S–0884

Sub. Code
23MMA1C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

First Semester

Mathematics

ALGEBRAIC STRUCTURES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Prove that $N(a)$ is subgroup of G.

2. If G is a group of order 231, then prove that the 11–sylow subgroup in the centre of G.

3. Define solvable of a group.

4. Define the direct sum of R-module.

5. If $T \in A(V)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \ldots + \alpha_m T^m$, where the $\alpha_i \in F$ is invertible, if $\alpha_0 \neq 0$.

6. If $u \in V$ is such that $uT^{n_1 - k} = 0$, where $0 < K \leq n_1$, then prove that $u = u_0 T^k$ for some $u_0 \in V_1$.

7. Define the companion matrix.

8. Define the Jordan Canonical form.

9. Prove that $tr(A+B) = trA + trB$ and for $\lambda \in F$, $tr(\lambda A) = \lambda trA$.

10. Prove that congruence is an equivalence relation.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove the second part of Sylow's theorem.

Or

(b) If $O(G) = p^n$, where P is prime number then prove that, $Z(G) \neq (e)$.

12. (a) Show that G is solvable if and only if $G^{(K)} = (e)$ for some integer K.

Or

(b) Show that every finite abelian group is the direct product of cyclic groups.

13. (a) If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

Or

(b) Prove that there exists a subgroup W of V invariant under T, such that $V = V_1 \oplus W$.

14. (a) If S and T are nilpotent linear transformation which commute, then prove that ST and $S+T$ are nilpotent linear transformations.

Or

(b) Prove that for $f(x) \in F[x]$, $C(f(x))$ statistics $f(x)$ and has $f(x)$ as its minimal polynomial. What is its characteristic polynomial?

15. (a) If N is normal and $AN = NA$, then prove that $AN^* = N^* A$.

Or

(b) Find the rank and signature of the real quadratic form. $x_1^2 + x_1x_2 + 2x_1x_3 + 2x_2^2 + 4x_2x_3 + 2x_3^2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the third part of Sylow's theorem.

17. Let G be a group and suppose that G is internal direct product of $N_1, N_2, \ldots, N_n$. Let $T = N_1 \times N_2 \times \ldots \times N_n$, then prove that G and T are isomorphic.

18. (a) If $T \in A(v)$ and $\lambda \in F$ is a characteristic root of T in F. Let $U_\lambda = \{u \in v / vT = \lambda v\}$. If $S \in A(v)$ commutes with T, then prove that U_λ is invariant under S.

(b) If $T \in A(v)$ has only O as a characteristic root, then prove that T is nilpotent.

19. Prove that the elements S and T in $A_F(v)$ are similar in $A_F(v)$ if and only if they have the same elementary divisors.

20. If F is a field of characteristic O and if $T \in A_F(v)$ is such that $trT^i = 0$ for all $i \geq 1$ then prove that T is nilpotent.

S–0885

Sub. Code
23MMA1C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

First Semester

Mathematics

REAL ANALYSIS – I

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. What is meant by rearrangement series?

2. Define Partition of $[a,b]$.

3. State comparison theorem.

4. State Riemann-Stieltjes theorem.

5. State first mean-value theorem.

6. Write the statement of Lebesgue's criterion for Riemann-Integrability.

7. What is meant by double series?

8. Write the statement of Tauber theorem.

9. Define boundedly convergent sequence.

10. Write the statement of Weierstrass M-test.

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) If f is continuous on $[a,b]$ and if f' exists and is bounded in the interior say $|f'(x)| \leq A \forall x \in (a,b)$ then prove that f is of bounded variation on $[a,b]$.

Or

(b) State and prove Abel's test.

12. (a) If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a,b]$, then prove that $C_1 f + c_2 g \in \mathcal{R}(\alpha)$ on $[a,b]$, for any two constants $c_1 \,\&\, c_2$. Also prove that $\int_a^b (c_1 f + c_2 g) d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$.

Or

(b) Assume that α is increasing on $[a,b]$. If $f \in \mathcal{R}(\alpha)$ on $[a,b]$, then prove that $|f| \in \mathcal{R}(\alpha)$ on $[a,b]$ and $\left| \int_a^b f(x)\, d\alpha(x) \right| \leq \int_a^b |f(x)\, d\alpha(x)|$.

13. (a) State and prove second mean-value theorem for Riemann-Stieltjes theorem.

Or

(b) Assume $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a,b]$, where α is increasing on $[a,b]$. Define $F(x) = \int_a^x f(t)\, d\alpha(t)$ and $G(x) = \int_a^x g(t)\, d\alpha(t)$ if $x \in [a,b]$. Prove that $f \in R(G), g \in R(F)$ and the product $f \cdot g \in \mathcal{R}(\alpha)$ on $[a,b]$. Also prove that $\int_a^b f(x) g(x)\, d\alpha(x) = \int_a^b f(x)\, d$ $G(x) = \int_a^b g(x)\, d\, F(x)$.

14. (a) Assume that $\lim_{p,q\to\infty} f(p,q) = a$. For each fixed p, assume that $\lim_{q\to\infty} \hat{J}(p,q)$ exists. Then prove that $\lim_{p\to\infty}\left(\lim_{q\to\infty} f(p,q)\right)$ also exists and has the value a.

Or

(b) If a series is convergent with sum S, then prove that $(c,1)$ is summable with cesaro sum S.

15. (a) Let α be of bounded variation on $[a,b]$. Assume that each term of the seq. $\{f_n\}$ is a real-valued function such that $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$ for each n = 1, 2, ... Assume that $(f_n) \to f$ uniformly on $[a,b]$ and define $g_n(x) = \int_a^x f_n(t)\, d\alpha(t)$ if $x \in [a,b]$, n = 1, 2, ... prove that

(i) $f \in \mathcal{R}(\alpha)$ on $[a,b]$

(ii) $(g_n) \to g$ uniformly on $[a,b]$, where $g(x) = \int_a^x f(t) d\alpha(t)$.

Or

(b) State and prove Dirichlet's test for uniform convergence theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let $\sum a_n$ be an absolutely convergent series having sum S. Prove that every rearrangement of $\sum a_n$ also converges absolutely and has sum S.

17. Assume $f \in \mathscr{R}(\alpha)$ on $[a,b]$ and assume that α has a continuous derivative α' on $[a,b]$. Prove that the Reimann integral $\int_a^b f(x)\alpha'(x)\,dx$ exists and also prove that $\int_a^b f(x)\,d\alpha(x) = \int_a^b f(x)\alpha'(x)\,dx$.

18. Assume that α is of bounded variation on $[a,b]$. Let $v(x)$ denote the total variation of α on $[a,x]$ if $a < x \le b$, and let $v(a) = 0$. Let f be defined and bounded on $[a,b]$. If $f \in \mathscr{R}(\alpha)$ on $[a,b]$, then prove that $f \in R(v)$ on $[a,b]$.

19. Assume that $\sum a_n (z - z_0)^n$ converges for each z in $B(z_0; r)$. Prove that the function f defined by the equation $f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ if $z \in B(z_0; r)$ has a derivative $f'(z)$ for each z in $B(z_0; r)$ given by $f'(z) = \sum_{n=1}^{\infty} n a_n (z - z_0)^{n-1}$.

20. Let $\{f_n\}$ be a boundedly convergent sequence on $[a,b]$. Assume that each $f_n \in R$ on $[a,b]$, and that the limit function $f \in R$ on $[a,b]$. Assume also that there is a partition p of $[a,b]$ such that on every subinterval $[c,b]$ not containing any of the points x_k, the sequence $\{f_n\}$ converges uniformly to f. Prove that $\lim_{n\to\infty} \int_a^b f_n(t)\,dt = \int_a^b \lim_{n\to\infty} f_n(t)\,dt = \int_a^b f(t)\,dt$.

S–0886

Sub. Code
23MMA1C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A — $(10 \times 2 = 20)$

Answer **all** the questions.

1. State uniqueness theorem for linear equation with constant coefficients.

2. Check whether the following functions are linearly independent (or) not. $\phi_1(x) = \cos x$, $\phi_2(x) = \sin x$ exist for $-\infty < x < \infty$.

3. Compute the Wronskian of the solutions of the equation $y''' - 4y' = 0$.

4. If the function $\sin ax, \cos ax (a\ \ real)$, then write down the characteristic polynomial of an annihilator.

5. State the Hermite equation.

6. Prove that $P_n(-x) = (-1)^n P_n(x)$.

7. Define a singular point of the equation.

8. Compute the indicial polynomial and its roots of the equation $x^2y'' + (\sin x)y' + (\cos x)y = 0$.

9. Determine whether the equation $e^x dx + (e^y(y+1))dy = 0$ is exact or not.

10. Let $f(x,y) = 4x^2 + y^2$ on $s : |x| \leq 1,\ |y| \leq 1$. Show that of satisfy Lipschitz condition on the set S.

Part B (5 × 5 = 25)

Answer **all** questions. Choosing either (a) or (b).

11. (a) Find all solutions ϕ of $y'' + y = 0$ satisfying $\phi(0) = 1,\ \phi(\pi/2) = 2$.

Or

(b) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0, then prove that

$$w(\phi_1, \phi_2)(x) = e^{-a_1, (x-x_0)} w(\phi_1, \phi_2)(x_0).$$

12. (a) Find the solution ϕ of the initial value problem.

$$y''' + y = 0,\ y(0) = 0,\ y'(0) = 1,\ y''(0) = 0,$$

Or

(b) Find a particular solution of the equation $y'' - y' - 2y = x^2 + \cos x$ using the annihilator method.

13. (a) Let $\phi_1, \phi_2, \ldots \phi_n$ be the n solutions of $L(y) = 0$ on I satisfying $\phi_i^{(i-1)}(x_0) = 1,\ \phi_i^{(j-1)}(x_0) = 0,\ j \neq i$. If ϕ is any solution of $L(y) = 0$ on I, then prove that there are n constants $c_1, c_2, \ldots c_n$ such that $\phi = c_1\phi_1 + c_2\phi_2 + \ldots c_n\phi_n$.

Or

(b) One solution of $x^2 y''' - 3x^2 y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

14. (a) Find all solutions of the following equation for $x > 0$ $x^2 y'' - (2+i)xy' + 3iy = 0$.

Or

(b) Obtain two linearly independent solutions of the equation $x^2 y'' - 2x(x+1)y' + 2(x+1)y = 0$ which are valid near $x = 0$.

15. (a) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$: $y' = y^2$, $y(0) = 1$.

Or

(b) Consider the problem $y' = 1 + y^2$, $y(0) = 0$

(i) Using separation of variables, find the solution ϕ of this problem.

(ii) Show that $\phi_k(x) \to \phi(x)$ for each x satisfying $|x| \le 1/2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let ϕ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0. Prove that for all x in I $\|\phi(x_0)\| e^{-k(x-x_0)} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k(x-x_0)}$ where $\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2 \right]^{1/2}$ $k = 1 + |a_1| + |a_2|$.

17. Consider the equation $y''' - 4y' = 0$.

(a) Compute three linearly independent solutions.

(b) Find the wronskian of the solutions found in (i).

(c) Find that solution ϕ satisfying

$$\phi(0) = 0,\ \phi'(0) = 1,\ \phi''(0) = 0.$$

18. (a) Find two linearly independent power series solutions (in powers of x) of the equation $y'' + 3x^2 y' - xy = 0$.

(b) Show that $\int_{-1}^{1} P_n^2(x)dx = 2/2n+1$.

19. Establish the Bessel function of order x of the first kind $J_\alpha(x)$.

20. Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle $R : |x - x_0| \leqq a,\ |y - y_0| \leqq b$. Prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if $\dfrac{\partial M}{\partial Y} = \dfrac{\partial N}{\partial x}$ in R.

S–0887

Sub. Code
23MMA1E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

First Semester

Mathematics

Elective : NUMBER THEORY AND CRYPTOGRAPHY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define pairwise relatively prime.
2. Find [87, 27[?
3. What is canonical factorization?
4. State the Bertrand's posulate.
5. Define Fermat's number.
6. State Wilson's theorem.
7. Define the Legendre symbol.
8. Define Dirichlet product.
9. Define enciphering and deciphering.
10. Define cipher text.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove principle of mathematical induction.

Or

(b) Find x, y such that $(418, 165) = 418x + 165y$.

12. (a) Prove that there exist arbitrarily large gaps between consecutive primes.

Or

(b) Prove that if n is composite, and if p is the least prime factor of n, then $p \leq \sqrt{n}$.

13. (a) State and prove Fermat's little theorem.

Or

(b) Let p be an odd prime. Prove that $x^2 \equiv a^2 \pmod{p}$ if and only if $x \equiv \pm a \pmod{p}$.

14. (a) Prove that if f is multiplicative and $g(n) = \sum_{d/n} f(d)$, then g is multiplicative.

Or

(b) State and prove and three properties of Legendre symbol.

15. (a) Write a short note on digital signature standard.

Or

(b) Working in the 26-letter alphate use the matrix $A = \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix}$.

(i) Find encipher the message unit "NO"

(ii) Find decipher the ciphertext "FWMDIQ"

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let $\alpha = \frac{1+\sqrt{5}}{2}$. If a and b are integers such that $a > b > 0$ and n is the number of iterations needed to compute (a, b), using Euclid's Algorithm, prove that $n \leq 1 + \log_\alpha b$.

17. Prove that for any natural number n, $n! = \prod_{p \leq n} p^{\Sigma} k \geq 1^{[n/p^k]}$.

18. Prove that if p is an odd prime then the congruence $x^2 \equiv -1 (\text{mod}\, p)$ has the solutions $x \equiv \pm[(p-1)/2]\,!(\text{mod}\, p)$ if $p \equiv 1(\text{mod}\, 4)$ and has no solutions if $p \equiv 3(\text{mod}\, 4)$.

19. State and prove Euclid-Euler theorem.

20. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(Z/NZ)$ and set $D = ad - bc$. Prove that the following are equivalent :

(a) g.c.d. $(D, N) = 1$;

(b) A has an inverse matrix;

(c) If x and y are not both 0 in Z/NZ, then $A\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$;

(d) A gives a 1-to-1 correspondence of $(Z/NZ)^2$ with itself.

S–0888

Sub. Code
23MMA1E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

First Semester

Mathematics

Elective : GRAPH THEORY AND APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define a complete bipartite graph with an example.

2. State the Cayley's formula.

3. What is meant by edge connectivity? Give an example.

4. Draw the Petersen graph.

5. Define a perfect matching with an example.

6. Determine the edge chromatic number of $K_{3,3}$.

7. Define a covering of a graph G. Give an example.

8. State the Hajos conjecture.

9. State Jordan curve theorem in the plane.

10. How many orientations does a simple graph G have?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If a k-regular bipartite graph with $k > 0$ has bipartition (X, Y), then prove that $|x| = |y|$.

Or

(b) Prove that a graph is bipartite if and only if it contains no odd cycle.

12. (a) With the usual notations, prove that $k \leq k' \leq \delta$.

Or

(b) Prove that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.

13. (a) State and prove the Berge theorem.

Or

(b) State the Vizing's theorem. Also prove that $\chi' = \Delta$ if G is bipartite.

14. (a) If $\delta > 0$, then prove that $\alpha' + \beta' = \gamma$.

Or

(b) If G is 4-chromatic, then prove that G contains a subdivision of K_4

15. (a) State and prove the Euler's formula for a connected plane graph. Using this theorem, prove that $K_{3,3}$ is nonplanar.

Or

(b) Prove that each vertex of a disconnected tournament D with $\gamma \geq 3$ is contained in a directed K-cycle, $3 \leq k \leq \gamma$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Show that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

    (b) Prove that a connected graph is a tree if and only if every edge is a cut edge.

17. (a) State and prove the chvatal theorem.

    (b) If G is a simple graph with $\gamma \geq 3$ and $E > \binom{\nu - 1}{2} + 1$, then prove that G is Hamiltonian, Moreover, the only nonhamiltonian simple graphs with γ vertices and $\binom{\gamma - 1}{2} + 1$ edges are $C_{1,\gamma}$ and $\gamma = 5, C_2, 5$.

18. (a) State and prove the Hall's theorem.

    (b) Prove that every 3-regular graph without cut edges has a perfect matching.

19. With the usual notations, prove the following :

    (a) $r(k, l) \leq \binom{k + l - 2}{k - 1}$; (b) $r(k, k) = 2^{k/2}$.

20. Show that every planar graph is 5-vertex-colourable.

S–0890

Sub. Code
23MMA1E4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

First Semester

Mathematics

***Elective* — MATHEMATICAL PROGRAMMING**

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Mixed Integer programming problem.

2. Write down the Gomory's Algorithm properties.

3. Define Dynamic Programming problem.

4. State Bellman's principle of Optimality.

5. Differentiate Linear programming and Goal programming.

6. Write down the General Goal Programming model.

7. Define NLPP.

8. What is meant by Quadratic programming problem?

9. What is meant by simulation according to Shannon?

10. Why we can use Monte-Carlo simulation?

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) Solve the following integer programming problem using Gomory's cutting plane algorithm

Maximize $Z = 7x_1 + 6x_2$

Subject to $2x_1 + 3x_2 \leq 12$

$6x_1 + 5x_2 \leq 30$

and $x_1, x_2 \geq 0$ and integers

Or

(b) Explain the Branch and Bound method.

12. (a) Use Dynamic programming to show that

$$\sum_{i=1}^{n} P_i \log P_i$$

Subject to the constraint

$$\sum_{i=1}^{n} P_i = 1$$

and $P_i \geq 0$ for all i is minimum when $P_1 = P_2 = \ldots = P_n = \frac{1}{n}$.

Or

(b) Find the minimum value of

$Z = y_1^2 + y_2^2 + \ldots + y_n^2$

Subject to the constraint

$y_1 \cdot y_2 \cdot \ldots y_n = C (C \neq 0)$

and $y_j \geq 0, j = 1, 2, \ldots n$

13. (a) A manufacturing firm produces two types of products : A and B. The unit profit from product A is Rs.100 and that of product B is Rs.50. The goal of the firm to earn a total profit of exactly Rs.700 in the next week. Let us suppose that the manager is addition to the profit goal of Rs.700 also wants to achieve sales volume of products A and B close to 5 and 4, respectively. Formulate this problem as a goal programming model.

Or

(b) Explain the Graphical solution method for solving a Goal programming.

14. (a) Solve the following non-linear programming problem graphically

Maximize $Z = x_1 + 2x_2$

Subject to $x_1^2 + x_2^2 \leq 1$

$2x_1 + x_2 \leq 2$ and

$x_1, x_2 \geq 0.$

Or

(b) Derive the Kuhn-Tucker conditions for the quadratic programming problem

15. (a) The investment corporation wants to study the investment projects based on three factors. Market demand in units; price per unit minus cost per unit and investment required. These factors are felt to be independent of each other. In analysing a new consumer product, corporation estimates the following probability distributions.

Annual Demand		Price minus cost per unit		Investment Required	
Units	Probability	Rs.	Probability	Rs.	Probability
20,000	0.05	3.00	0.10	17,50,000	0.25
25,000	0.10	5.00	0.20	20,00,000	0.50
30,000	0.20	7.00	0.40	25,00,000	0.25
35,000	0.30	9.00	0.20		
40,000	0.20	10.00	0.10		
45,000	0.10				
50,000	0.05				

Using simulation process, repeat the trial 10 times, compute the return on investment for each trial taking these three factors into account. What is the most likely return?

Or

(b) The materials Manager of a firm wishes to determine the expected (mean) demand for a particular item in stock during the reorder lead time. This transformation is needed to determine how far in advance to reorder, before the stock level is reduced to zero. However, both the lead time (in days) and the demand per day for the item are random variables, described by the probability distribution given below :

Lead Time (days)	Probability of occurrence	Demand/Day (units)	Probability
1	0.50	1	0.10
2	0.30	2	0.30
3	0.20	3	0.40
		4	0.20

Manually simulate the problem for 30 reorders to estimate the demand during lead time.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Solve the following integer Linear Programming problem by using the cutting plane algorithm

Maximize $z = 2x_1 + 20x_2 - 10x_3$

Subject to the constraints

$2x_1 + 20x_2 + 4x_3 \leq 15$

$6x_1 + 20x_2 + 4x_3 = 20$ and

$x_1, x_2, x_3 \geq 0$ and are integers.

Also show that it is not possible to obtain a feasible integer solution by using the method of simple rounding off.

17. The owner of a chain of four grocery stores has purchased 6 crates of fresh strawberries. The estimated probability distribution of potential sales of the strawberries before spoilage differs among the four stores. The following table gives the estimated total expected profit at each store, when it is allocated various number of crates

Number of crates	Stores			
	1	2	3	4
0	0	0	0	0
1	4	2	6	2
2	6	4	8	3
3	7	6	8	4
4	7	8	8	4
5	7	9	8	4
6	7	10	8	4

For administrative reasons, the owner does not wish to split crates between stores. However, he is willing to distribute zero creates to any of his store. Find the allocation of 6 crates to 4 stores so as to maximize the expected profit.

18. A company produces motorcycle seats. The company has two production lines. The production rate of line 1 is 50 seats per hour and for line 2 it is 60 seats per hour. The company has entered into a contract to supply 1,200 seats daily to another company. Currently, the normal operation period for each line is 8 hours. The production manager of the company is trying to determine the best daily operation hours for the two lines. He sets the priorities to achieve the goals as given below:

P1: Produce and deliver 1,200 seats daily

P2: Limit the daily overtime operation hours of line 2 to 3 hours

P3: Minimize under utilization of the regular daily operation hours of each line. Assign differential weights based on the relative productivity rate.

P4: Minimize the daily overtime operation hours of each line as much as possible. Assign differential weights based on the relative cost of overtime. It is assumed that the cost of operation is identical for the two production lines.

Formulate the problem as GP model and then solve it by using the graphical method.

19. Use Wolfe's method to solve the Quadratic programming problem

Maximize $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to the constraints

$x_1 + 2x_2 \leq 2$ and

$x_1, x_2 \geq 0.$

20. A book store wishes to carry a particular book in stock. Demand is not certain and three is a lead time of 2 days for stock replenishment. The probabilities of demand are as given below:

Demand (unit/day) :	0	1	2	3	4
Probability :	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of Rs. 10 per order. The store also incurs a carrying cost of Re. 0.5 per book per day. The inventory carrying cost is calculated on the basis of stock at the end of each day.

The Managers of the book store wishes to compare two options for his inventory decision.

A : Order 5 books when present inventory plus any outstanding order falls below 8 books.

B : Order 8 books when present inventory plus any outstanding order falls below 8 books.

Currently (beginning of first day) the stock has a stock of 8 books plus 6 books ordered two days ago and expected to arrive next day. Carry out simulation run for 10 days to recommend an appropriate option. You may use random numbers in the sequences using first number for day one 89, 34, 78, 63, 61, 89, 39, 16, 13, 73.

S–0891

Sub. Code
23MMA1E5

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

First Semester

Mathematics

Elective – FUZZY SETS AND THEIR APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define level set of a fuzzy set.

2. What do you mean by connected set?

3. Define marginal basic assignment.

4. Define basic probability assignment.

5. Prove that $H(X) \geq H(X/Y)$.

6. Write a short notes on Hartley information.

7. Define granulation fuzzy set.

8. Write the formula for defuzzified value in discrete case.

9. Define feasible set.

10. What do you mean by Hamming distance?

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) Prove that a fuzzy set A on R is convex if and only if $A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{A(x_1), A(x_2)\}$.

Or

(b) Determine the transitive max-min closure $R_T(X, x)$ for a fuzzy relation $R(X, x)$ defined by the membership matrix $R = \begin{pmatrix} 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{pmatrix}$.

12. (a) Prove that $pl(A_1 \cap A_2 \cap A_3) \leq pl(A_1) + pl(A_2) + pl(A_3) - pl(A_1 \cup A_2) - pl(A_1 \cup A_3) - pl(A_2 \cup A_3) + pl(A_1 \cup A_2 \cup A_3)$.

Or

(b) Show that

(i) $Bel(A) + Bel(\overline{A}) \leq 1$

(ii) $pl(A) + pl(\overline{A}) \geq 1$

13. (a) Prove that the function $I(N) = \log_2 N$ is the only function that satisfies Hartley information.

Or

(b) State and prove Gibb's inequality.

14. (a) Explain the following terms in defuzzification method.

(i) Center of area method

(ii) Center of maxima method

Or

(b) Explain in detail about fuzzification function.

15. (a) Describe the concept of individual decision making with suitable example.

Or

(b) Narrate the multiperson decision making with an illustration.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let $\mu_p = \begin{pmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{pmatrix}$; $\mu_Q = \begin{pmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{pmatrix}$

Compute (a) $\mu_{p\circ Q}$ (b) μ_{p*Q}

17. Prove that a belief measure Bel on a finite power set $\wp$ (X) is a probability measure if and only if its basis assignment m is given by $M(\{x\}) = Bel(\{x\})$ and $m(A) = 0$ for all subsets of X that are not single tons.

18. Using the measure of fuzziness defined by

$f_{c,w}(A) = (b-a)^{1/w} - \left(\int_a^b \int_{c,A}^{w} (x)\, dx \right)^{1/w}$ for $w-1$ and the standard fuzzy complement. Calculate the degree of fuzzyness of the fuzzy set defined on the interval $[0, 10]$ of real numbers by the following membership functions

(a) $\mu_A(x) = \dfrac{x}{10}$

(b) $\mu_B(x) = 2^{-x}$

19. Discuss about stabilizing an inverted pendulum in fuzzy controller.

20. Solve the following fuzzy linear programming problem

Max $z = 6x_1 + 5x_2$

Subject to $(5,3,2)\, x_1 + (6,4,2)x_2 \leq (25, 6, 9)$

$(5, 2, 3)x_1 + (2,1.5, 1)x_2 \leq (13, 7, 4)$

$x_1, x_2 > 0$

S–0892

Sub. Code
23MMA1E6

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025.

First Semester

Mathematics

Elective — DISCRETE MATHEMATICS

(CBCS – 2023 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part A　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by quantifiers?

2. Check whether the following statement is true (or) false. Justify your answer.

   $(\exists x)\ (\exists y)\ Q(x, y)$ where $Q(x, y)$: $x + y = 10$; x and y are integers.

3. Consider D, defined by $D(K) = 5.2^k$, $k \geq 0$. Find the recurrence relation on D.

4. Define the following terms :

   (a) Zero function ;

   (b) Successor function ;

   (c) Projection function.

5. Draw the Hasse diagram for $P(\{1, 2, 3\}, \subseteq)$

6. Define switching circuit. Give an example.

7. Define the Hamming distance with an example.

8. What is meant by group code? Give an example.

9. There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

10. Find the next larger 4-combination of the set

$\{1, 2, 3, 4, 5, 6\}$ after $\{1, 2, 5, 6\}$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Obtain a disjunctive normal form of

$\neg(P \vee Q) \leftrightarrow (P \wedge Q)$.

Or

(b) Verify the validity of the following argument :

"Lions are dangerous animals. There are lions. Therefore there are dangerous animals".

12. (a) Find the generating function of Fibonacci sequence.

Or

(b) When will you say that a set A is said to be partial recursive? Also if A denotes Ackermann's function, evaluate :

(i) $A(1, 1)$;

(ii) $A(2, 1)$;

(iii) $A(3, 2)$.

13. (a) Prove that every chain is a lattice.

Or

(b) (i) Expand the following function into the canonical sum-of-products forms : $f(x, y, z) = xy + y\bar{z}$.

(ii) Write down the minterm normal form of

$f(x_1, x_2) = \bar{x}_1 \vee \bar{x}_2$.

14. (a) Show that $(m, m+1)$ parity check code $e : B^m \to B^{m+1}$ is a group code.

Or

(b) For $H = \begin{bmatrix} 110 \\ 101 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix}$ find $e_H : B^3 \to B^6$. Form the decoding table.

15. (a) What is meant by the pigeonhole principle? Also prove that a function f from a set with $k+1$ or more elements to a set with k elements is not one-to-one.

Or

(b) Let n and k be positive integers with $n \geq k$. Prove that $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that if $p \to q$, $q \to r$, $\neg(p \wedge r)$ and $p \vee r$, then r.

17. Solve the recurrence relationship

$S(K) - 4S(K-1) = 11S(K-2) + 30S(K-3) = 0$,

$S(0) = 0$, $S(1) = -35$, $S(2) = -85$.

18. Simplify the following using Karnaugh diagrams :

$f(x_1, x_2, x_3, x_4) = x_1x_3 + x_1'x_3x_4 + x_2x_3'x_4 + x_2'x_3x_4$.

19. Suppose e is an (m, n) encoding function and d is a maximum likelihood decoding function associated with e. Prove that (e, d) can correct k or fewer errors if and only if the minimum distance of e is at least $2k+1$.

20. (a) How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

(b) What is the next permutation in lexicographic order after 362541?

S–0893

Sub. Code
23MMA2C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Second Semester

Mathematics

ADVANCED ALGEBRA

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define algebraic over F.

2. What is the degree of $\sqrt{2} + \sqrt{3}$ over Q?

3. Let F be the field of rational numbers. Find the degree of the splitting field of the polynomial $x^6 + 1$ over F.

4. Define a simple extension of F.

5. Define the fixed field of G.

6. What is meant by the Galois group?

7. Find the primitive roots of : 17, 23, 31.

8. When will you say that a complex number θ is said to be a primitive n^{th} root of unity?

9. Define algebraic over a field F.

10. Define the norm of x.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

Or

(b) If $m > 0$ and n are integers, prove that $e^{m/n}$ is transcendental.

12. (a) Prove that there is an isomorphism τ^{**} of $F[x]/(f(x))$ onto $F'[t]/(f'(t))$ with the property that for every $\alpha \in F,\ \alpha\tau^{**} = \alpha'$, $(x + (f(x)))\tau^{**} = t + (f'(t))$.

Or

(b) If $f(x) \in F[x]$ is irreducible, then prove the following :

(i) If the characteristic of F is 0, $f(x)$ has no multiple roots.

(ii) If the characteristic of F is $p \neq 0$, $f(x)$ has a multiple root only if it is of the form $f(x) = g(x^p)$.

13. (a) Prove that the fixed field of G is a subfield of k.

Or

(b) If $\alpha_1, \alpha_2, \alpha_3$ are the roots of the cubic polynomial $x^3 + 7x^2 - 8x + 3$ find the cubic polynomial whose roots are

(i) $\alpha_1^2, \alpha_2^2, \alpha_3^2$;

(ii) $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \frac{1}{\alpha_3}$.

14. (a) If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F, then find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

Or

(b) If the field F has p^n elements prove that the automorphisms of F form a cyclic group of order n.

15. (a) Let C be the field of complex numbers and suppose that the division ring D is algebraic over C. Prove that $D = C$.

Or

(b) State and prove the Lagrange identity.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. With the usual notations, prove that the number e is transcendental.

17. If $p(x)$ is a polynomial in $F[x]$ of degree $n \geq 1$ and is irreducible over F, then prove that there is an extension E of F, such that $[E:F] = n$, in which $p(x)$ has a root.

18. Establish the fundamental theorem of Galois theory.

19. Prove that a finite division ring is necessarily a commutative field.

20. State and prove the Frobenius theorem.

S–0894

Sub. Code
23MMA2C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Second Semester

Mathematics

REAL ANALYSIS – II

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Show that every countable set has measure zero.

2. Define ess supf.

3. Show that $\int_1^{\infty} dx/x = \infty$.

4. Show that if f and g are measurable, $|f| \leq |g|$ a.e., ang g is integrable, than f is integrable.

5. State Riesz – Fischer theorem.

6. State Jordan's test.

7. Prove that if f is differentiable at c, than f is continuous at c.

8. Write the matrix form of the chain rule.

9. Define open mapping.

10. State inverse function theorem.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that every interval is measurable.

Or

(b) Prove that the following statements are equivalent.

(i) f is a measurable function

(ii) $\forall \alpha, [x : f(x) \geq \alpha]$ is measurable

(iii) $\forall \alpha, [x : f(x) < \alpha]$ is measurable

(iv) $\forall \alpha, [x : f(x) \leq \alpha]$ is measurable.

12. (a) State and prove Lebesgue's dominated convergence theorem.

Or

(b) Let f be a bounded measurable function defined on the finite interval (a, b). Show that $\lim_{\beta \to \infty} \int_a^b f(x) \sin \beta x \, dx = 0$.

13. (a) State and prove Dini's theorem.

Or

(b) State and prove the Weierstrass Approximation theorem.

14. (a) Let u and v be two real-valued functions defined on a subset s of the complex plane. Assume also that u and v are differentiable at a interior point c of s and that the partial derivatives satisfy cauchy – Riemann equations at c. Then prove that the function $f = u + iv$ has a derivative at c. Moreover $f'(c) = D_1\, u(c) + i\, D_1\, v(c)$.

Or

(b) Let S be an open connected subset of $\mathbb{R}^n$ and let $f : S \to \mathbb{R}^m$ be differentiable at each point of S. If $f'(c) = 0$ for each c in S, then prove that f is constant on S.

15. (a) Assume that $f = (f_1, \ldots, f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in $\mathbb{R}^n$ and that the Jacobian determinant $J_f(a) \neq 0$ for some point a in S. Then prove that there is an n-ball $B(a)$ on which f is one-to-one.

Or

(b) State and prove second-derivative test for extrema.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Prove the following statements regarding the set E are equivalent :

    (a) E is measurable

    (b) $\forall \in > 0$, ∄0, an open set, $0 \supseteq E$ such that $M^*(0 - E) \leq \varepsilon$.

    (c) ∄G, a G_δ –set, $G \geq E$ such that $M^*(G - E) = 0$.

    (d) $\forall E > 0$, ∄F, a closed set, $F \subseteq E$ such that $M^*(E - F) \leq \varepsilon$

    (e) ∄F, a F_σ –set, $F \subseteq E$ such that $M^*(E - F) = 0$.

17. State and prove Fatou's lamma.

18. State and prove Fejer's theorem.

19. Assume that one of the partial derivatives $D_1, f, \ldots D_n f$ exists at C and that the remaining $n - 1$ partial derivatives exists in some n-ball $B(c)$ and are continuous at C. Then prove that f is differentiable at C.

20. State and prove the implicit function theorem.

S–0895

Sub. Code
23MMA2C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2023-2024 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the Schrodinger equation in quantum physics.

2. Classify the partial differential equation $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$

3. Define Domain of dependence.

4. State the Cauchy-Kowalesky theorem.

5. Write the statement for the partial differential equation with initial conditions with transverse vibration of a beam.

6. Mention the uniqueness theorem for heat conduction problem.

7. Identify the Poisson Integral formula for interior of a unit circle.

8. Prove that the solution of the Dirichlet problem if it exists, is unique.

9. Define Dirac delta function.

10. Prove that $\lim_{\in \to 0} \int_{C_\in} \frac{\partial G}{\partial n} ds = 1$.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) What are the assumptions made by deriving the vibrating membrane?

Or

(b) Determine canonical form corresponding to the partial differential equation $u_{xx} + u_{yy} + u_{xy} + u_x = 0$.

12. (a) Obtain the solution of the Initial-Boundary value problem $u_{tt} = 4u_{xx}, x > 0, t > 0$ $u(x,0) = |\sin x|$, $x > 0; u_t(x,0) = 0$ $x > 0$ and $u(0,t) = 0$, $t > 0$.

Or

(b) Obtain the solution of the equation $u_{xx} - u_{yy} = 1$ with $u(x,0) = \sin x, u_y(x,0) = x$.

13. (a) Find the solution of $u_t = ku_{xx}, 0 < x < l, t > 0$ $u(0,t) = 0, u(l,t) = u_0$, for $t \geq 0$ and $u(x,0) = f(x), 0 \leq x \leq l$.

Or

(b) Explain the Struck string problem for the vibration of a stretched string in 1-D, wave equation.

14. (a) State and prove the maximum principle theorem for Harmonic function.

Or

(b) Obtain a solution of the Neumann problem for a circle.

15. (a) Find the solution of the Dirichlet problem in a rectangular domain : $\nabla^2 u = h$ in D and $u = 0$ on B by the method of Green's function.

Or

(b) Prove that the Green's function is symmetric.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find the characteristic equations and characteristics and then reduce to the equations $u_{xx} \mp (\operatorname{sech}^4 x)\, u_{yy} = 0$ to the canonical forms.

17. Explain the Riemann's method of solving linear hyperbolic partial differential equation.

18. Solve the partial differential equation

$$\nabla^2 u = 0, 0 \le x < a, 0 \le y < b \qquad u(x,0) = f(x), 0 \le x \le a,$$
$$u(x,b) = 0, 0 \le x \le a, u_x(0,y) = u_x(a,y) = 0, \ 0 \le y \le b.$$

19. Obtain the solution of the Neumann problem for a rectangle.

20. Find the solution of the Robin's problem on the quarter infinite plane using the method of images :

$$\nabla^2 u = h(\zeta, \eta) \text{ in } \zeta > 0, \eta > 0$$

$$u = f(\eta) \text{ on } \zeta = 0$$

$$u_x = g(\zeta) \text{ on } \eta = 0.$$

S–0897

Sub. Code
23MMA2E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Second Semester

Mathematics

Elective — MATHEMATICAL STATISTICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define random variable.

2. If C_1 and C_2 are events such that $C_1 \subset C_2$, then prove that $P(C_1) \leq P(C_2)$.

3. Define moment generating function.

4. Let X have pmf $P(x) = \begin{cases} \frac{x}{6}; & x = 1,2,3 \\ 0 & \text{elsewhere} \end{cases}$. Find $E(X^3)$.

5. Define negative Binomial distribution.

6. Let X be $N(2,25)$. Find $P(-8 < X < 1)$.

7. Write down the mean and variance of the t-distribution.

8. Let X be $\chi^2(50)$. Approximate $\Pr(40 < X < 60)$.

9. Define the statistic.

10. Suppose $n_1 = 10$, $n_2 = 7$, $\bar{x} = 4.2$, $\bar{y} = 3.4$, $s_1^2 = 49$, $s_2^2 = 32$. Find 90% confidence interval for $\mu_1 - \mu_2$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) From a bowl containing 5 red, 3 white and 7 blue chips. Select 4 at random and without replacement. Compute the conditional probability of 1 red, 0 white and 3 blue chips, given that there are at least 3 blue chips in this sample of 4 chips.

Or

(b) Show that for any random variable, $P[X = x] = F_X(x) \quad - F_X(x-)$, $\forall x \in R$, where $F_X(x-) = \lim_{z \uparrow x} F_X(z)$.

12. (a) State and prove the Markov's inequality.

Or

(b) Let X have the pdf $f(x) = \frac{x+2}{18}$, $-z < x < 0$, zero elsewhere. Find $E(X), E\left[(X+2)^3\right], E\left[6X - 2(X+2)^3\right]$.

13. (a) Compute the measure of skewness and kurtosis of the binomial distribution $b(n, p)$.

Or

(b) If the random variable X is $N\left(\mu, \sigma^2\right), \sigma^2 > 0$, then prove that the random variable, $V = \frac{(x-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

14. (a) Suppose $\{Y_n\}$ is a sequence of random variables which is bounded in probability. Suppose $X_n = O_p(Y_n)$. Then prove that $X_n \xrightarrow{P} O$, as $n \to \infty$.

Or

(b) Let Z_n be $\chi^2(n)$. Find the limiting distribution of $Y_n = \frac{Z_n - n}{\sqrt{2n}}$.

15. (a) Suppose the number of customers X that enter a store between the hours 9:00 AM and 10 AM follows a Poisson distribution with parameter θ. Suppose a random sample of the number of customer for 10 days results in the values

9 7 9 15 10 13 11 7 2 12

Based on these data, obtain an unbiased point estimate of θ. Explain the meaning of this estimate in terms of the number of customers.

Or

(b) Let X have a binomial distribution with the number of trails $n = 10$ and with p either $\frac{1}{4}$ or $\frac{1}{2}$. The sample hypothesis $H_0 : P = \frac{1}{2}$ is rejected and the alternative simple hypothesis $H_1 : P = \frac{1}{4}$ is accepted, if the observed value of X_1, a random sample of size 1, is less than or equal to 3. Find the significance level and the power of the set.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the Boole's inequality.

(b) A bowl contains 8 chips. Three of the chips are red and 5 are blue. Four chips are to be drawn successively at random and without replacement.

(i) Compute the probability that the chips are all blue.

(ii) Compute the probability that the first blue chips appears on the third draw.

17. Let $f(x_1, x_2) = 21x_1^2x_2^3$, $0 < x_1 < x_2 < 1$, zero elsewhere, be the joint pdf of X_1 and X_2.

(a) Find the conditional mean and variance of X_1, given $X_2 = x_2$, $0 < x_2 < 1$.

(b) Find the distribution of $Y = E(x_1 / x_2)$.

(c) Determine $E(Y)$ and $\text{var}(Y)$ and compare there to $E(X_1)$ and $\text{var}(X_1)$, respectively.

18. Find the moment generating function, mean and variance of the gamma distribution.

19. State an prove the central limit theorem.

20. Two different teaching procedures were used on two different groups of students.

Each group contained 100 students of about the same ability. At the end of the term, an evaluating team assigned a letter grade to each student. The results were tabulated as follows

Grade

Group	A	B	C	D	F	Total
I	15	25	32	17	11	100
II	9	18	29	28	16	100

If we consider these data to be independent observations from two respective multinomial distributions with $k = 5$, test at the 5 percent significance level the hypothesis that the two distributions are the same (and hence the two teaching procedures are equally effective).

S–0899

Sub. Code
23MMA2E4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025.

Second Semester

Mathematics

Elective — CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. State the fundamental lemma of the calculus of variations.

2. State Euler's Poisson equation.

3. Define an extremal field.

4. Write the Jacobi's equation.

5. Define isoperimetric condition.

6. Write the control function.

7. Define degenerate kernel.

8. State Fredholm Alternative theorem.

9. Write the Fredholm integral equation.

10. State Fredholm's first theorem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the extremals of the functional $V[y(x)] = \int_{x_0}^{x_1} \frac{y'^2}{x^3} dx$.

Or

(b) Test for an extremum the functional

$$V[y(x)] = \int \left(y^2 + 2xyy'\right) dx \;;\; y(x_0) = y_0;\; y(x_1) = y_1.$$

12. (a) Find the broken line extremals of the functional

$$v = \int_0^a \left(y'^2 - y^2\right) dx.$$

Or

(b) Test for an extremum of the functionals

$$V[y(x)] = \int_0^{\pi/4} \left(4y^2 - y'^2 + 8y\right) dx \;;\; y(0) = -1,\; y(\pi/4) = 0.$$

13. (a) Find an approximate solution of the problem of the extremum of the functional

$$V(y(x)) = \int_0^1 \left(x^2 y''^2 + 100xy^2 - 20xy\right) dx \;;\; y(1) = y'(1) = 0.$$

Or

(b) Find the extremals of the isoperimetric problem

$V(y(x)) = \int_{x_0}^{x_1} y'^2 dx$ given that $\int_{x_0}^{x_1} y\, dx = c$, where c is constant.

14. (a) Show that the integral equation

$$g(s) = \lambda \int_0^{\pi} (\sin s \sin 2t) g(t) dt \text{ has no eigen values.}$$

Or

(b) Solve the integral equation

$$g(s) = f(s) + \lambda \int_0^{2\pi} (\sin s . \cos t) g(t) dt .$$

15. (a) Solve the integral equation

$$g(s) = s + \lambda \int_0^1 (st + (st))^{1/2} g(t) dt .$$

Or

(b) Solve the in homogeneous Fredholm integral equation of the second kind,

$$g(s) = 2s + \lambda \int_0^1 (s + t) g(t) dt .$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that a necessary condition for the extremum of a functional of the form

$J(y) = \int_a^b F(x, y, y_1', \ldots.., y^{(n)}) dx$ where F is differentiable w.r. to each of its arguments is $F_y - d/dx \; F_{y'} + d^2/dx^2 F_{y''} + \ldots \; + (-1)^n d^n/dx^n \; F_{y^{(n)}} = 0$.

17. Find the equation of geodesics on a surface on which the element of length of the curve is of the form.

$$ds^2 = (\varphi_1(x) + \varphi_2(y)) \; (dx^2 + dy^2).$$

18. Find an approximate solution of the problem of the extremum of the functional

$$V(y(x)) = \int_1^2 \left(xy'^2 - \frac{x^2 - 1}{x} y^2 - 2x^2 y \right) dx \; ; \quad y(1) = y(2) = 0 \text{ and}$$

compare it with the exact solution.

19. Solve the integral equation

$$y(x) = \cos x + \lambda \int_0^{\pi} \sin(x - t) y(t) \, dt$$

20. Solve the integral equation

$$g(s) = f(s) + \lambda \int_0^{s} e^{s-t} g(t) dt \quad \text{and evaluate the resolvant}$$

kernel.

S–0902

Sub. Code
23MMA3C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. State the Cauchy's representation formula.

2. Define zero and pole. Give an example for each.

3. Define homologous to zero.

4. Find the residue of $\cot z$ at $z = 0$.

5. Write down the Laplace's equation in polar coordinates.

6. Compute $\int\limits_{|z|=R} \frac{R^2 - |z|^2}{|z-a|^2} d\theta$, where $z = \mathrm{Re}^{i\theta}$.

7. Write down the power series expansion of arc $\sin z$.

8. What is Laurent series of an analytic function around a point at which the function is analytic?

9. Define $\Gamma_{(z)}$ and find the value $\Gamma_{(1/2)}$.

10. State the Jensen's formula.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the Taylor's theorem.

Or

(b) State and prove the Schwarz lemma.

12. (a) State and prove the residue theorem.

Or

(b) State and prove the Rouche's theorem.

13. (a) Evaluate $\int_0^\pi \frac{d\theta}{a+\cos\theta}, a>1$.

Or

(b) State and prove the Mean-value property.

14. (a) State and prove the Hurwitz theorem.

Or

(b) If $f(z)$ is analytic in the region Ω, containing z_0, then prove that the representation $f(z)=f(z_0)$ + $\frac{f'(z_0)}{1!}(z-z_0)+\frac{f''(z_0)}{2!}(z-z_0)^2+\ldots+\frac{f^{(n)}(z_0)}{n!}(z-z_0)^n+..$ is valid in the largest open disk of center z_0 contained in Ω.

15. (a) Prove that for $|z|<1$, $(1+z)\quad(1+z^2)+(1+z^4)$

$(1+z^8)\ldots=\dfrac{1}{1-z}$.

Or

(b) Define an entire function with an example. Also prove that every function which is meromorphic in the whole plane is the quotient of two entire functions.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Suppose that $\phi(r)$ is continuous on the arc γ . Prove that the function $F_n(z)=\int_\gamma \dfrac{\phi(r)\,dr}{(r-z)^n}$ is analytic in each of the regions determined by γ, and its derivative is $F_n'(z)=nF_{n+1}(z)$.

17. State and prove the general form of Cauchy's theorem.

18. Derive the Poisson's formula and show that if $u(z)$ is harmonic for $|z|<R$ and continuous for $|z|<R$ then $u(a)=\dfrac{1}{2\pi}\displaystyle\int_{|z|=R}\dfrac{R^2-|a|^2}{|z-a|^2}u(z)\,d\theta$ for all $|a|<R$.

19. State and prove the Schwarz's theorem.

20. Derive the Legendre's duplication formula.

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S–0903

Sub. Code
23MMA3C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025.

Third Semester

Mathematics

PROBABILITY THEORY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Show that the probability of the impossible event in zero.

2. Define truncated distribution of X.

3. Suppose that the random variable X can take on two value $x_1 = -1$ with probability $p_1 = 0.1$ and $x_2 = +1$ with probability $p_2 = 0.9$. Find the expected value of X.

4. What is meant by regression of the first type?

5. Define characteristic function.

6. What is meant by probability generating function of X?

7. Define a normal distribution.

8. Define a beta distribution.

9. State the De-Moivre-Lapace theorem.

10. Define limit distribution function.

Part B (5 × 5 = 25)

Answer **all** questions. Choosing either (a) or (b).

11. (a) Compute the probability that heads appear atleast twice in three successive tosses of a coin.

Or

(b) State the prove Bayes theorem.

12. (a) The random variable X has the normal distribution with density $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. Find the value of $E(X)$ and $E(X^2)$.

Or

(b) Find the regression curves for the two-dimensional normal distribution is $f(x, y) = \frac{1}{2\pi} \exp\left(-\frac{[x^2 - 2xy + 2y^2]}{2}\right)$.

13. (a) The joint distribution of the random variable (X, Y) is given by the density

$$f(x, y) = \begin{cases} \frac{1}{4}\left[1 + xy(x^2 - y^2)\right] & \text{for } |x| \le 1,\ |y| \le 1 \\ 0 & \text{for all other points} \end{cases}.$$

Show that the random variable X and Y are dependent.

Or

(b) The characteristic function of the random variable X is given by the $\phi(t) = \exp\left(-\frac{t^2}{2}\right)$. Find the density function of this random variable.

14. (a) The random variable X has the distribution $N(1;2)$. Find the probability that X is greater than 3 in absolute value.

Or

(b) Describe the Cauchy distribution.

15. (a) A box contains a collection of IBM cards corresponding to the workers from same branch of industry of the workers 20% are minors and 80% adults, we select one IBM card in a random way and mark the age given on this card. Before choosing the next card, we return the first one to the box, so that the probability of selecting the card corresponding to a minor remains 0.2 we observe n cards in this manner. What value should n have in order that the probability will be 0.95 that the frequency of cards corresponding to minor lies between 0.18 and 0.22?

Or

(b) State and prove the Kolmogorov inequality.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that a real single-valued function $F(x, y)$ is a distribution function of a certain two-dimensional random variable if and only if $F(x, y)$ is non decreasing and continuous atleast from the left with respect to both arguments x and y satisfies equalities $F(-\infty, y) = F(x, -\infty) = 0$, $F(+\infty, +\infty) = 1$ and the inequality $F(x_2, y_2) - F(x_2 + y_1) - F(x_1, y_2) + F(x_1, y_1) \geq 0$ holds for every $(x_1, y_1)(x_2, y_2)$, were $x_1 < x_2$ and $y_1 < y_2$.

17. Show that the expected value of the product of an arbitrary finite number of independent random variables, whose expected values exist, equals the product of the expected values of these variables.

18. Let $F(x)$ and $\phi(t)$ denote respectively the distribution function and the characteristic function of the random variable X. If $a+h$ and $a-h$ are continuity points of the distribution function $F(x)$, then prove that

$$F(a+h)-F(a-h)=\lim_{T\to\infty}\frac{1}{\pi}\int_{-T}^{T}\frac{\sinh t}{t}\,e^{-ita}\phi(t)dt\,.$$

19. State and prove poisson's theorem.

20. State and prove the Lindeberg-Levy theorem.

S–0904

Sub. Code
23MMA3C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2023 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** the questions.

1. If $X=\{a, b, c\}$, let $J_1=\{\phi, x, \{a\}, \{a, b\}\}$ and $J_2=\{\phi, x,\{a], \{b, c\}\}$. Find the smallest topology containing J_1 and J_2 and the largest topology contained in J_1 and J_2

2. Define the order topology. Give an example.

3. Define a continuous function on topological spaces.

4. What is meant by the square metric?

5. Define the term linear continuum.

6. What are the components and path components of $\mathbb{R}_\ell$?

7. When will you say that the space X is said to be compact? Give an example.

8. State the uniform continuity theorem.

9. Is the product of two Lindelof spaces need not be lindelof? Justify your answer.

10. State the tietz extension theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that the topologies of $\mathbb{R}_{\ell}$ and $\mathbb{R}_{K}$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another.

Or

(b) Prove that the collection

$S = \{\pi_1^{-1}(u) / u \text{ open in } x\} \cup \{\pi_2^{-1}(v) / v \text{ open in } y\}$ is a subbasis for the product topology on $x \times y$.

12. (a) Let x and y be topological spaces and let $f : x \rightarrow y$. Prove the following are equivalent :

(i) f is continuous

(ii) for every subset A of X, one has $f(\overline{A}) \subset \overline{f(A)}$.

(iii) For very closed set B of Y, then set $f^{-1}(B)$ is closed in X.

Or

(b) Let $f : A \rightarrow \underset{\alpha \in J}{\pi} X_{\alpha}$ be given by the equation $f(a) = (f_{\alpha}(a))_{\alpha \in J}$, where $f_{\alpha} : A \rightarrow X_{\alpha}$ for each α. Let πx_{α} have the product topology. Prove that the function f is continuous if and only if each function f_{α} is continuous.

13. (a) Prove that the image of a connected space under a continuous map is connected.

Or

(b) Show that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

14. (a) Prove that every compact subspace of a Hausdorff space is closed.

Or

(b) Show that compactness implies limit point compactness but not conversely.

15. (a) Suppose that X has a countable basis. Prove that every open covering of X contains a countable subcollection covering X.

Or

(b) Prove that every compact Hausdorff space is normal.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Let X be a topological space. Prove the following conditions hold :

(i) ϕ and X are closed.

(ii) Arbitrary intersections of closed sets are closed.

(iii) Finite unions of closed sets are closed.

(b) Let A be a subset of the topological space X and let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

17. Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

18. If L is a linear continuum in the order topology, then prove that L is connected and so are intervals and rays in L.

19. Let X be a simply ordered set having the least upper bound property. prove that in the order topology, each closed interval in X is compact.

20. State the prove the Urysohn lemma.

S–0905

Sub. Code
23MMA3C4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Third Semester

Mathematics

INDUSTRIAL STATISTICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define biased and unbiased estimator. Give an example for each.

2. Define two-sided test.

3. State the properties of sufficient statistic.

4. Define the exponential class of probability density functions of the continuous type.

5. Write a short notes on Bayes confidence interval.

6. Define an efficient estimator of θ.

7. What is meant by best critical region of size α?

8. Define likelihood ratio test.

9. Define a central chi-square variable.

10. Write down the formula for correlation coefficient of the random sample.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11\. (a) Let $\overline{X}$ be the mean of a random sample of size n from a distribution that is $N(\mu, 9)$. Find n such that $P_r(\overline{X} - 1 < \mu < \overline{X} + 1) = 0.90$, approximately.

Or

(b) Let two independent random samples, each of size 10, from two normal distributions $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\overline{x} = 4.8$, $s_1^2 = 8.64$, $\overline{y} = 5.6$, $s_2^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$.

12\. (a) Let $X_1, X_2, ..., X_n$ denote a random sample from a normal distribution with mean zero and variance θ, $0 < \theta < \infty$. Show that $\sum_{i=1}^{n} X_i^2 / n$ is an unbiased estimator of θ and has variance $\frac{2\theta^2}{n}$.

Or

(b) Let $X_1, X_2, ..., X_n$ denote a random sample from a distribution that is $N(0, \theta)$. Prove that $Y = \sum X_i^2$ is a complete sufficient statistic for θ. Also find the unbiased minimum variance estimator of θ^2.

13\. (a) Let $X_1, X_2, ..., X_n$ denote a random sample from a distribution which is $b(1, \theta)$, $0 < \theta < 1$. Find the decision function δ which is a Baye's solution.

Or

(b) Derive the Rao-Cramer inequality.

14. (a) Let $X_1, X_2, ..., X_{25}$ denote a random sample of size 25 from a normal distribution $N(\theta, 100)$. Find a uniformly most powerful critical region of size $\alpha = 0.10$ for testing $H_0 : \theta = 75$ against $H_1 : \theta > 75$.

Or

(b) Enumerate the sequential probability ratio test.

15. (a) Discuss the role of the distributions of certain quadratic forms in the technique of analysis of variance.

Or

(b) Compute the mean and variance of a random variable that is $\chi^2(r, \theta)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X have a p.d.f. of the form $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, zero elsewhere, where $\theta \in \{\theta : \theta = 1, 2\}$. To test the simple hypothesis $H_0 : \theta = 1$ against the alternative simple hypothesis $H_1 : \theta = 2$, use a random sample X_1, X_2 of size $n = 2$ and define the critical region to be $C = \left\{(x_1, x_2) : \frac{1}{4} \le x_1 x_2\right\}$. Find the power function of the test.

17. Let $X_1, X_2, ..., X_n$ denote a random sample of size n from a distribution with p.d.f. $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, zero else where, and $\theta > 0$.

(a) Show that the geometric mean $(X_1, X_2, ..., X_n)^{\frac{1}{n}}$ of the sample is a complete sufficient statistic for θ.

(b) Find the maximum likelihood estimator of θ and observe that it is a function of this geometric mean.

18. Let X have a gamma distribution with $\alpha = 4$ and $\beta = \theta > 0$.

(a) Find the Fisher information $I(\theta)$.

(b) If $X_1, X_2, ..., X_n$ is a random sample from this distribution, show that the maximum likelihood estimator of θ is an efficient estimator of θ.

19. State and prove the Neyman – Pearson theorem.

20. Describe the analysis of variance for two-way classification.

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S–0906

Sub. Code
23MMA3E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Third Semester

Mathematics

***Elective* — ALGEBRAIC NUMBER THEORY**

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an Unimodular.

2. Find the degrees of the field extensions $Q(\sqrt{7})$.

3. Find $Q\left(\sqrt{2}\ \sqrt[3]{5}\right)$.

4. Define ring of integers.

5. Find an integral basis for $Q\left(\sqrt{5}\right)$.

6. Suppose $\alpha_1, \alpha_2, \ldots, \alpha_n \in D$ form a Q-basis for k. Prove that if $\Delta[\alpha_1, \alpha_2, \ldots, \alpha_n]$ is square free then $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is an integral basis.

7. Define Cyclotomic field.

8. Find discriminants for $Q(3)$.

9. Write the maximal condition for noetherian.

10. Write a statement of fermat.

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) Prove that every finite integral domain is a field.

Or

(b) Prove that every subgroup H of a free abelian group G of rank n is free of rank $s \leq n$. Moreover there exists a basis $u_1, u_2, \ldots, u_n$ for G and positive integers $\alpha_1, \alpha_2, \ldots, \alpha_s$ such that $\alpha_1, u_2, \ldots, \alpha_s u_s$ is a basis for H.

12. (a) Prove that the set of algebraic numbers is a subfield of the complex field C.

Or

(b) Prove that the algebraic integers form a subring of the field of algebraic numbers.

13. (a) Find the ring of integers of $Q(\sqrt{2}, i)$.

Or

(b) Let G be an additive subgroup of D of rank equal to the degree of k, with z-basis $\{\alpha_1, \ldots, \alpha_n\}$. Prove that $\left|D/G^2\right|$ divides $\Delta[\alpha_1, \ldots, \alpha_n]$.

14. (a) Let d be a square free rotational integer. Prove that the integers of $Q(\sqrt{d})$ are :

(i) $z[\sqrt{d}]$ if $d \not\equiv 1 \pmod 4$;

(ii) $z\left[\frac{1}{2} + \frac{1}{2}\sqrt{d}\right]$ if $d \equiv 1 \pmod 4$.

Or

(b) Prove that the quadratic fields are precisely those of the form $Q(\sqrt{d})$ for d a square free rational integer.

15. (a) Prove that if a domain D is noetherian, then factorization into irreducibles is possible in D.

Or

(b) Prove that the ring of integers D in a number field k is noetherian.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let G be a free abelian group of rank r and H is a subgroup of G. Prove that G/H is finite if and only if the ranks of G and H are equal.

17. Let θ be a complex number satisfying a monic polynomial equation whose coefficients are algebraic integers. Prove that θ is an algebraic integer.

18. (a) Find the ring of integers of $Q\left(\sqrt[3]{175}\right)$.

(b) Show that it has no z-basis of the form $\{1, \theta, \theta^2\}$.

19. Prove that the discriminant of $Q(\zeta)$, where $\zeta = e^{2\pi i/p}$ and p is an odd prime is $(-1)^{(p-1)/2}, p^{p-2}$.

20. Prove that the group of units U of the integers in $Q(\sqrt{d})$ where d is negative and squarefree is as follows :

(a) For $d = -1,\ U = \{\pm 1, \pm i\}$

(b) For $d = -3,\ U = \{\pm 1, \pm w, \pm w^2\}$ where $w = e^{2\pi i/3}$

(c) For all other $d < 0, U = \{\pm 1\}$.

S–0908

Sub. Code
23MMA3E3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

Third Semester

Mathematics

Elective – STOCHASTIC PROCESSES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What are the possible types of specification of a one-dimensional processes?

2. Define a Stochastic graph.

3. Define ergodic.

4. Define transient state.

5. Define a counting process.

6. Suppose that the customer arrives in a bank according to a poisson process with a mean of 3 per minutes find the probability that is duration of 2 minutes, the number of customer arriving is exactly 4.

7. Define a lattice variable.

8. What is meant by Stopping time?

9. What is meant by Markov renewal branching process?

10. What is meant by probability of extinction?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Let $x(t) = A\cos wt + B\sin wt,$ where A, B are uncorrelated random variables with mean 0 and variance 1 and w is a positive constant. Show that the process is covariance stationary.

Or

(b) Suppose that the probability of a dry day (State 0) following a rainy day (State 1) is 1/3 and that the probability of a rainy day following a dry day is 1/2. Given that May 1 is a dry day, find the probability that (i) May 3 is dry day : (ii) May 5 is a dry day.

12. (a) State and prove first entrance theorem.

Or

(b) If state j is persistent, then prove that for every stack k that can be reached from state j, $F_{kj} = 1$.

13. (a) State and prove additive properties of poisson process.

Or

(b) Describe the Erlang's formula.

14. (a) Show that the renewal function M satisfies the equation $M(t) = F(t) + \int_0^t M(t-x)\,dF(x)$.

Or

(b) State and prove Wald's equation.

15. (a) If $m = E(X_1) = \sum_{k=0}^{\infty} k\, p_k$ and $\sigma^2 = \text{var}(xi)$ then prove that $E\{X_n\} = m^n$.

Or

(b) Show that the generating function $f(t,s) = \sum_{k=0}^{\infty} PhX(t) = k\}S^k$ of an age-dependent branching process $\{X(t), t \geq 0\}; X0 = 1$ satisfies the integral equation $F(t,s) = [1 - G(t)] + \int_0^t P(F(t-u,s))dG(u)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Derive the Chapman-Kolmogorov equation for higher transition probability.

17. State and prove Ergodic theorem.

18. Describe the Birth and Death process.

19. State and prove elementary renewal theorem.

20. State and prove Yaglom's theorem.

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S–0910

Sub. Code
23MMA4C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2025.

Fourth Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define are length of a curve.

2. Write down the formula for the radius of spherical curvature.

3. Define the anchor ring.

4. Define a double family of curves and its differential equation.

5. Write down the canonical equations for geodesics.

6. State the Gauss-Bonnet theorem.

7. Write a short notes on Dupin's indicatrix.

8. What is a developable? Write down its general equation.

9. State the characterization of complete surfaces.

10. State the Sturm's theorem.

Part B (5 × 5 = 25)

Answer **all** questions. Choosing either (a) or (b).

11. (a) Find the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $x=a\cosh(z/a)$, from the point $(a, 0, 0)$ to the point (x,y,z).

Or

(b) If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma\rho')+\frac{\rho}{\sigma}=0$.

12. (a) Calculate the first fundamental magnitudes for the surface $\vec{r}=(u\cos v, u\sin v, f(u))$.

Or

(b) On the paraboloid $x^2-y^2=z$, find the orthogonal trajectories of the sections by the planes Z=constant.

13. (a) Prove that the curves of the family $\frac{v^3}{u^2}=$ constant are geodesics on a surface with metric $v^2du^2-2uvdudv+2u^2dv^2 (u>0, v>0)$.

Or

(b) Establish the Christoffel symbols of the second kind.

14. (a) State and prove the Euler's theorem.

Or

(b) State and prove the Monge's theorem.

15. (a) State and establish the Hilbert's lemma.

Or

(b) Prove that the Gaussian curvature at any point on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is given by $p^4/a^2b^2c^2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadratic surfaces $ax^2+by^2+cz^2=1$, $a'x^2+b'y^2+c'z^2=1$.

17. (a) Show that on a right helicoid, the family of curves orthogonal to the curves $u\cos v=$ constant is the family $(u^2+a^2)\sin^2 v$ = constant.

(b) Find a surface of resolution which is isometric with a region of the right helicoid.

18. Establish the Liouville's formula for K_g.

19. Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

20. State and prove the Hilbert's theorem.

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