conservative system. Also find the Jacobi integral.

13. (a) Find the stationary values of the function $f = z$, subject to the constraints $x^2 + y^2 + z^2 - 4 = 0$ and $xy - 1 = 0$.

Or

(b) Using Hamiltonian H, find the equation of motion for a charged particle in an electromagnetic field.

14. (a) Explain the following terms :

(i) Hamilton's principal function;

(ii) Pfaffian differential forms.

Or

(b) Derive the Hamilton – Jacobi equation.

15. (a) Show that the transformation $Q = \frac{1}{2}(q^2 + p^2)$, $P = -\tan^{-1}(q/p)$ is canonical.

Or

(b) Enumerate the point transformation.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. A particle of mass m is suspended by a massless wire of length $r = a + b\cos\omega t$ $(a > b > 0)$ to form a spherical pendulum. Find the equation of motion.

17. Obtain the differential equations of motion for a spherical pendulum of length l. Reduce the problem to quadratures and obtain the integrals of motion.

18. Establish the principle of least action. Deduce the Jacobi's form of the principle of least action.

19. State and prove the Stackel's theorem.

20. Define Lagrange bracket $[u, v]$ and show that $[q_j, q_k] = [p_j, p_k] = 0$, while $[q_i, p_k] = \delta_{jk}$.

F–0623

Sub. Code
7MMA2E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Second Semester

Mathematics

Elective – GRAPH THEORY

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the following terms:

   (a) Vertex – transitive graph;

   (b) Edge-transitive graph.

2. What is meant by a bond? Give an example.

3. Define the k-connected graph. Give an example for a 3-connected graph.

4. Define a Hamiltonian graph with an example.

5. State the Hall's theorem.

6. Define an edge chromatic number,

7. Define the Ramsey numbers. Also find $r(2,l)$.

8. Define k-critical graph. Give an example.

9. Embed $k_{3,3}$ on mobius band.

10. Write short notes on the four-colour conjecture.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If a k-regular bipartite graph with $k > 0$ has bipartition (X, Y), then prove that $|X| = |Y|$.

Or

(b) Define a spanning tree with an example. Also prove that every connected graph contains a spanning tree.

12. (a) With the usual notations, prove that $K \leq K' \leq \delta$.

Or

(b) Prove that a connected graph has an Euler trial if and only if it has at most two vertices of odd degree.

13. (a) Define a perfect matching. If G is a k-regular bipartite graph with $k > 0$, then prove that G has a perfect matching.

Or

(b) If G is bipartite, then prove that $\chi' = \Delta$.

14. (a) With the usual notations, prove that $r(k,l) \leq \binom{k+l-2}{k-1}$.

Or

(b) Prove that in a critical graph, no vertex cut is a clique.

15. (a) Let v be a vertex of a planar graph G. Prove that G can be embedded in the plane in such a way that v is on the exterior face of the embedding.

Or

(b) If two bridges overlap, then prove that either they are skew or else they are equivalent 3-bridges.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Show that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

(b) If e is a link of G, then prove that $\tau(G) = \tau(G-e) + \tau(G.e)$.

17. Prove that a grapy G with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths.

18. If G is simple, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

19. State and prove that Brook's theorem.

20. Prove that following three statements are equivalent:

(a) Every planar graph is 4-vertex-colourable;

(b) Every plane graph is 4-face colourable;

(c) Every Simple 2-edge-connected 3-regular planar graph is 3-edge-colourable.

F–0624

Sub. Code
7MMA3C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Write down the Hadamard's formula.
2. Distinguish between translation, rotation and inversion.
3. Compute $\int_{|z|=1} e^z z^{-n} dx$.
4. State the Liouville's theorem.
5. Define an isolated singularity. Give an example.
6. Find the poles of $\frac{1}{\sin z}$.
7. Find the residue of the function $\frac{1}{z^m (1-x)^n}$ (m, n positive integers).
8. State the argument principle theorem.
9. Obtain the series expansion for arc tan z and arc sin z.
10. What is the genus of $\cos\sqrt{z}$?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that a harmonic function satisfies the formal differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$.

Or

(b) Show that the function $f(z)$ is analytic if $\frac{\partial f}{\partial \bar{z}} = 0$.

12. (a) Prove that the line integral $\int_\gamma p\,dx + q\,dy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p$, $\frac{\partial U}{\partial y} = q$.

Or

(b) Define the winding number of γ with respect to a. Also prove the Cauchy's estimate theorem.

13. (a) State and prove the Taylor's theorem.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the residue theorem.

Or

(b) Evaluate $\int_0^\infty \frac{x^2}{x^4 + 5x^2 + 6}\,dx$.

15. (a) State and prove the Hurwitz theorem.

Or

(b) What is meant by entire function? Also prove that every function which is mesomorphic in the whole plane is the quotient of two entire functions.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Abel's theorem.

17. State and prove Cauchy's theorem for a rectangle.

18. State and prove the local mapping theorem.

19. Show that $\int_0^{\pi} \log \sin x \, dx = \pi \log\left(\frac{1}{2}\right)$.

20. Obtain the Laurent expansion $\sum_{n=-\infty}^{\infty} A_n (z-a)^n$ for the function $f(z)$ analytic in $R_1 < |z-a| < R_2$.

F–0625

Sub. Code
7MMA3C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Give an example, for a set which is opening in the subspace need not be open in the whole space.

2. Define the Interior point with an example.

3. Define Product Space.

4. State the pasting Lemma.

5. What are components and path components of $\mathbb{R}_l$?

6. Determine whether or not $\mathbb{R}^w$ is connected in the uniform topology.

7. State the extreme value theorem.

8. Define compact space, with an example.

9. Give an example, for a 2^{nd} – countable is 1^{st} – countable, but not the converse.

10. Whether the space $\mathbb{R}_K$ is regular or not? Justify your answer.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let x be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of x such that for each open set U of x and each x in U, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Prove that $\mathcal{C}$ is a basis for the topology of x.

Or

(b) Let A be a subset of the topological space x. Let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

12. (a) Let x be a metric space with metric d. Define $\overline{d} : x \times x \to \mathbb{R}$ by the equation $\overline{d}\,(x,y) = \min\{\, d(x,y), 1\}$. Prove that $\overline{d}$ is a metric that induces the same topology as d.

Or

(b) State and prove uniform limit theorem.

13. (a) Prove that a space x is locally path connect if and only if for every open set U of x, each path component of U is open in x.

Or

(b) Show that the union of a collection of connected subspaces of x that have a point in common is connected.

14. (a) State and prove the tube lemma.

Or

(b) Prove : Compactness implies limit point compactness, but not conversely.

15. (a) Prove that, a subspace of a Hausdroff space is Hausdroff; a product of Hausdroff is Hausdroff.

Or

(b) Suppose that x has a countable basis. Prove that every open covering of x contains a countable sub collection covering x.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Prove that the collection $S = \{\pi_1^{-1}(U) / u \text{ is open in } x\} U$ $\{\pi_2^{-1}(V) / v \text{ is open in } y\}$ is a subbasis for the product topology on $x \times y$.

(b) Show that y is a subspace of x, and A is a subset of y, then the topology A inherits as a subspace of y is the same as the topology it inherits as a subspace of x.

17. Let x and y be topological spaces. Let $f : x \to y$. Then the following are equivalent :

(a) f is continuous.

(b) for every subset A of x, one has $f(\overline{A}) \subset \overline{f(A)}$.

(c) for every closed set B of y, the set $f^{-1}(B)$ is closed in x.

(d) for each $x \in X$ and each neighborhood v of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

18. Let L be a linear continuum in the order topology. Prove that L is connected, and so are intervals and rays in L.

19. State and prove the Lebesque number lemma.

20. State and prove the Urysohn metrization theorem.

F–0626

Sub. Code
7MMA3C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Third Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Bowl I contains 3 red chips and 7 blue chips. Bowl II contains 6 red chips and 4 blue chips. A bowl is selected at random and then I chips is drawn from this bowl. Compute the probability that this chip is red.

2. Find the p.d.f., mode of the distribution $F(x) = \begin{cases} 1 - e^{-x} - x e^{-x}, & 0 \le x < \infty \\ 0, & elsewhere \end{cases}$.

3. Let x_1 and x_2 have the joint p.d.f., $f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$ find the marginal p.d.f. of x_1 and x_2.

4. Define a negative binomial distribution.

5. If the m.g.f. of a random variable x is $\left(\frac{2}{3}+\frac{1}{3}e^t\right)^5$, find $\Pr(0 \le x \le 3)$..

6. If x is $N(\mu, \sigma^2)$, find b so that $\Pr\left[-b < \frac{(x-\mu)}{\sigma} < b\right] = 0.90$.

7. Let x have the p.d.f. $f(x) = \begin{cases} \frac{x^2}{9}, & 0 < x < 3 \\ 0, & elsewhere \end{cases}$. Find the p.d.f. of $y = x^2$.

8. Write down the mean and variance of the F-distribution.

9. Let x be $\chi^2(50)$. Approximate $\Pr(40 < x < 60)$.

10. When the say that a sequence of random variables $x_1, x_2, ...$ converges in probability to a random variable X?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let x have the p.d.f. $f(x) = \begin{cases} 2(1-x), & -2 < x < 4 \\ 0, & elsewhere \end{cases}$.

Find $E(x)$, $E(x^2)$ and $E[bx + 2(x+2)^3]$.

Or

(b) Cast a die a number of independent times until a six appears on the up side of the die.

(i) Find the p.d.f. $f(x)$ of x, the number of casts needed to obtain that first six.

(ii) Show that $\sum_{x=1}^{\infty} f(x) = 1$.

12. (a) Let x_1 and x_2 have the joint p.d.f. $f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1 \\ 0 & elsewhere \end{cases}$. Find the conditional mean and variance of x_1, given $x_2 = x_2, 0 < x_2 < 1$.

Or

(b) Show that the random variables x_1 and x_2 with joint p.d.f. $f(x_1, x_2) = \begin{cases} 12 x_1 x_2 (1 - x_2), & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$ are independent.

13. (a) Let x and y have a bivariate normal distribution with parameters $\mu_1 = 3, \mu_2 = 1, \sigma_1^2 = 16, \sigma_2^2 = 25$, and $\rho = \frac{3}{5}$. Determine the following probabilities:

(i) $\Pr(3 < y < 8)$.

(ii) $\Pr(3 < y < 8 \mid x = 7)$

(iii) $\Pr(-3 < x < 3)$.

Or

(b) Compute the measures of skewness and kurtosis of a gamma distribution with parameters α and β.

14. (a) Let x_1 and x_2 have the joint p.d.f.

$$f(x_1,x_2)\begin{cases} \dfrac{x_1 x_2}{36}, & x_1 = 1,2,3 \text{ and } x_2 = 1,2,3. \\ 0 & elsewhere \end{cases}.$$

(i) Find the joint p.d.f. of $y_1 = x_1 x_2$ and $y_2 = x_2$

(ii) Find the marginal p.d.f. of y_1

Or

(b) Let $y_1 < y_2 < y_3 < y_4 < y_5$ denote the order statistics of a random sample of size 5 from a distribution having p.d.f. $f(x) = e^{-x}, 0 < x, \infty$, zero elsewhere. Show that $z_1 = y_2$ and $z_2 = y_4 - y_2$ are independent.

15. (a) Let $\overline{x_n}$ denote the means of a random sample of size n from a distribution $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{x_n}$.

Or

(b) Let $F_n(u)$ denote the distribution function of a random variable U_n whose distribution depends upon the positive integer n. Further, Let U_n converge in probability to the positive constant c and let $\Pr(U_n < 0) = 0$ for every n. Prove that the random variable $\sqrt{U_n}$ converges in probability to $\sqrt{c}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Given the distribution function $F(x) = \begin{cases} 0 \ , & x < -1 \\ \frac{x+2}{4} \ , & -1 \le x < 1 \\ 1 \ , & 1 \le x \end{cases}$.

(a) Sketch the graph of $F(x)$.

(b) $\Pr(-\frac{1}{2} < x \le \frac{1}{2})$.

(c) $\Pr(x = 0)$.

(d) $\Pr(x = 1)$

(e) $\Pr(2 \le x \le 3)$.

17. Let x and y have the joint p.d.f. described as follows

(x,y)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
f(x,y)	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{4}{15}$

And $f(x,y)$ is equal to zero, elsewhere.

(a) Find the means μ_1 and μ_2, the variances σ_1^2 and σ_2^2, and the correlation coefficient ρ.

(b) Compute $E(y \mid x = 1)$, $E(y \mid x = 2)$, and the line $\mu_2 + \rho\left(\frac{\sigma_2}{\sigma_1}\right)(x - \mu_1)$.

(c) Do the points $[k, E(y / x = k)]$, $k - 1,2$, lie on the line?

18. Find the m.g.f. of a normal distribution and hence find mean and variance of a normal distribution.

19. Derive the p.d.f. of F–distribution.

20. State and prove the central limit theorem.

F–0627

Sub. Code
7MMA3E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Third Semester

Mathematics

Elective – DISCRETE MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Find a set of three numbers which is closed under multiplication.

2. Define semigroup homomorphism.

3. Is the sequence $D(k)=5.2^k, k \geq 0$ homogeneous relation?

4. Using recursion and iteration process to find the F_6 of the Fibonacci numbers.

5. Find $T(27)$ where $T(n)$ denotes the worst time for binary search of a file with n records.

6. Define a primitive recursive function.

7. Find least and greast element for the poset $(D(24), \leq)$ where $\leq$ is the order " divisor of ".

8. Define distributive lattice with an example.

9. Is the lattice of divisor of 32 a Boolean algebra?

10. Draw the circute diagram for the function $P=(x_1' x_2)' + x_3$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that the semigroups $(z,+)$ and $(T,+)$ are isomorphic where T is the set of all even integers.

Or

(b) (i) If $(a*b)^2 = a^2 * b^2$, for all $a,b \in G$, then prove that G is abelian.

(ii) Define subsemigroup.

12. (a) Show that $a^n - b^n$ is divisible by $(a-b)$ for all $n \in N$.

Or

(b) Find $f(n)$ if $f(n) = 7f(n-1) - 10f(n-2)$ given that $f(0)=4$ and $f(1)=17$

13. (a) Solve $S(k) - S(k-1) - 6S(k-2) = -30$ where $S(0)=20$, $S(1)=-5$.

Or

(b) If A denotes Ackermann's function, evaluate

(i) $A(2,2)$;

(ii) $A(3,2)$.

14. (a) Show that the distributive and modular inequalities are satisfied the elements of an arbitrary lattice.

Or

(b) Draw the Hasse diagrams of $D(10), D(15), D(32)$ and $D(45)$ where $D(n)$ denotes the lattices of all +ve divisors of the integer n.

15. (a) Find the principal disjunctive normal form of $P(x_1, x_2, x_3) = (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_1 \vee \overline{x}_2)$.

Or

(b) Write down the minimization algorithm for Boolean polynomials.

Part C (3 × 10 = 30)

Answer any **three** qssuestions.

16. (a) If G is an abelian group, then for all $a, b \in G$, show that $(a * b)^n = a^n * b^n$ for every integer n.

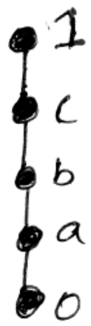
(b) Construct the Cayley table for the set z_5 under the binary operation module addition $+_5$.

17. Solve, using characteristic equation method, the recurrence relations $S(k) - 7S(k-2) + 6S(k-3) = 0$ where $S(0) = 8, S(1) = 6, S(2) = 22$.

18. Using the generating function, solve the difference equation $y_{n+2} - y_{n+1} - 6y_n = 0$ given $y_1 = 1, y_0 = 2$.

19. (a) Show that a lattice L is distributive if and only if for all a,b,c in $L, (a \vee b) \wedge c \leq av(b \wedge c)$.

(b) Determine the operation table for $\wedge$ and $\vee$ for the lattices whose Hasse diagrams is given in figure



20. Let B be a finite Boolean algebra and let A be the set of all atoms of B. Prove that the Boolean algebra B is isomorphic to the Boolean algebra $P(A)$.

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F–0629

Sub. Code
7MMA3E4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Third Semester

Mathematics

Elective – FUZZY MATHEMATICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Differentiate between a crisp set and a fuzzy set.

2. Define the following terms:
   (a) Scalar cardinality;
   (b) Fuzzy cardinality.

3. Write down the axiomatic Skeleton for fuzzy complements.

4. Define the union of two fuzzy sets. Give an example.

5. Write a short notes on Cartesian product of fuzzy sets.

6. When will you say that the relation is said to be antitransitive?

7. Define a fuzzy measure. Give an example.

8. Write a short notes on the lattice of possibility distributions of length n.

9. What is meant by grouping requirement?

10. State the Gibbs' inequality.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Narrate the following terms with suitable example for each:

(i) L – fuzzy sets;

(ii) Normalized fuzzy set;

(iii) Fuzzy number.

Or

(b) Suppose that f is a function mapping ordered pairs from $x_1 = \{a, b, c\}$ and $x_2 = \{x, y\}$ to $Y = \{p, q, r\}$. Let f be specified by the following matrix:

$$\begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{pmatrix} p & p \\ q & r \\ r & p \end{pmatrix} \end{array}$$

find the extension principle $f(A_1, A_2)$.

12. (a) What is meant by a dual point? For each $a \in [0,1]$, prove that ${}^d a = c(a)$ if and only if $c(c(a)) = a$.

Or

(b) For all $a, b \in [0,1]$, prove that $i(a,b) \geq i_{\min}(a,b)$.

13. (a) Narrate the following terms with an illustration for each:

(i) Cylindric extension;

(ii) Sagittal diagram;

(iii) Fuzzy equivalence relation.

Or

(b) Determine the transitive max-min closure $R_T(x,x)$ for a fuzzy relation $R(x,x)$ defined by the membership matrix $M_R = \begin{pmatrix} 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{pmatrix}$.

14. (a) Let $X = \{a,b,c,d\}$. Given the basic assignment $m(\{a,b,c\}) = 0.5$, $m(\{a,b,d\}) = 0.2$, and $m(x) = 0.3$. Find the corresponding belief and plausibility.

Or

(b) Show that a belief measures that satisfies $\eta(A \cap B) = \min[\eta(A), \eta(B)]$ is based on nested focal elements.

15. (a) Consider two fuzzy sets, A and B, defined on the set of real numbers $x = [0,4]$ by the membership grade functions $\mu_A(x) = \frac{1}{1+x}$ and $\mu_B(x) = \frac{1}{1+x^2}$. Draw graphs for these functions and their standard classical complements.

Or

(b) Enumerate the Boltzmann entropy.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Consider the fuzzy sets A, B and C defined on the interval $x = [0,10]$ of real numbers by the membership grade functions $\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = 2^{-x}$, $\mu_C(x) = \frac{1}{1+10(x-2)}$.

Determine mathematical formulas and graphs of the membership grade functions of each of the following:

(a) $\overline{A}, \overline{B}, \overline{C}$;

(b) $A \cup B$, $B \cup C$;

(c) $A \cap B$, $A \cap C$;

(d) $A \cap B \cap C$.

17. Prove that fuzzy set operations of union, intersection, and continuous complement that satisfy the law of excluded middle and the law of contradiction are not idempotent or distributive.

18. Solve the following fuzzy relation equation:

$$p_0 \begin{bmatrix} 0.5 & 0.7 & 0 & 0.2 \\ 0.4 & 0.6 & 1 & 0 \\ 0.2 & 0.4 & 0.5 & 0.6 \\ 0 & 0.2 & 0 & 0.8 \end{bmatrix} = [0.5 \quad 0.5 \quad 0.4 \quad 0.2].$$

19. Prove: Given a consonant body of evidence ($\mathfrak{F}$, m), the association consonant belief and plausibility measure possess the following properties:

(a) $Bel(A \cap B) = \min[Bel(A), Bel(B)]$ for all $A, B \in \mathcal{P}(x)$;

(b) $Pl(A \cup B) = \max[Pl(A), Pl(B)]$ for all $A, B \in \mathcal{P}(x)$.

20. Let m_x and m_y marginal basic assignments on set x and y, respectively, and let m be a joint basic assignment on $x \times y$ such that $m(A \times B) = m_x(A) \cdot m_y(B)$ for all $A \in \mathcal{P}(x)$ and $B \in \mathcal{P}(y)$. Prove that $E(m) = E(m_x) + E(m_y)$.

F–0632

Sub. Code
7MMA4C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023.

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the Euclidean norm. Give an example.

2. When will you say that a normed space x is said to be strictly convex?

3. Define a Hamel Basis for a normed space x with an example.

4. What is meant by a support functional?

5. Define a continuous seminorm.

6. Define the graph of F.

7. What do you mean by dual of normal space?

8. State the Riesz representation theorem for L^p.

9. Define an inner product on a linear space.

10. What is meant by the orthogonal projection?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let Y be a closed subspace of a normed space x. For $x+y$ in the quotient space x/y, let $\|x+y\| = \inf\{\|x+y\| : y \in y\}$. Prove that $\| \ \|$ is a norm on x/y.

Or

(b) Let x any y be normed spaces. If x is finite dimensional, then prove that every linear map from x to y is continuous. Conversely, if x is infinite dimensional and $y \neq \{0\}$, then prove that there is a discontinuous linear map from x to y.

12. (a) State and prove the Hahn-Banach extension theorem.

Or

(b) Let x be a normed space any y be a Banach space. Let X_0 be a dense subspace of x and $F_0 \in BL(x_0, y)$. Prove that there is a unique $F \in BL(x, y)$ such that $Fl_{x_0} = F_0$. Also prove $\|F\| = \|F_0\|$.

13. (a) State and prove the Banach – Steinhaus theorem.

Or

(b) Let x any y be Banach spaces and $F : x \to y$ be a linear map which is closed and surjective. Prove that F is continuous and open.

14. (a) Let x be a normed space. If x' is separable, then prove that x is also separable.

Or

(b) Let x be a normed space and $A \in BL(x)$. Prove that $\sigma(A') \subset \sigma(A)$.

15. (a) State the schwarz inequality. Also prove that parallelogram law.

Or

(b) Derive the Parseval formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let x denote a subspace of B(T) with the sup norm, $1 \in x$ and f be a linear functional on x. If f is continuous and $\|f\| = f(1)$, then prove that f is positive. Conversely, if $\operatorname{Re} x \in x$ Whenever $x \in x$ and if f is positive, then prove that f is continuous and $\|f\| = f(1)$.

17. State and prove the Taylor-Foguel theorem.

18. State and prove closed graph theorem.

19. State and prove the Riesz representation theorem for $C([a,b])$.

20. State and prove projection theorem.

F–0633

Sub. Code
7MMA4C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a connected network. Give an example.

2. What is meant by pessimistic time estimate?

3. Define holding cost.

4. Define the effective lead time.

5. Write short notes on queue discipline.

6. Define a Poisson distribution. Give an example.

7. Draw the transition rate diagram.

8. Find P_n of the model $(M / M / \infty) : (GD / \infty \mid \infty)$.

9. Write short notes on golden section method.

10. Define separable function.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Enumerate the maximal flow algorithm.

Or

(b) Construct the project network comprised of activities A to L with the following precedence relationships :

(i) A, B and C, the first activities of the project can be executed concurrently.

(ii) A and B precede D.

(iii) B precedes E, F and H

(iv) F and C precede G

(v) E and H precede I and J

(vi) C, D, F and J precede K

(vii) K precedes L

(viii) I, G and L are the terminal activities of the project

12. (a) Enumerate the multi item EOQ with storage limitation.

Or

(b) A company uses annually 12,000 units of a raw material costing Rs. 1.25 per unit. Placing each order costs 45 paise, and the carrying costs are 15% per year per unit of the average inventory. Find the economic order quantity.

13. (a) Explain the role of exponential distribution.

Or

(b) Derive the Poisson distribution from the difference differential equations of the pure birth model.

14. (a) Derive L_s, W_s, W_q and L_q for $(M/M/1):(GD/\infty/\infty)$ queuing model.

Or

(b) Kleen all is a service company that performs a variety of odd jobs, such as yard work, tree pruning. The company's four employees leave the office with the first assignment of the day. After completing an assignment, the employee calls the office requesting instruction for the next job to be performed. The time to complete an assignment is exponential, with a mean of 45 minutes. The travel time between jobs is also exponential, with a mean of 20 minutes.

(i) Determine the average number of employees who are travelling between jobs.

(ii) Compute the probability that no employee is on the road.

15. (a) Carry out atmost three iterations of the following using the method of steepest-ascent. Also assume that $x_0 = (1,1)$.

Maximize $f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Or

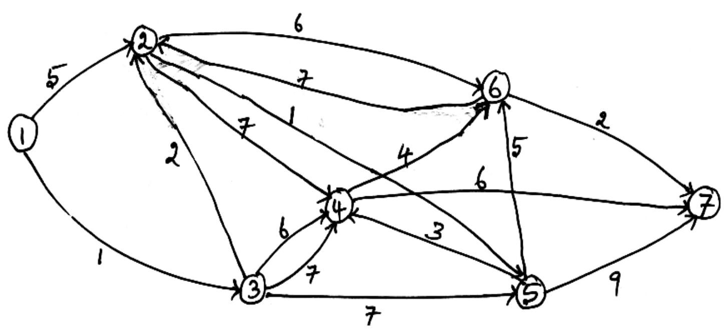
(b) Find the maximum value of the following function by dichotomous search by taking $\Delta = 0.05$.

$$f(x) = \frac{1}{\left|(x-3)^3\right|},\ 2 \le x \le 4$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Enumerate Dijkstra's algorithm. Use Dijkstra's algorithm to find the shortest route between node 1 and every other node in the network.



17. Find the optimal inventory policy for the following five-period model. The unit production cost is Rs. 10 for all periods. The unit holding cost is Rs. 1 per period.

Period i	Demand D_i (units)	Setup cost k_i ($)
1	50	80
2	70	70
3	100	60
4	30	80
5	60	60

18. Inventory is withdrawn from a stock of 80 items according to a Poisson distribution at the rate of 5 items per day. Determine the following :

(a) The probability that 10 items are withdrawn during the first 2 days.

(b) The probability that no items are left at the end of 4 days.

(c) The average number of items withdrawn over a 4 day period.

19. B and K groceries operates with three check-out counters. The manager uses the following schedule to determine the number of counters in operation, depending on the number of customers in store :

No. of customers in store	No. of customers in operation
1 to 3	1
4 to 6	2
More than 6	3

Customers arrived in the counters area according to a Poisson distribution with a mean rate of 10 customers per hour. The average check-out time per customer is exponential with mean 12 minutes. Determine the steady-state probability P_n of n customers in the check-out area.

20. Use separable convex programming to solve the NLPP :

Maximize $z = x_1 - x_2$

Subject to

$3x_1^4 + x_2 \leq 243$

$x_1 + 2x_2^2 \leq 32$

$x_1 \geq 2.1$

$x_2 \geq 3.5$

F–0634

Sub. Code
7MMA4C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Fourth Semester

Mathematics

TOPOLOGY–II

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the one-point compactification.

2. Is the rationals Q is locally compact? Justify your answer.

3. Define a completely regular space.

4. What is meant by the Stone-Cech compactification?

5. Define a G_s – set with an example.

6. What is meant by locally discrete?

7. Is the space (–1, 1) in R complete? Justify.

8. Define the point-open topology.

9. What do you mean by the compact open topology?

10. When will you say that the space is said to be Bair space?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let x be locally compact Hausdorff space and let A be a subspace of x. If A is closed in x or open in x, then prove that A is locally compact.

Or

(b) Let x be a set and let $\mathcal{D}$ be a collection of subsets of x that is maximal with respect to the finite intersection property. Prove that any finite intersection of elements of $\mathcal{D}$ is an element of $\mathcal{D}$.

12. (a) Show that a product of completely regular spaces is completely regular.

Or

(b) Let x be completely regular. Prove that x is connected if and only if $\beta(x)$ is connected.

13. (a) Let $\mathcal{A}$ be a locally finite collection of subsets of x. Prove the following:

(i) Any sub collection of $\mathcal{A}$ is locally finite.

(ii) The collection $\mathcal{B} = \{\overline{A}\}_{A \in \mathcal{A}}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

Or

(b) Find a non discrete space that has a countably locally finite basis but does not have a countable basis.

14. (a) Prove that Euclidean space $\mathbb{R}^k$ is complete in either of its usual metrics, the Euclidean metric d or square metric ρ.

Or

(b) If x is locally compact, or if x satisfies the first countability axiom, then prove that x is compactly generated.

15. (a) Let x be locally compact Hausdorff space and let $\mathcal{C}(x, y)$ have the compact open topology. Prove that the map $e : x \times \mathcal{C}(x, y) \to y$ defined by the equation $e(x, f) = f(x)$ is continuous.

Or

(b) If x is a compact Hausdorff space or a complete metric space, then prove that x is a Baire space.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the Tychonoff theorem.

17. Let x be a completely regular space. If y_1 and y_2 are two compactifications of x satisfying the extension property. Prove that y_1 and y_2 are equivalent.

18. Prove that a space x is metrizable if and only if x is regular and has a basis that is countably locally finite.

19. Let $I = [0,1]$. Prove that there exists a continuous map $f : I \to I^2$ whose image fills up the entire square I^2.

20. State and prove a nowhere-differentiable function theorem.

F–0635

Sub. Code
7MMA4E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023

Fourth Semester

Mathematics

Elective – ADVANCED STATISTICS

(CBCS – 2017 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an unbiased estimator of the parameter.

2. What is meant by the two-sided test?

3. State the properties of sufficient statistic.

4. Define a complete family of probability density functions.

5. What is meant by efficient estimator of θ?

6. State the Fisher information in the random sample.

7. Define a best critical region of size α.

8. Define the sequential probability ratio test.

9. State the assumptions of analysis of variance.

10. Define non central Chi square distribution.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let the observed value of the mean $\overline{X}$ of a random sample of size 20 from a distribution that is $N(\mu, 80)$ be 81.2. Find a 95% confidence interval for μ.

Or

(b) Let two independent random samples, each of size 10, from normal distributions $N(\mu, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\overline{x} = 4.8, S_1^2 = 8.64, \overline{y} = 5.6$, $S_2^2 = 7.88$. Determine 95% confidence interval for $\mu_1 - \mu_2$.

12. (a) Let the random variables x and y have the joint p.d.f. $f(x, y) = (2/\theta^2)e^{-(x+y)\theta}$, $0 < x < y < \infty$, zero elsewhere. Show that the mean and variance of y are, respectively, $3\theta/2$ and $5\theta\ 2/4$.

Or

(b) Let $X_1, X_2, \ldots\ldots X_n$ denote a random sample of size n from a distribution with p.d.f. $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, zero elsewhere, and $\theta > 0$. Show that the geometric mean $(X_1, X_2, \ldots\ldots X_n)^{1/n}$ of the sample is a complete sufficient statistic for θ.

13. (a) Discuss the Bayesian estimation.

Or

(b) Enumerate the limiting distribution of maximum likelihood estimator.

14. (a) (i) State the Neyman-Pearson theorem.

(ii) Define an uniformly most powerful test. Does it exist always? Justify.

Or

(b) Let X be $N(0, \theta)$ and let $\theta' = 4$, $\theta'' = 9$, $\alpha_a = 0.05$ and $\beta_a = 0.10$. Show that the sequential probability ratio test can be based upon the statistic $\sum_{i=1}^{n} x_i^2$. Also determine $C_0(n)$ and $C_1(n)$.

15. (a) Let Y_i, $i = 1, 2, \ldots, n$, denote independent random variables that are, respectively, $\chi^2(r_i, \theta_i)$, $i = 1, 2, \ldots n$. Prove that $Z = \sum_{i=1}^{n} Y_i$ is $\chi^2\left(\sum_{i=1}^{n} r_i, \sum_{i=1}^{n} \theta_i\right)$.

Or

(b) State and prove the Boole's inequality.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let $X_1, X_2, \ldots X_n$ be i.i.d., each with the distribution having p.d.f. $f(x; \theta_1, \theta_2) = \left(1/\theta_2\right) e^{-(x-\theta_1)/\theta_2}$, $\theta_1 \le x < \infty$, $-\infty < \theta_1 < \infty$, $0 < \theta_2 < \infty$, zero elsewhere. Find the maximum likelihood estimators of θ_1 and θ_2.

17. State and prove the Neyman factorization theorem.

18. (a) Derive the Rao-Cramer inequality.

(b) Given the p.d.f. $f(x;\theta)=\frac{1}{\pi\left[1+(x-\theta)^2\right]}$, $-\infty < x < \infty$, $-\infty < \theta < \infty$. Show that the Rao-Cramer lower bound is $2/n$, where n is the size of a random sample from this Cauchy distribution.

19. Enumerate the procedure of likelihood ratio test and state its properties.

20. Describe the analysis of variance for two-way classification.