S–3131

Sub. Code
23MMA1C1

M.Sc. DEGREE EXAMINATION, APRIL 2026

First Semester

Mathematics

ALGEBRAIC STRUCTURES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Prove that $n(k) = 1 + p + p^2 + \ldots + p^{k-1}$.

2. If $O(G) = p^2$, where p is a prime number, then prove that G is abelian.

3. Define the direct sum of R-module.

4. If A and B are group, then prove that $A \times B$ is isomorphic to $B \times A$.

5. Define the index of nilpotence.

6. If M of dimension m is cyclic with respect to T, then show that the dimension of MT^K is $M - K$ for all $K \leq m$.

7. Define the basic Jordan block.

8. Define the characteristic polynomial.

9. If $(vT, vT) = (v, v)$ for all $v \in V$, then prove that T is unitary.

10. If $T \in A(v)$ is Hermitian, then prove that all its characteristic roots are real.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that the number of conjugate classes in S_n in $p(n)$, the number of partitions of n.

Or

(b) If G is a finite group, then prove that $C_a = O(G)/O(N(a))$.

12. (a) Suppose that G is the internal direct product of $N_1, N_2, \ldots, N_n$, then $i \neq j, N_i \cap N_j = (e)$, and if $a \in N_i$, $b \in N_j$ then $ab = ba$.

Or

(b) Show that S_n is not solvable for $n \geq 5$.

13. (a) If V is n-dimensional over F and if $T \in A(v)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

Or

(b) If $T \in A(v)$ is nilpotent of index of nilpotence n_1, then prove that a basis of v can be found such that the matrix of T in this basis has the form

$$\begin{pmatrix} M_{n_1} & O & \cdots & O \\ O & M_{n_2} & \cdots & O \\ \vdots & & \ddots & \\ O & O & \cdots & M_{n_r} \end{pmatrix}$$

Where $n_1 \geq n_2 \geq \ldots n_r$ and $n_1 + n_2 + \ldots + n_r = \dim_F v$.

14. (a) Suppose the two matrices *A, B* in F_n are similar in K_n, where *K* is an extension of *F*. Then prove that A and B are already similar in F_n.

Or

(b) Prove for each $i = 1, 2, ..., K$, $V_i \neq (0)$ and $v = v_1 \oplus v_2 \oplus ... \oplus v_k$. Also prove that the minimal polynomial of T_i is $q_i(x)^{l_i}$.

15. (a) Show that the linear transformation *T* and *V* is unitary if and only if it takes an orthonormal basis of *V* into an orthonormal basis of *V*.

Or

(b) Find the rank and signature of the real quadratic form $x_1^2 + 2x_1x_2 + x_2^2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Cauchy's theorem.

17. Let *R* be a Euclidean ring, then prove that for any finitely generated, *R*-Modulo *M*, is the direct sum of a finite number of cyclic submodules.

18. Show that two nilpotent linear transformations are similar if and only if they have the same invariants.

19. Let *V* and *W* be two vector spaces over *F*, and suppose that ψ is a vector space isomorphism of *V* onto *W*. Suppose $S \in A_F(v)$ and $T \in A_F(W)$ are such that for any $v \in V$, $(vS)\psi = (v\psi)T$, then prove that *S* and *T* have the same elementary divisors.

20. Given the real symmetric matrix A, there is an invertible matrix T such that $TAT' = \begin{pmatrix} I_r & & \\ & -I_s & \\ & & O_t \end{pmatrix}$

where I_r and I_s are the $r \times r$ and $s \times s$ unit matrices and O_t is the $t \times t$ zero matrix. The integers $r+s$, which is the rank of A, $r-s$, which is the signature of A, characterize the congruence class of A. Then prove that two real symmetric matrices are congruent if and only if they here the same rank and signature.

S–3132

Sub. Code
23MMA1C2

M.Sc. DEGREE EXAMINATION, APRIL 2026

First Semester

Mathematics

REAL ANALYSIS - I

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define total variation.

2. What is meant by absolutely convergent series?

3. Define a Riemann-Stielties integral.

4. State Riemann - Stielties theorem.

5. State second mean value theorem.

6. Define Oscillation function.

7. What is meant by double sequence function?

8. Define Cesaro Summable series.

9. What is meant by uniformly convergent sequence?

10. State Arzela's theorem.

Part B $(5 \times 5 = 25)$

Answer **all** the questions, choosing either (a) or (b).

11. (a) If f is monotonic on $[a\ b]$, then prove that f is of bounded variation on $[a\ b]$.

Or

(b) State and prove Dirichlet's test.

12. (a) If $f \in \mathcal{R}(\alpha)$ on $[a\ b]$ then prove that $\alpha \in \mathcal{R}(f)$ on $[a\ b]$ and $\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a)$.

Or

(b) Assume that $\alpha \nearrow$ on $[a, b]$. If $f \in \mathcal{R}(\alpha)$ and $g \in \mathcal{R}(\alpha)$ on $[a\ b]$ and if $f(x) \leq g(x) \forall x$ in $[a, b]$ then prove that $\int_a^b f(x)d\alpha(x) \leq \int_a^b g(x)d\alpha(x)$.

13. (a) State and prove First mean-value theorem for Riemann-Stielties integrals.

Or

(b) Let α be of bounded variation on $[a, b]$ and assume that $f \in \mathcal{R}(\alpha)$ on $[a\ b]$. Prove that $f \in \mathcal{R}(\alpha)$ on every subinterval $[c, d]$ of $[a,b]$.

14. (a) State and prove Mertens theorem.

Or

(b) State and prove Able's limit theorem.

15. (a) Let $\{f_n\}$ be a sequence of function defined on a set S. Prove that there exists a function f such that $f_n \to f$ uniformly on S if and only if for every $\in > 0$ $\exists$ as N: $m > N$ and $n > N$ implies $|f_m(x) - f_n(x)| < \in$ for every x in S.

Or

(b) If $\lim_{n\to\infty} f_n = f$ on $[a,b]$. If $g \in R$ on $[a,b]$ define $h(x) = \int_a^x f(t)g(t)dt$ $h_n(x) = \int_a^x f_n(t)g(t)dt$. If $x \in [a,b]$. Prove that $h_n \to h$ uniformly only $[a.b]$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let f be of bounded variation on $[a,b]$. If $x \in (a, b)$, let $V(x) = V_f(a, x)$ and put $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of V. Is Converse true? Justify your answer.

17. Assume that $C \in (a,b)$. If $f \in R(\alpha)$ on $[a,b]$. Prove that $f \in R(\alpha)$ on $[a,c]$ and on $[c,b]$. Also prove that $\int_a^b f d\alpha = \int_a^c f d\alpha + \int_c^b f d\alpha$.

18. State and prove Lebesguels criterion for Riemann - Integrability.

19. State and prove Bernstein theorem.

20. Assume that each term of $\{f_n\}$ is a real-valued function having a finite derivative at each point of an open interval (a,b). Assume that for atleast one pt x_0 in (a,b) the sequence $\{f_n(x_n)\}$ converges. Assume further that there exists a function g such that $(f_n^{'}) \to g$ uniformly on (a,b) . Prove that

(a) $\exists$ a function $f . \exists$ $(f_n) \to f$ uniformly on (a,b).

(b) for each x in (a,b) the derivatives $f'(x)$ exists and equals $g(x)$.

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S–3133

Sub. Code
23MMA1C3

M.Sc. DEGREE EXAMINATION, APRIL 2026.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Find all solutions of the following equation $y'' + (3i - 1)y' - 3iy = 0$.

2. Compute the Wronskian of $\phi_1(x) = \sin x$, $\phi_2(x) = e^{ix}$.

3. State the existence theorem for linear equations with constant coefficients.

4. If the function $x^k e^{ax}$, then write down the characteristic polynomial of an annihilator.

5. State the Chebyshev equation.

6. Prove that $p_n(-1) = (-1)^n$.

7. Define a regular singular point of the equation.

8. Compute the indicial polynomial and its roots of the equation $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$.

9. Find an integrating factor of the equation

$$\left(e^y + xe^y\right)dx + xe^y dy = 0$$

10. Write down the Lipschitz condition.

Part B (5 × 5 = 25)

Answer **all** questions. Choosing either (a) or (b).

11. (a) Find the solutions of the following initial value problem:

$$y'' + (4i + 1)y' + y = 0,\ y(0) = 0,\ y'(0) = 0$$

Or

(b) Find all solutions of the equation $y'' - y' - 2y = e^{-x}$.

12. (a) Find four linearly independent solutions of the equation $y^{(4)} + \lambda y = 0$, in case :

(i) $\lambda = 0$

(ii) $\lambda > 0$

(iii) $\lambda < 0$.

Or

(b) If f, g are two functions with k derivatives then prove that $D^k(f\ g) = \sum_{l=0}^{k} \binom{k}{l} D^l(f) D^{k-l}(g)$, where $\binom{k}{l} = \frac{k!}{(k-l)!\,l!}$.

13. (a) Consider the equation $y'' + \frac{1}{x} y' - \frac{1}{x^2} y = 0$ for $x > 0$. Find two linearly independent solutions for $x > 0$, and prove that they are linearly independent.

Or

(b) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solution of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

14. (a) Show that –1 and 1 are regular singular points for the Legendre equation $(1 - x^2) y'' - 2xy' + \alpha(\alpha + 1) y = 0$.

Or

(b) Show that $x^{\frac{1}{2}} J_{-\frac{1}{2}}(x) = \frac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)} \cos x$.

15. (a) Find all real valued solution of the equation $y' = y^2 / xy + x^2$. Also find the solutions of $y' = 2y^{\frac{1}{2}}$ passing through the point (x_0, y_0), where $y_0 > 0$.

Or

(b) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3 : y' = x^2 + y^2,\ y(0) = 0$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that two solutions ϕ_1, ϕ_2 for $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

17. Let ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = 0$ on an interval I containing a point x_0. Prove that for all x in I, $\|\phi(x_0)\| e^{-k|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k|x-x_0|}$, where $k = 1 + |a_1| + |a_2| + \ldots + |a_n|$.

18. (a) Find two linearly independent power series solutions (in powers of x) of the equation $y'' + x^2 y' + x^2 y = 0$.

(b) Prove that $\int_{-1}^{1} P_n(x) p_m(x) dx = 0, \ (n \neq m)$.

19. With the usual notations, prove that $J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{x}{2}\right)^{2m}$.

20. Show that the necessary and sufficient condition for an equation to be exact.

S–3134

Sub. Code
23MMA1E1

M.Sc. DEGREE EXAMINATION, APRIL 2026

First Semester

Mathematics

Elective – NUMBER THEORY AND CRYPTOGRAPHY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Lucas number.

2. Find $[87,27]$.

3. Define prime counting function.

4. State the Bertrand's postulate.

5. Define multiplication modulom.

6. Prove that if $a \equiv b(\bmod m)$ and $c \geq 1$ then $ac \equiv bc(\bmod mc)$.

7. Define multiplicative function.

8. Define quadratic residue $(\bmod p)$.

9. Define discrete logarithm.

10. Find the inverse of $A = \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix} \in M_2(z \mid 26z)$.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove Euclid's lemma.

Or

(b) Prove that the following statements of greatest common divisor

(i) If $a \mid b$ and $c \mid d$, then $ac \mid bd$.

(ii) If $a \mid b$ and $a \mid c$, then $a \mid bx + cy$ for all x, y.

12. (a) Let $f(x)$ be a polynomial of degree $r > 1$ with integer coefficients and positive lead co-efficient, that is $f(x) = \sum_{k=0}^{r} a_k x^k$ with $a_r > 0$. Prove that there are infinitely many n such that $f(n)$ is composite.

Or

(b) Prove that if n is composite and if p is the least prime factor of n, then $p \leq \sqrt{n}$.

13. (a) State and prove chinese remainder theorem.

Or

(b) Prove that if $m \neq n$, then $(f_m, f_n) = 1$.

14. (a) State and prove Euler's criterion.

Or

(b) Prove that if f and g are both multiplicative functions, then so is $f * g$.

15. (a) Find the discrete log of 28 to the base 2 in F_{37}^{*} using the Silver-Pohlig-Hellman algorithm (2 is a generator of F_{37}^{*}).

Or

(b) Write a short note on the ElGamal crypto system.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove any five properties of least common multiple.

17. State and prove fundamental theorem of Arithmetic.

18. (a) State and prove Wilson's theorem.

(b) Prove that if p is an odd prime, $p \mid (a^2 + b^2)$ and $(a,b) = 1$ then $p \equiv 1 \pmod 4$.

19. State and prove Euclid – Euler theorem.

20. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2(z/NZ)$ and set $D = ad - bc$. Prove that the following are equivalent :

(a) g.c.d. $(D, N) = 1$;

(b) A has an inverse matrix;

(c) If x and y are not both 0 in $z|NZ$, then $A\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$;

(d) A gives a 1-to-1 correspondence of $(z|NZ)^2$ with itself.

S–3135

Sub. Code
23MMA1E2

M.Sc. DEGREE EXAMINATION, APRIL 2026

First Semester

Mathematics

Elective – GRAPH THEORY AND APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Show that $\delta \leq 2E/v \in \Delta$, using usual notations.

2. Define a cut vertex. Give an example.

3. What is meant by a block of a graph? Give an example.

4. Draw a graph which is Hamiltonian but not Eulerian.

5. State the Berge theorem.

6. When will you say that a graph G is said to be K-edge chromatic? Give an example.

7. Define the Ramsey numbers. Also find $r(2, l)$.

8. What is meant by girth of a graph? Give an example.

9. State the Kuratowski's theorem.

10. Draw all the graphs with the tournaments on four vertices.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Define the incidence and adjacency matrices. Give an example for each. Also let M be the incidence matrix and A the adjacency matrix of a graph G. Prove that every column sum of M is 2.

Or

(b) Define a spanning tree with an example. Also prove that every connected graph contains a spanning tree.

12. (a) If G is a block with $\gamma \geq 3$, then prove that any two edges of G lie on a common cycle.

Or

(b) If G is a simple graph with $\gamma \geq 3$ and $\delta \geq \frac{\gamma}{2}$, then prove that G is Hamiltonian.

13. (a) State and prove the Hall's theorem.

Or

(b) Prove that every 3-regular graph without cut edges has a perfect matching.

14. (a) If G is simple and contains no K_{m+1}, then prove that $\varepsilon(G) \leq \varepsilon(T_{m, v})$. Moreover, $\varepsilon(G) = \varepsilon(T_{m, v})$ only if $G \cong T_{m, v}$.

Or

(b) If G is simple, then prove that $\pi_K(G) = \pi_K(G-e) - \pi_K(G \cdot e)$ for any edges e of G.

15. (a) Define a planar graph with an example. Also prove that K_5 is non planar.

Or

(b) What is meant by a directed graph? Give an example. Also prove that $\sum_{v \in V} d^{-1}(v) = \varepsilon = \sum_{v \in V} d^{+}(v)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. With the usual notations, prove that $\tau(K_n) = n^{n-2}$.

17. Prove that $K \leq K' \leq \delta$ using usual notations.

18. State and prove the Vizing's theorem.

19. (a) Explain Hajos conjecture.

(b) If G is 4-chromatic, then prove that G contains a subdivision of K_4.

20. Show that a digraph D contains a directed path of length $\chi - 1$.

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S–3136

Sub. Code
23MMA1E5

M.Sc. DEGREE EXAMINATION, APRIL 2026

First Semester

Mathematics

Elective — FUZZY SETS AND THEIR APPLICATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define Level-2 fuzzy set.

2. What do you mean by fuzzy partial order set?

3. Define Belief measure.

4. If $A \subseteq B$ then prove that Bel $(A) \leq$ Bel (B).

5. Define measure of fuzziness.

6. Prove that $H(X \mid Y) = H(X, Y) - H(Y)$.

7. Write the four modules in fuzzy controllers.

8. Write the formula for defuzzified value in continuous case.

9. Define constraint matrix.

10. What do you mean by priority set?

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) Find a scalar and fuzzy cardinality of the fuzzy sets

$\mu_A(x) = \dfrac{x}{x+1}$, $x \in \{1, 2, \ldots\ldots 10\}$.

Or

(b) Explain the following terms in fuzzy relation :

(i) Reflexive

(ii) Irreflexive

(iii) Symmetric

(iv) Antisymmetric

(v) Transitive

(vi) Antitransitive.

12. (a) Let $X = \{a, b, c, d\}$ be the universal set giving the basic assignment $m\{a, b, c\} = 0.5$, $m(\{a, b, d\}) = 0.2$, $m(X) = 0.3$. Determine the corresponding plausibility measure.

Or

(b) Explain the following terms :

(i) Fuzzy measure ;

(ii) Plausibility measure.

13. (a) Show that the measure of fuzziness defined by the equations $f(A) = -\sum_{x \in X} \{\mu_A(x) \log_2 \mu_A(x) + 1(1 - \mu_A(x))$ $\log_2(1 - \mu_A(x))\}$ and $f_C(A) = |X| - \sum_{X \in x} |\mu_A(x) - C\mu_A(x)|$. Express fuzziness in bits (or) hats $(\log_e)$.

Or

(b) Prove that $H(X, Y) \leq H(X) + H(Y)$.

14. (a) Explain in detail about general dynamic system.

Or

(b) Discuss about fuzzy controller process.

15. (a) Enumerate the individual decision making with an illustration.

Or

(b) What is meant by Hamming distance of fuzzy numbers? Explain with suitable example.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find the Resolution form of the given membership matrix

$$M_R = \begin{pmatrix} 1 & 0 & 0.7 \\ 0.3 & 0.2 & 0 \\ 0 & 0.5 & 1 \end{pmatrix}.$$

17. Calculate the joint basis assignment $M_{1,2}$ for the focal elements C, RUD and DUC.

Focal element	C	RUD	DUC
M_1	0.05	0.15	0.05
M_2	0.05	0.05	0.05

Also determine $Bel_{1,2}$ and $Pl_{1,2}$ for these focal elements.

18. Consider the fuzzy sets A and B defined on the set of real number $X = [0, 4]$ by the membership grade functions $\mu_A(x) = \frac{1}{1+x}$ and $\mu_B(x) = \frac{1}{1+x^2}$. To calculate the actual degree of fuzzy sets of these sets, let us use the hamming distance $\omega = 1$.

19. Describe the basic features of fuzzy neural networks.

20. Solve the following fuzzy linear programming problem.

Max. $Z = 5x_1 + 4x_2$

Subject to

$(4, 2, 1)x_1 + (5, 3, 1)\, x_2 \leq (24, 5, 8)$

$(4, 1, 2)x_1 + (1, 5, 1)x_2 \leq (12, 6, 3)$

$x_1, x_2 \geq 0.$

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S–3137

Sub. Code
23MMA1E6

M.Sc. DEGREE EXAMINATION, APRIL 2026

First Semester

Mathematics

Elective — DISCRETE MATHEMATICS

(CBCS – 2023 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   (10 × 2 = 20)

Answer **all** questions.

1. Define the following terms :
   (a) PDNF ;
   (b) PCNF.

2. Show that $(\forall x)(P(x)) \rightarrow (\exists x)(P(x))$ is a logically valid statement.

3. Find the recurrence relation for the sequence $S(K) = 2K + 9$.

4. Show that $f(x) = x/2$ is partial recursive.

5. Is the lattice N_5 modular? Justify your answer.

6. Write short notes on Karnaugh map.

7. Draw the block diagram for coding.

8. Define decoding function. Give an example.

9. State the generalized pigeonhole principle.

10. What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Obtain the principal disjunctive normal form of

$P \to ((P \to Q) \wedge \neg(\neg Q \vee \neg P))$.

Or

(b) Verify the validity of the following inference :

"If one person is more successful than another, then he has worked harder to deserve success. John has not worked harder than Peter. Therefore, John is not more successful than Peter".

12. (a) Find $f(n)$ if $f(n) = 7f(n-1) - 10f(n-2)$ given that $f(0) = 4$ and $f(1) = 17$.

Or

(b) Show that $f(x, y) = x^y$ is primitive recursive.

13. (a) With the usual notations, prove that $(L \times M, \wedge, \vee)$ is a lattice.

Or

(b) In a Boolean algebra L, prove the following :

(i) $(a \wedge b)' = a' \vee b'$;

(ii) $(a \vee b)' = a' \wedge b'$ for all $a, b \in L$.

14. (a) Establish the properties of distance function δ.

Or

(b) Let $e : B^m \to B^n$ be a group code. Prove that the minimum distance of e is the minimum weight of a non-zero code word.

15. (a) How many different strings can be made by reordering the letters of the word SUCCESS?

Or

(b) Generate the permutations of the integers 1, 2, 3 in lexicographic order.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$.

17. Solve the recurrence relation $S(n) = S(n-1) + 2(n-1)$ with $S(0) = 3$, $S(1) = 1$ by finding its generating function.

18. For the formula $(P \wedge Q) \vee (\neg R \wedge \neg P)$ draw a corresponding circuit using

(a) NOT, AND and OR gates

(b) NAND gates only.

19. (a) Find the minimum distance of the encoding function $e: B^2 \rightarrow B^4$ given by

$e(00) = 0000$, $e(10) = 0110$, $e(01) = 1011$, $e(11) = 1100$.

(b) For $H = \begin{bmatrix} 110 \\ 101 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix}$ find $e_H : B^3 \rightarrow B^6$. Form the decoding table.

20. (a) If n is a positive integer and r is an integer with $1 \leq r \leq n$, then prove that there are $p(n, r) = n(n-1)(n-2) \ldots\ldots\ldots (n-r+1)$, r-permutations of a set with n distinct elements.

Also prove $P(n, r) = n!/(n-r)!$ if n and r are integers with $0 \leq r \leq n$.

(b) Let n and r be non negative integers with $r \leq n$. Prove that $C(n, r) = C(n, n-r)$.

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S–3138

Sub. Code
23MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2026

Second Semester

Mathematics

ADVANCED ALGEBRA

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Algebraic over F.

2. What is the degree of $\sqrt{2}\ \ \sqrt{3}$ over Q?

3. If $a \in K$ is a root of $p(x) \in F[x]$, where $F \subset K$, then prove that in $K[x]$, $(x-a)/p(x)$.

4. Define a simple extension of F.

5. Define the fixed field of G.

6. Define a normal extension of F.

7. Is any two finite fields having the same number of elements are isomorphic? Justify your answer.

8. When will you say that a complex number θ is said to be a primitive n^{th} root of unity?

9. Define solvable group.

10. State the Lagrange identity.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) If L is an algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

Or

(b) Prove that the number e is transcendental.

12. (a) Let $f(x) \in F[x]$ be of degree $n \geq 1$. Prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.

Or

(b) Prove that $(f(x)+g(x))' = f'(x)+g'(x)$ and $(\alpha f(x))' = \alpha f'(x)$ for $f(x), g(x) \in F[x]$ and $\alpha \in F$.

13. (a) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

Or

(b) Express the following as polynomials in the elementary symmetric functions in x_1, x_2, x_3 :

(i) $x_1^3 + x_2^3 + x_3^3$

(ii) $(x_1 - x_2)^2 (x_1 - x_3)^2 (x_2 - x_3)^2$.

14. (a) For every prime number p and every positive integer m, prove that there exists a field having p^m elements.

Or

(b) If R is a ring in which $px = 0$ for all $x \in R$, where p is a prime number, then prove that $xT_a^{pm} = x\,a^{pm} - a^{pm}x$.

15. (a) (i) State the Frobenius theorem.

(ii) State the Lift - Division algorithm.

(iii) If A is a ring algebraic over a field F and A has no zero divisions prove that A is a division ring.

Or

(b) If $a \in H$ then prove that $a^{-1} \in H$ if and only if $N(a) = 1$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Also prove that $[L : F]=[L : K][K : F]$.

17. Show that the polynomial $f(x) \in F[x]$ has a multiple root if and only it $f(x)$ and $f'(x)$ have a non trivial common factor.

18. State and establish the fundamental theorem of Galois theory.

19. State and prove the Wedderburn theorem.

20. Prove that every positive integer can be expressed as the sum of squares of four integers.

S–3139

Sub. Code
23MMA2C2

M.Sc. DEGREE EXAMINATION, APRIL 2026

Second Semester

Mathematics

REAL ANALYSIS – II

(CBCS – 2023 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part A　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Define the outer measure. Give an example.

2. Prove that continuous functions are measurable.

3. When will you say that a measurable function f is said to be integrable?

4. State the Riemann integrable over $[a, b]$.

5. Define an orthogonal system on I.

6. State the Jordan theorem.

7. Define the directional derivative of $\vec{f}$.

8. Write down the Jacobian matrix.

9. What is meant by open mapping?

10. Find and classify the extreme values of the following function :

    $f(x, y) = y^2 + x^2 y + x^4$.

Part B

(5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that the class $\mathfrak{M}$ is a $\sigma-$ algebra.

Or

(b) Show that there exists a non-measurable set.

12. (a) Show that $\int_0^1 \frac{x^{1/3}}{1-x} \log\frac{1}{x}\, dx = 9\sum_{n=1}^{\infty} \frac{1}{(3n+4)^2}$.

Or

(b) Let f be a bounded measurable function defined on the finite interval (a, b). Show that $\lim_{\beta\to\infty} \int_a^b f(x) \sin \beta x\, dx = 0$.

13. (a) State and prove the Parseval's formula.

Or

(b) State and establish the Fejer theorem.

14. (a) If $\vec{f}$ is differentiable at $\vec{c}$, then prove that $\vec{f}$ is continuous at $\vec{c}$.

Or

(b) State and prove the mean-value theorem.

15. (a) Let A be an open subset of $\mathbb{R}^n$ and assume that $\vec{f}: A \to \mathbb{R}^n$ has continuous partial derivatives $D_j f_i$ on A. If $J_{\vec{f}}(x) \neq 0$ for all x in A, then prove that $\vec{f}$ is an open mapping.

Or

(b) State and establish second-derivative test for extreme theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let c be any real number and let f and g be real-valued measurable functions defined on the measurable set E. Prove that $f + c, cf, f + g, f - g$ and fg are also measurable.

17. State and prove the Fatou's lemma.

18. State and establish the Weierstrass approximation theorem.

19. Derive the Taylor's formula.

20. State and prove the inverse function theorem.

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S–3140

Sub. Code
23MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2026

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Write any two assumptions in the vibrating membrane model.

2. Determine the region in which the equation $u_{xx} + u_{xy} - xu_{yy} = 0$ is parabolic (or) elliptic.

3. Determine the solution of the initial boundary value problem $u_{tt} = 4u_{xx}$, $x > 0$, $t > 0$, $u(x, 0) = |\sin x|$, $x > 0$, $u_t(x, 0) = 0$, $x \geq 0$, $u(0, t) = 0$, $t \geq 0$.

4. Find the solution of IVP $u_{tt} = c^2 u_{xx}$; $x \in R$, $t > 0$; $u(x, 0) = \sin x$; $u_t(x, 0) = \cos x$.

5. State the uniqueness theorem for wave equation.

6. State the condition for existence of solution of the heat conduction problem.

7. Define second boundary value problem.

8. State the maximum principle of the boundary value problem.

9. Define Dirac Delta function.

10. If $\frac{\partial G}{\partial n}$ is discontinuous at (ξ, η), then prove that $\xi \lim_{\to 0} \int_{C_\in} \frac{\partial G}{\partial n} dS = 1$ where $C_\in : (x-\xi)^2 + (y-\eta)^2 = \in^2$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Derive the one dimensional wave equation from the vibrating string.

Or

(b) Find the general solution of $u_{xx} + 4u_{yy} - 4u_{xy} = e^y$.

12. (a) Derive the characteristic equation for the Euler equation

$$Au_{xx} + Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y).$$

Or

(b) Find the solution of initial value problem for semi-infinite string with a free end.

13. (a) Explain the plucked string of length l.

Or

(b) State and prove the uniqueness theorem for Heat conduction problem.

14. (a) Show that the solution of the Dirichlet problem depends continuously on the boundary data.

Or

(b) State and prove the minimum principle of the boundary value problem.

15. (a) Prove that the Green's function is symmetric.

Or

(b) Show that the method of images well is the Robin's problem on the quarter infinite plane.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Derive the 1^{st} canonical form of hyperbolic equations for 2^{nd} order equation in two independent variables and hence reduce the equation $y^2 u_{xx} - x^2 u_{yy} = 0$ into canonical form.

17. Find the D'Alembert solution of the Cauchy problem for one dimensional wave equation. Also interpret the solution when

(a) the initial velocity is zero and

(b) initial displacement is zero.

18. Find the solution of vibrating string problem with fixed ends by method of separation of variables.

19. Derive the solution of the Dirichlet problem for a rectangle.

20. Using method of Green's function derive the solution of the Dirichlet problem for the Helmholtz operator.

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S–3141

Sub. Code
23MMA2E2

M.Sc. DEGREE EXAMINATION, APRIL 2026

Second Semester

Mathematics

Elective — MATHEMATICAL STATISTICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** the questions.

1. Define σ-field.

2. Prove that the probability of the impossible event is zero.

3. Let X have the pdf $f(x)\begin{cases} 2(1-x), & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$ find $E(X^2)$.

4. State Markov's inequality.

5. If the mgf of a random variable X is $\left(\frac{1}{3}+\frac{2}{3}e^t\right)^5$, find $P(X=2 \text{ or } 3)$.

6. Suppose that 10 percent of the probability for a certain distribution that is $N(\mu, \sigma^2)$ is below 60 and that 5 percent is above 90. What are the values of μ and σ?

7. Write down the mean and variance of the F-distribution.

8. State slut sky's theorem.

9. Define type II error and give an example.

10. Let the observed value of the mean $\overline{X}$ of a random sample of size 20 from a distribution that is $N(\mu, 80)$ be 81.2. Find a 95% confidence interval for μ.

Part B (5 × 5 = 25)

Answer **all** questions. Choosing either (a) or (b).

11. (a) State and prove Boole's inequalilty.

Or

(b) A bowl contains 8 chips. Three of the chips are red and 5 are blue. Four chips are to be drawn successively at random and without replacement

(i) Compute the probability that the chips are al blue

(ii) Compute the probability that the first blue chips appears on the third draw.

12. (a) State and prove chebyshev's inequality.

Or

(b) Find the mean and variance for the following distribution $f(x) = 6x(1-x), 0 < x < 1$, zero elsewhere.

13. (a) Compute the measures of skewness and kurtosis of the Poisson distribution with mean μ.

Or

(b) Derive the pdf of the Binomial distribution.

14. (a) If X_n converges to X in probability, then show that X_n converges to X in distribution.

Or

(b) Let Z_n be $\chi^2(n)$ and let $W_n = \frac{z_n}{n^2}$. Find the limiting distribution of W_n.

15. (a) Explain Zea May's Dala problem.

Or

(b) Assume that the weight of cereal in a "10-ounce box" is $N(\mu, \sigma^2)$. To test $H_0 : \mu = 10.1$ against $H_1 : \mu > 10.1$, we take a random sample of size $n = 10$ and observe that $\bar{x} = 10.4$ and $s = 0.4$

(i) Do we accept or reject H_0 at the 5% significance level?

(ii) What is the approximate p-value of this test?

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the cumulate distribution function properties.

17. Let (X_1, X_2) be a random vector such that the variance of X_2 is finite. Then prove that

(a) $E[E(X_2 | X_1)] = E(X_2)$

(b) $\text{var}\,[E(X_2 | X_1)] \leq \text{var}(X_2)$.

18. Find the moment generating function, mean and variance of the normal distribution.

19. State and prove the central limit theorem.

20. A die was $n=120$ independent times and the following data resulted

Set up	1	2	3	4	5	6
Frequency	b	20	20	20	20	40-b

If we use a chi-square test, for what value of b would the hypothesis that the die is unbiased be rejected at the 0.025 significant level.

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| S–3142 |

Sub. Code
23MMA2E4

M.Sc. DEGREE EXAMINATION, APRIL 2026

Second Semester

Mathematics

Elective – CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define variation of a functional.

2. State Poisson's equation.

3. What is transversality condition?

4. Write short notes on weak extremum.

5. What is an isoperimetric problem?

6. What is Newton's method for functional equations Kantorovich?

7. Define singular integral equation.

8. Define resolvent Kernel for Fredholm integral equation.

9. Find the value of C_0, C_1 and C_2 in the integral equation

$$g(s) = s + \lambda \int_0^1 \left[st + (st)^{1/2}\right] g(t)dt.$$

10. Write down Volterra integral equation.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Solve the Brachistochrone problem.

Or

(b) Find the extremals of the functional

$$v[y(x), z(x)] = \int_{x_0}^{x_1} F(y', z')\, dx$$

12. (a) Test for an extremum the functional $\int_0^{x_1} \frac{\sqrt{1 + y'^2}}{y} dx$ given that $y(0) = 0$ and $y_1 = x_1 - 5$.

Or

(b) Is the Jacobi condition fulfilled for extremal of the functional

$v[y(x)] = \int_0^a \left(y'^2 + y^2 + x^2\right) dx$, that passes through the points $A(0,0)$ and $B(a,0)$?

13. (a) Find the geodesics of a circular cylinder $r = R$.

Or

(b) Using the Ritz method, find an approximate solution of the problem of the minimum of the functional $v\left[y(x) = \int_0^2 \left(y'^2 + y^2 + 2xy\right)dx; y(0) = y(2) = 0\right]$ and compare it with the exact solution.

14. (a) Invert the integral equation

$$g(s) = f(s) + \lambda \int_0^{2\pi} (\sin s \cos t) g(t) dt$$

Or

(b) Solve the Fredholm integral equation of the second kind $g(s) = s + \lambda \int_0^1 \left(st^2 + s^2 t\right) g(t)\, dt$.

15. (a) Solve the inhomogeneous Fredholm integral equation of the second kind,

$g(s) = 2x + \lambda \int_0^1 (s + t) g(t) dt$, by the method of successive approximations to the third order.

Or

(b) Evaluate the resolvent for the integral equation

$$g(s) = f(s) + \lambda \int_0^1 (s - t) g(t) dt.$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove fundamental lemma of calculus of variations.

    (b) Derive Euler equation.

17. Test for an extremum for the functional $v[y(x)] = \int_0^a \left(y'^2 + 2yy' - 16y^2\right)dx; a > 0, y(0) = 0; y(a) = 0$.

18. Find the form of an absolutely flexible, non extensible homogeneous rope of length l suspended at the points A and B.

19. Solve the integral equation $g(s) = f(s) + \lambda\int_0^1 (1 - 3st)g(t)dt$.

20. Describe the solution of Volterra integral equation by the method of successive approximations.

S–3143

Sub. Code
23MMA2E5

M.Sc. DEGREE EXAMINATION, APRIL 2026

Second Semester

Mathematics

Elective – WAVELETS

(CBCS – 2023 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define discrete Fourier transform.

2. What is meant by convolution operator?

3. Define upsampling operator.

4. Define p^{th}-stage wavelet filter bank.

5. What is the Hilbert space?

6. Define Fourier Series.

7. If $z, \omega \in l^2(z)$, then prove that $\left(z^*\right)^\wedge(\theta) = \hat{z}(\theta + \pi)$.

8. Define a first-stage wavelet.

9. What is meant by Fourier transform of f ?

10. Define wavelet identity.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove Fourier inversion formula.

Or

(b) Suppose $T : l^2(\mathbb{Z}_N) \to l^2(\mathbb{Z}_N)$ is a linear transformation. Let $A_{T,E}$ be the matrix representing T in the standard basis E. If T is translation invariant, then prove that $A_{T,E}$ is circulant.

12. (a) Let $w \in l^2(\mathbb{Z}_N)$. Then prove that $\{R_k\omega\}_{k=0}^{N-1}$ is an orthonormal basis for $l^2(\mathbb{Z}_N)$ iff $|\hat{w}(n)| = 1$ for all $nt\,\mathbb{Z}_N$.

Or

(b) State and prove the folding lemma.

13. (a) Suppose H is a Hilbert space, $\{a_j\}_{j\in\mathbb{Z}}$ is an orthonormal set in H and $f \in H$. Then prove that the sequence $\{\langle f, a_j\rangle\}_{j\in\mathbb{Z}}$ belongs to $l^2(\mathbb{Z})$ with $\sum_{j\in\mathbb{Z}} |\langle f, a_j\rangle|^2 \le \|f\|^2$.

Or

(b) Suppose $f : [-\pi, \pi] \to C$ is continuous and bounded, say $|f(\theta)| \le M$ for all θ. If $\langle f, e^{in\theta}\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)e^{-in\theta}d\theta = 0$ for all $n \in \mathbb{Z}$, then prove that $f(\theta) = 0$ for all $\theta \in [-\pi, \pi]$.

14. (a) Suppose $z t l^2(\mathbb{Z})$ and $w \in l'(\mathbb{Z})$. Then prove that $z * w \in l^2(\mathbb{Z})$ and $\|z * w\| \leq \|w\|_1 \|z\|$.

Or

(b) Suppose $u t l^2(\mathbb{Z})$ and $\{R_{2k}u\}_{k \in \mathbb{Z}}$ is orthonormal in $l^2(\mathbb{Z})$. Define a sequence $u \in l^2(\mathbb{Z})$ by $\theta(k) = (-1)^{k-1}\overline{u(1-k)}$. Then prove that $\{R_{2k}v\}_{k \in \mathbb{Z}}$ $U\{R_{2k}u\}_{k \in \mathbb{Z}}$ is a complete orthonormal system in $l^2(\mathbb{Z})$

15. (a) Define $G : R \to R$ by $G(x) = \frac{1}{\sqrt{2\pi}} e^{-x/2}$. Then prove that $\int_R G(x)dx = 1$.

Or

(b) Suppose $f, g \in L^2(R)$. Then prove that $\langle \hat{f}, \hat{g} \rangle = 2\pi \langle f, g \rangle$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Suppose $z t l^2(\mathbb{Z}_N)$ and $k \in \mathbb{Z}$. Then prove that for any $m t \mathbb{Z}$, $(R_k z)^\wedge(m) = e^{-2\pi i m k / N} \hat{z}(m)$.

17. Suppose N is even, say $N = 2M$, $Z \in l^2(\mathbb{Z}_N)$ and $x, y, w t l^2(\mathbb{Z}_{N/2})$. Then prove that $D(z) * w = D(z * U(W))$ and $U(x) * V(y) - V(x * y)$.

18. Suppose $f \in L^1([-\pi, \pi])$ and $\langle f, e^{in\theta} \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) e^{-in\theta} d\theta = 0$ for all $n \in \mathbb{Z}$. Then prove that $f(\theta) = 0$ a.e.

19. Suppose $w, z \in l^2(\mathbb{Z})$. The set $\{R_{Qk} w\}_{k \in \mathbb{Z}}$ is orthonormal iff $|\hat{w}(\theta)|^2 + |\hat{w}(\theta + \pi)|^2 = 2$ for all $\theta \in \{0, \pi)$.

20. State and prove Mallat's theorem.

S–3144

Sub. Code
23MMA2E6

M.Sc. DEGREE EXAMINATION, APRIL 2026

Second Semester

Mathematics

Elective — MACHINE LEARNING AND ARTIFICIAL INTELLIGENCE

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define General Boundary and specific boundary.

2. How do you design a checker's learning problem?

3. What is Artificial Neural Network?

4. What is meant by Genetic algorithm?

5. What are Bayesian belief nets?

6. Give the Baye's rule equation.

7. What is adversarial search?

8. List the properties of task environments.

9. What are the different types of planers?

10. Define stochastic dominance.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain the inductive biased hypothesis space and unbiased learner.

Or

(b) What is the difference between find – S and candidate elimination algorithm?

12. (a) Discuss the perceptron training rule.

Or

(b) Brief the origin of genetic algorithm.

13. (a) Explain Brute force MAP hypothesis learner. What is minimum description length principle?

Or

(b) Discuss about Naive Bayes classifier with an example.

14. (a) Define the following terms :

(i) Intelligence

(ii) Artificial intelligence

(iii) Agent

(iv) Rationality

(v) Logical reasoning

Or

(b) Explain Alpha-Beta pruning using example.

15. (a) Write forward state-space search algorithm.

Or

(b) Explain about the exact inference in Bayesesian networks.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Describe hypothesis space search in ID3 and contrast it with candidate – Elimination algorithm.

17. Derive the back propagation rule considering the training rule for output unit weights and training rule for Hidden Unit Weights.

18. Explain the concept of Bayes theorem with an example.

19. Is AI is a science (or) is it engineering? Or neither or both? Explain.

20. Describe the Hierarchical planning method with an example.

S–3145

Sub. Code
23MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2026

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2023 onwards)

Time : 3 Hours　　　　　　　　Maximum : 75 Marks

Part A　　　　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Define the winding number of γ with respect to the point 'a'.

2. Show that the function $\sin z$ have essential singularity at ∞.

3. Define simple connected region.

4. When is the differential $f(z)\,dz$ exact in a region Ω?

5. State the mean-value property.

6. Write down the Schwarz's formula.

7. State the Weierstrass's theorem on power series.

8. Write down the power series expansion of are $\tan z$.

9. Define the genus of an entire function.

10. Find the value of $\Gamma(2)$ and $\Gamma(1/2)$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the Cauchy's integral formula.

Or

(b) State and prove the maximum principle theorem.

12. (a) State and prove the residue theorem.

Or

(b) State and prove the argument principle theorem.

13. (a) Define harmonic function. If u_1 and u_2 are harmonic in a region Ω, then prove that $\int_{\gamma} u_1 * du_2 - u_2 * du_1 = 0$.

Or

(b) Derive the Poisson formula.

14. (a) State and prove the Schwarz's theorem.

Or

(b) State and prove the Taylor series.

15. (a) With the usual notations, prove that $\frac{\pi^2}{\sin^2 \pi z} = \sum_{-\infty}^{\infty} \frac{1}{(z-n)^2}$.

Or

(b) Show that $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right) = \frac{1}{2}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the local mapping theorem.

17. If $pdx + qdy$ is locally exact in Ω, then prove that $\int_{\gamma} pdx + qdy = 0$ for every cycle $\gamma \sim 0$ in Ω.

18. Show that $\int_{0}^{\pi} \log \sin x \, dx = \pi \log\left(\frac{1}{2}\right)$.

19. Obtain the Laurent expansion $\sum_{n=-\infty}^{\infty} A_n (z-a)^n$ for the function $f(z)$ analytic in $R_1 < |z-a| < R_2$.

20. Establish the Jensen's formula

$$\log|f(0)| = -\sum_{i=1}^{n} \log\left(\frac{\rho}{|a_i|}\right) + \frac{1}{2\pi} \int_{0}^{2\pi} \log\left|f\left(\rho e^{i\theta}\right)\right| d\theta .$$

S–3146

Sub. Code
23MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2026

Third Semester

Mathematics

PROBABILITY THEORY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. If events A and B are satisfy the condition $A < B$, then prove that $P(A) \leq P(B)$.

2. Define a continuity interval.

3. The random variable X can take on two values 2 and 4, the probability of $P(X=2)=0.2$ and $P(X=4)=0.8$. Find the value of $E(X^2)$.

4. What is meant by quantile of order p?

5. Define semi-invariants.

6. The characteristic function of the random variables X is given by the $\phi(t)=\exp\left(\frac{-t^2}{2}\right)$. Find the density function of this random variable.

7. Define a polya distribution.

8. What is meant by the uniform distribution?

9. State the Lapunov theorem.

10. Define strong law of large numbers.

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) Let $\{A_n\}$, $n = 1, 2, \ldots$ be a non increasing sequence of events and let A be their product. Then prove that $P(A) = \lim_{n \to \infty} P(A_n)$.

Or

(b) Guns 1 and 2 are shooting at the same target. It has been found that gun 1 shoots on the average nine shots during the same time gun 2 shoots ten shots. The precision of these two guns is not the same ; on the average, out of ten shots from gun 1 eight hit the target, and from gun 2, only seven. During the shooting the target has been hit by a bullet, but it is not known which gun shot this bullet. What is the probability that the target was hit by gun 2?

12. (a) If for a random variable X the absolute moment of order n exists, for arbitrary $k (k = 1, 2, \ldots n - 1)$ the following inequality is then prove that $\beta_k^{\frac{1}{k}} \leq \beta_{k+1}^{1(k+1)}$.

Or

(b) Prove that the probability that the random variables $X_1, X_2, \ldots X_N$ whose variances exist, satisfy atleast one linear relation, equals 1 if and only if $|M| = \theta$.

13. (a) Find the characteristic function and the moments of a normal distribution is $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$.

Or

(b) Find the density function of the random variable X, whose characteristic function is $\phi(t) = \begin{cases} 1 - |t| & for\ |t| \le 1 \\ 0 & for\ |t| > 1 \end{cases}$.

14. (a) Explain the uniform distribution.

Or

(b) (i) Define a beta distribution.

(ii) The random variable X has the beta distribution with $p = q = 2$, hence its density $f(y)$ has the form $f(y) = \begin{cases} 0 & for\ y \le 0\ and\ y \ge 1 \\ 6y(1-y) & for\ 0 < y < 1 \end{cases}$. What is the probability that X is not greater than 0.2?

15. (a) Show that the sequence of random variables $\{X_n\}$ given by $P\left(Y_n = \frac{r}{n}\right) = \binom{n}{r} p^r (1-p)^{n-r}$ and $X_n = Y_n - P$ is stochastically convergent to θ, that is, for any $\varepsilon > \theta$ we have $\lim_{n \to \infty} p\left(|X_n| > \varepsilon\right) = 0$.

Or

(b) State and prove Borel - Cantelli lemma.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the single - valued function $F(x)$ is a distribution function if and only if it is non decreasing, continuous at least from the left and satisfies conditions $F(-\infty)-0$ and $F(+\infty)=1$.

17. State and prove the Chebyshev inequality.

18. Let $F(x,y)$, $F(x)$, $F_2(y)$, $\phi(t, u)$, $\phi_1(t)$ and $\phi_2(u)$ denote the distribution functions and the characteristic functions of the random variables (X, Y), X and Y, respectively. The random variables X and Y are then prove that independent iff the equation $\phi(t, u)=\phi_1(t)\phi_2(u)$ holds for all real t and u.

19. If for N = 1, 2, ... equality $p=\frac{b}{N}=$ constant is satisfied and $\lim_{N\to\infty} \alpha=0$, then prove that the probability function of the random variable X with the polya distribution tends to the probability function of the binomial distribution as $N\to\infty$.

20. State and prove the Levy-Cramer theorem.

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S–3147

Sub. Code
23MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2026

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the finite complement topology. Give an example.

2. Define open rays on an ordered set.

3. Prove that the function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = 3x + 1$ is a homeomorphism.

4. Define the box topology with an example.

5. Is the space $\mathbb{R}_l$ connected? Justify your answer.

6. Define locally path connected with suitable example.

7. Prove that every closed subspace of a compact space is compact.

8. Define sequentially compact.

9. Define first countability axiom on a topological space x.

10. State the imbedding theorem.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ', respectively, on X. Prove the following are equivalent :

(i) τ' is finer than τ

(ii) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.

Or

(b) Let A be a subset of the topological space x and let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

12. (a) State and prove the pasting lemma.

Or

(b) State and prove the sequence lemma.

13. (a) State and prove intermediate value theorem.

Or

(b) If x is a topological space, each path component of x lies in a component of x. If x is locally path connected, then prove that the components and the path components of x are the same.

14. (a) Prove that a subspace A of $\mathbb{R}^n$ is compact if and only if it is closed and is bounded in the Euclidian metric d or the square metric ρ.

Or

(b) Define limit point compact with an example. Also prove that [0, 1] is not limit point compact as a subspace of $\mathbb{R}_l$.

15. (a) Show that every matrizable space is normal.

Or

(b) Prove that a product of completely regular spaces is completely regular.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) If $\mathcal{B}$ is a basis for the topology of x and $\mathcal{C}$ is a basis for the topology of y, then prove that the collection $\mathcal{D} = \{B \times C / B \in \mathcal{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

(b) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

17. Let $\bar{d}(a, b) = \min\{|a-b|, 1\}$ be the standard bounded metric on $\mathbb{R}$. If x and y are two points of $\mathbb{R}^w$, define $D(x, y) = \sup\left\{\frac{\bar{d}(x_i, y_i)}{i}\right\}$. Prove that D is a metric that induces the product topology on $\mathbb{R}^w$.

18. (a) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

(b) Show that a finite Cartesian product of connected spaces is connected.

19. Prove that the product of finitely many compact spaces is company.

20. State and prove the Tietze extension theorem.

S–3148

Sub. Code
23MMA3C4

M.Sc. DEGREE EXAMINATION, APRIL 2026

Third Semester

Mathematics

INDUSTRIAL STATISTICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an Unbiased and biased estimator of the parameter. Give an example for each.

2. What is meant by composite statistical hypothesis?

3. Define the minimum mean-square-error estimator.

4. Define complete family of probability density functions.

5. Define an efficient estimator of θ.

6. State the robust estimator.

7. What do you mean by an uniformly most powerful test?

8. Define likelihood ratio test.

9. Define a noncentral chi-square distribution.

10. What is meant by analysis of variance?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Find the maximum likelihood estimators of the parameters θ_1, θ_2 of the distribution $N(\theta_1, \theta_2)$. Are they biased or unbiased?

Or

(b) Let two independent random samples, each of size 10, from two normal distribution $N(\mu, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\overline{x} = 4.8$, $s_1^2 = 8.64$, $\overline{y} = 5.6$, $s_2^2 = 7.88$. Find 95% confidence interval for $\mu_1 - \mu_2$.

12. (a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the normal distribution $N(0, \theta)$, $0 < \theta < \infty$. Show that $\sum_1^n x_i^2$ is a sufficient statistic for θ.

Or

(b) Let $\overline{x}$ denote the mean of the random sample $x_1, x_2, \ldots, x_n$ from a gamma-type distribution with parameters $\alpha > 0$ and $\beta = \theta > 0$. Compute $E[x_1/\overline{x}]$.

13. (a) Discuss the Bayesian estimation.

Or

(b) With the usual notations, prove that the Fisher information $I(\theta) = \int_{-\infty}^{\infty} \left[\frac{\partial \ln f(x;\theta)}{\partial \theta}\right]^2 f(x;\theta)\, dx$.

14. (a) Let $x_1, x_2, \ldots, x_{25}$ denote a random sample of size 25 from a normal distribution $N(\theta, 100)$. Find a uniformly most powerful critical region of size $\alpha = 0.10$ for testing $H_0 : \theta = 75$ against $H_1 : \theta > 75$.

Or

(b) Narrate the sequential probability ratio test.

15. (a) Discuss the role of the distributions of a certain quadratic forms in the technique of analysis of variance.

Or

(b) With the usual notations, prove that

$$R = \frac{\sum_1^n (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_1^n (x_i - \overline{x})^2 \sum_1^n (y_i - \overline{y})^2}} = \frac{\sum_1^n x_i y_i - n\,\overline{x}\,\overline{y}}{\sqrt{\left(\sum_1^n x_i^2 - n\overline{x}^2\right)\left(\sum_1^n y_i^2 - n\overline{y}^2\right)}}.$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Discuss χ^2 test.

17. (a) Let $x_1, x_2, \ldots, x_n$ denote a random sample from a normal distribution with mean zero and variance θ, $0 < \theta < \infty$. Show that $\sum_1^n x_1^2 / n$ is an unbiased estimator of θ and has variance $2\theta^2/n$.

(b) State and prove the Rao and Blackwell theorem.

18. State and establish the Rao-Cramer inequality.

19. Prove the Neyman-Pearson theorem with statement.

20. Enumerate the following terms :

    (a) Least squares method.

    (b) Correlation coefficient.

S–3149

Sub. Code
23MMA3E1

M.Sc. DEGREE EXAMINATION, APRIL 2026

Third Semester

Mathematics

Elective — ALGEBRAIC NUMBER THEORY

(CBCS – 2023 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define finitely generated abelian group.

2. What is unimodular?

3. Define algebraic integer.

4. Define integral basis.

5. If K is a number field then prove that $K = Q(\theta)$, for an algebraic integer θ.

6. If $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ is any Q-basis of K, then prove that $\Delta[\alpha_1, \alpha_2, ..., \alpha_n] = \det(T(\alpha_i \alpha_j))$.

7. Define quadratic field.

8. Find $Q(\sqrt{-3})$.

9. Define noetherian.

10. State fermat's theorem.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that every finite integral domain is a field.

Or

(b) Let G be a free abelian grouo of rank r and H a subgroup of G. Then prove that G/H is a finite if and only if the ranks of G and H are equal. If this is the case and if G and H have bases $x_1,\ldots,x_r$ and $y_1,\ldots,y_r$ with $y_i = \sum_j a_{ij}\, x_j$ then prove that $|G|H| = |\det(a_{ij})|$.

12. (a) Prove that the co-efficients of the field polynomial are rotational numbers, so that $f_\alpha(t) \in Q[t]$.

Or

(b) Prove that an algebraic number α is an algebraic integer if and only if its minimum polynomial over Q has co-efficients in z.

13. (a) Let $K = Q(\theta)$ be a number field where θ has minimum polynomial p of degree n. Prove that the Q – basis $\{1, \theta, \theta^2, \ldots., \theta^{n-1}\}$ has discrimination $\Delta[1, \theta, \ldots., \theta^{n-1}] = (-1)^{n(n-1)/2} N(Dp(\theta))$, where Dp is the formal derivative of p.

Or

(b) Suppose that $G \neq D$. Prove that there exists an algebraic integer of the form

$\frac{1}{p}(\lambda_1\alpha_1 + \lambda_2\alpha_2 + \ldots. + \lambda_n\alpha_n)$ where

$0 \le \lambda_i \le p-1, \lambda_i \in z$ and p is a prime such that p^2 divides Δ_G.

14. (a) If $d \not\equiv 1 (\bmod 4)$ then prove that $Q(\sqrt{d})$ has an integral basis of the form $\{1, \sqrt{d}\}$ and discriminant $4d$.

Or

(b) Prove that the discriminant of $Q(\zeta)$, where $\zeta = e^{2\pi i/p}$ and p is an odd prime is $(-1)^{(p-1)/2} \times p^{p-2}$.

15. (a) Prove that for a domain D,

(i) x is a unit if and only if $x/1$.

(ii) x, y are associates if and only if x/y and y/x.

(iii) x is irreducible if and only if every divisor of x is an associate of x (or) a unit.

Or

(b) Prove that the ring of integers D in a number of field K is noetherian.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that every subgroup H of a free abelian group G of rank n is free of rank $s \le n$. Moreover there exists a basis $u_1, u_2, \ldots, u_n$ for G and positive integers $\alpha_1, \alpha_2, \ldots, \alpha_s$ such that $\alpha_1 u_1, \ldots, \alpha_s u_s$ is a basis for H.

17. Prove that if K is a number field then $K = Q(\theta)$ for some algebraic number θ.

18. (a) Find the ring of integers of $Q\left(\sqrt[3]{175}\right)$.

(b) Show that it has no z-basis of the form $\{1, \theta, \theta^2\}$.

19. Let d be a square free rational integer. Prove that the integers of $Q\left(\sqrt{d}\right)$ are

(a) $z\left[\sqrt{d}\right]$ if $d \not\equiv 1 \pmod 4$,

(b) $z\left[\frac{1}{2} + \frac{1}{2}\sqrt{d}\right]$ if $d \equiv 1 \pmod 4$.

20. If D is a domain and x, y are non-zero elements of D then prove that

(a) x/y if and only if $\langle x \rangle \supseteq \langle y \rangle$;

(b) x and y are associates if and only if $\langle x \rangle = \langle y \rangle$;

(c) x is a unit if and only if $\langle x \rangle = D$;

(d) x is a irreducible if and only if $\langle x \rangle$ is maximal among the proper principal ideals of D.

S–3151

Sub. Code
23MMA3E3

M.Sc. DEGREE EXAMINATION, APRIL 2026

Third Semester

Mathematics

Elective – STOCHASTIC PROCESSES

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define stationary process.

2. What is meant by transition matrix?

3. State basic limit theorem of renewal theory.

4. State general Erogodic theorem.

5. What are the types of postulates for Poisson process?

6. Define transition density matrix.

7. Define mean recurrence time.

8. State Black Well's theorem.

9. What is meant by Bienayame-Galton-Watson process?

10. Show that $E\{X_{n+r}|X_n\} = X_n m^r$, for $r, n = 0,1,2\ldots$.

Part B

(5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) The process $\{X(t), t \in T\}$, where probability distribution under a certain condition, is given by

$$P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,\ldots \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$

Show that the process is not stationary.

Or

(b) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2, and with transition matrix $P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and the initial distribution $P\{X_0 = i\} = \dfrac{1}{z}$, $i = 0,1,2$.

Find (i) $P\{X_2 = 2, X_1 = 1 \mid X_0 = 2\}$

(ii) $P\{X_2 = 2, X_1 = 1, X_0 = 2\}$

(iii) $P\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$.

12. (a) If state j is persistent non-null, then prove that as $n \to \infty$

(i) $P_{jj}^{(nt)} \to t / \mu_{jj}$ when state j is periodic with period t.

(ii) $P_{jj}^{(n)} \to 1 / \mu_{jj}$ when state j is a periodic.

Or

(b) Let a finite Markov chain with state space $S = \{0,1,\dots l\}$ be also a martingale. Then prove that, as $n \to \infty$, $P_{ij}^{(n)} = 0$, $j = 1,2,\dots l-1$ $P_{il}^{(n)} = \frac{i}{l}$ and $P_{io}^{(n)} = 1 - \frac{i}{l}$ for $i = 1,2,,\dots l-1$.

13. (a) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that $P\{N(s) = K \mid |V|(t) = n\} = \binom{n}{k}\left(\frac{s}{t}\right)^k \left(1 - \left(\frac{s}{t}\right)\right)^{n-k}$.

Or

(b) Show that the interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter X has a negative exponential distribution with mean $1/\lambda$.

14. (a) State and prove basic Renewal theorem.

Or

(b) Show that the probability 1, $\frac{N(t)}{t} \to \frac{1}{\mu}$ as $t \to \infty$ where $\mu = E(X_n) \leq \infty$.

15. (a) Prove that the p.g.f. $R_n(S)$ of Y_n satisfies the recurrence relation $Rn(S) = SP(R_{n-1}(S))$, $P(S)$ being the p.d.f. of the offspring distribution.

Or

(b) Show that the Galton –Watson process with $m = 1$ and $\sigma^2 < \infty$, we have $\lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{1 - P_n(S)} - \frac{1}{1 - S}\right\} \to \frac{\sigma^2}{2}$ uniformly in $0 \leq s < 1$.

Part C

(3 × 10 = 30)

Answer any **three** questions.

16. Explain Polya's urn model.

17. Prove that state j is persistent if and only if $\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty$.

18. Show that the p.g.f of a non-homogeneous process $\{W(t), t \geq 0\}$ is given by $Q(s,t) = \exp\{m(t)(s-1)\}$ where $m(t) = \int_0^1 \lambda(x)dx$ is the expectation of $N(t)$.

19. State and prove Central Limit theorem.

20. Prove that $P_n(S) = P_{n-1}(P(S))$ and $P_n(S) = P(P_{n-1}(S))$.

S–3152

Sub. Code
23MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2026

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Define a normed linear space. Give an example.

2. Define the following terms:

   (a) Strong topology

   (b) Weak topology

3. Define a Hilbert space. Give an example.

4. What is meant by an orthonormal set?

5. Define eigen space of T.

6. Let T be an operator on H. If $T^k = 0$ for some positive integer k, show that $\sigma(T) = \{0\}$.

7. Define a Banach algebra with an example.

8. Write down the resolvent equation.

9. What is meant by maximal ideal space?

10. Define the following terms in B^*-algebra.

   (a) Self-adjoint;

   (b) Normal

   (c) Projector.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Show that the real linear space R and the complex linear space C are Banach space under the norm $\|x\| = |x|$, $x \in C$ or R.

Or

   (b) State and prove the Hahn-Banach theorem.

12. (a) If M is a closed linear subspace of the Hilbert space H, then prove that $H = M \oplus M^{\perp}$.

Or

   (b) When will you say that N is said to be normal in Hilbert space? I every self-adjoint operator normal? Justify your answer. Also prove that $\|N^2\| = \|N\|^2$ if N is a normal operator on H.

13. (a) If $B = \{e_i\}$ is a basis for H, then prove that the mapping $T \to [T]$, which assigns to each operator T its matrix relative to B, is an isomorphism of the algebra $\mathfrak{B}(H)$ onto the total matrix algebra A_n.

Or

(b) For a two dimensional Hilbert space H, let $B = \{e_1, e_2\}$ be a basis and T be an operator on H given by the matrix $A = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$. If T is given by $Te_1 = e_2$ and $Te_2 = -e_1$, find the spectrum of T.

14. (a) Prove that the boundary of S is a subset of Z.

Or

(b) With the usual notations, prove that $\sigma(x^n) = \sigma(x)^n$.

15. (a) If A is self-adjoint, then prove that $\hat{A}$ is dense in $C(\mathfrak{M})$.

Or

(b) If x is a normal element in a B^*-algebra, then prove that $\|x^2\| = \|x\|^2$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the open mapping theorem.

17. (a) If x, y are any two vectors in a Hilbert space H, then prove that $4(x, y) = \|x + y\|^2 - \|x - y\|^2 + i\|x + iy\|^2 - i\|x - iy\|^2$.

(b) State and establish the Bessel's inequality.

18. State and prove the spectral theorem.

19. Prove that G is an open set, and therefore S is a closed set.

20. State and prove the Gelfand mapping theorem.

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S–3153

Sub. Code
23MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2026

Fourth Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the equation of the osculating plane at a point of inflexion.

2. What is meant by the torsion of the curve?

3. Define a right helicoid.

4. Find the fundamental coefficients E, F, G for the paraboloid $\vec{r} = \left(u, v, u^2 - v^2\right)$.

5. State the canonical equations for geodesics.

6. Write down the Christoffel symbols of the second kind.

7. Define the following terms :

   (a) Mean curvature ;

   (b) Umbilic.

8. Define an edge of regression.

9. Define a metric $\rho(A, B)$ on a connected surface.

10. What is meant by field of geodesics?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the curvature and torsion of the cubic curve given by $\vec{r} = (u, u^2, u^3)$.

Or

(b) Show that the involute of a circular helix and plane curves.

12. (a) Find the coefficients of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

Or

(b) Show that on a right helicoid, the family of curves orthogonal to the curves $u \cos v =$ constant is the family $(u^2 + a^2)\sin^2 v =$ constant.

13. (a) Show that every helix on a cylinder is a geodesic.

Or

(b) State and prove the Gauss-Bonnet theorem.

14. (a) Establish the second fundamental form of a surface.

Or

(b) Show that the surface $e^z \cos x = \cos y$ is minimal.

15. (a) Prove that the only compact surfaces with constant Gaussian curvature are spheres.

Or

(b) State and prove the Jacobi theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and establish the Serret-Frenet formulae.

17. Explain the isometric surfaces. Also find a surface of resolution which isometric with a region of the right helicoid.

18. Derive the geodesic differential equations.

19. Establish the Rodrigue's formula.

20. State and prove the Hilbert's lemma.

S–3154

Sub. Code
23MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2026

Fourth Semester

Mathematics

MECHANICS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by degrees of freedom? Give an example.

2. Define the potential energy.

3. State the standard form of Lagrange's equation for a holonomic system.

4. Define an ignorable coordinates.

5. State the Hamilton's principle.

6. Write down the Jacobi's form of the principle of least action.

7. What is meant by Pfaffian differential forms?

8. Write down the modified Hamilton-Jacobi equation.

9. Define a point transformation.

10. Define Lagrange bracket.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Establish Konig's theorem concerning kinetic energy of a particle system in motion.

Or

(b) If the position of the particle is given by the spherical coordinates (r, θ, ϕ), find the components of generalized momentum.

12. (a) A particle of mass m can slide without function on the inside of a small tube which is bent in the form of a circle of radius r. The tube rotates about a vertical diameter with a constant angular velocity ω. Find the differential equation of motion.

Or

(b) Suppose a mass-spring system is attached to a frame which is translating with a uniform velocity v_0. Let l_0 be the unstressed spring length and use the elongation x as the generalized coordinate. Find the Jacobi integral for the system.

13. (a) Establish the Euler-Lagrange equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$.

Or

(b) A particle of mass m is attracted to a fixed point 0 by an inverse square force, that is $F_r = -\frac{\mu m}{r^2}$ where μ is the gravitational coefficient. The plane polar co-ordinates (r, θ) are used to described the position of the particle. Find the equations of motion.

14. (a) Use Hamilton Jacobi method to analyse the Kepler problem.

Or

(b) Define Lioville's system and show that the Lioville's conditions are sufficient for the separability of an orthogonal system.

15. (a) Define a homogeneous canonical transformation and analysis the generating functions associated with such a transformation.

Or

(b) Establish the Jacobi's identity.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) State and prove D'Alembert's principle.

(b) Explain the following terms :

(i) Virtual displacement ;

(ii) Principle of virtual work.

17. Define Lagrange's equations in the standard form for a nonholonomic system.

18. Using Jacobi's form of the principle of least action, obtain the orbit for the Kepler problem in the form

$r = \dfrac{c^2/\mu m^2}{1+\sqrt{1+2hc^2/\mu^2 m^3}\cos\theta}$, with an eccentricity $e = \sqrt{1+2hc^2/m^3\mu^2}$.

19. Establish the Hamilton-Jacobi equation in the forms

$$\frac{\partial s}{\partial t} + H\left(q, \frac{\partial s}{\partial q}, t\right) = 0 .$$

20. Prove that the transformation $Q = \log\left(\frac{\sin p}{p}\right)$, $p = q \cot p$ is canonical. Obtain the four major types of generating functions associated with this canonical transformation.

S–3155

Sub. Code
23MMA4E1

M.Sc. DEGREE EXAMINATION, APRIL 2026

Fourth Semester

Mathematics

Elective — ADVANCED NUMERICAL ANALYSIS

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is meant by Multipoint Iteration method?

2. Define asymptotic error constant.

3. What is matrix norm? Give the properties.

4. Under what condition power method is suitable to find the Eigen value of the matrix?

5. What is meant by finite elements and knots?

6. List the disadvantages of Quadratic splines.

7. What do you mean by Numerical differentiation?

8. Give the formula for Newton-Cotes Method.

9. State implicit singlestep method.

10. Give the multistep methods available for solving ordinary differential equation.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Perform two iterations of the Chebyshev method to find the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$.

Or

(b) Use synthetic division and perform two iterations of the Birge-Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 + 3x + 2 = 0$. Use the initial approximation $p_0 = 0.5$.

12. (a) Find A^{10} when $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$.

Or

(b) For the matrix $= \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$, find all the eigen values and the corresponding eigenvectors.

13. (a) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data.

x	1	2	4	8
$f(x)$	3	7	21	73

Hence, estimate the values of $f(3)$ *and* $f(7)$.

Or

(b) Obtain the cubic spline approximation for the function defined by the data.

x	0	1	2	3
$f(x)$	1	2	33	244

with $M(0) = 0$, $M(3) = 0$. Hence find an estimate of $f(2.5)$.

14. (a) Derive the formula for the first derivative of $y = f(x)$ of $O(h^2)$ using backward difference approximations.

Or

(b) Find the approximate value of

$$\int_0^1 \frac{\sin x}{x} dx.$$

using

(i) mid-point rule and

(ii) two-point open type rule.

15. (a) Find singlestep methods for the differential equation $y' = f(t, y,)$ which produce exact results for $y(t) = a + b\cos t + c\sin t$.

Or

(b) Determine a, b and c such that the formula

$\int_0^h f(x)dx = h\left\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right\}$ is exact for polynomials of as high order as possible, and determine the order of the truncation error.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the complex roots of the equation $f(z) = z^3 + 1 = 0$ correct to eight decimal places. Use the initial approximation to a root as $(x_0, y_0) = (0.25, 0.25)$. Compare with the exact values of the roots $\left(1 + i\sqrt{3}\right)/2$.

17. Find all the eigenvalue of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ using the Jacobi method. Iterate till the off-diagonal elements, in magnitude, are less than 0.0005.

18. Obtain the least squares polynomial approximation of degree one and two for $f(x) = x^{\frac{1}{2}}$ on [0.1].

19. Evaluate the integral $I = \int_1^2 \int_1^2 \frac{dx\, dy}{x + y}$ using the trapezoidal rule with $h = k = 0.5$ and $h = k = 0.25$. Improve the estimate using Romberg integration.

20. For the initial value problem $u' = t + u,\ u(0) = 1$ estimate $u(0.5)$ using the third order Adams-Moulton method.

S–3156

Sub. Code
23MMA4E2

M.Sc. DEGREE EXAMINATION, APRIL 2026

Fourth Semester

Mathematics

Elective — ALGEBRAIC TOPOLOGY

(CBCS – 2023 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say that a space x is said to be simply connected?

2. Define the following terms :

   (a) Covering map

   (b) Covering space.

3. Define antipode-preserving.

4. What is meant by the projective plane p^2?

5. Define the least normal sub group of G.

6. Define the wedge of the circles S_α.

7. Define n-fold torus.

8. What is meant by torus type and projective type?

9. Define the group of covering transformations.

10. When will you say that a space B is said to be semi locally simply connected?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) With the usual notations, prove that the relation $\simeq$ and $\simeq$ p are equivalence relations.

Or

(b) Prove that the fundamental group of S^1 is isomorphic to the additive group of integers.

12. (a) If $n \geq 2$, prove that the n-sphere S^n is simply connected.

Or

(b) With the usual notations, prove that $\pi_1(x \times y, x_0 \times y_0)$ is isomorphic with $\pi_1(x, x_0) \times \pi_1(y_1, y_0)$.

13. (a) State and prove uniqueness of direct sums theorem.

Or

(b) Let $G = G_1 * G_2$. Let N_i be a normal subgroup of G_i, for $i = 1, 2$. If N is the least normal subgroup of G that contains N_1 and N_2, then prove that $G/N \cong \left(G_1/N_1\right) * \left(G_2/N_2\right)$.

14. (a) If X is the m-fold connected sum of projective planes, then prove that the torsion subgroup $T(X)$ of $H_1(X)$ has order 2 and $H_1(X)/T(X)$ is a free abelian group of rank $m-1$.

Or

(b) Show that the closed unit ball in $\mathbb{R}^2$ is a 2-manifold with boundary.

15. (a) Let B be path connected and locally path connected. Let $p : E \to B$ be a covering map in the former sense. If E_0 is a path component of E, then prove that $p_0 : E_0 \to B$ obtained by restricting p is a covering map.

Or

(b) Show that the image of the map ψ equals the image under ϕ of the subgroup $N(H_0)/H_0$ of $\pi_1(B, b_0)/H_0$.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let $h : s^1 \to x$ be a continuous map. Prove the following conditions are equivalent :

(a) h is nul homotopic.

(b) h extends to a continuous map $k : B^2 \to X$.

(c) h_* is the trivial homomorphism of fundamental groups.

17. If $h :\to S^1 \to S^1$ is continuous and antipode-preserving, then prove that h is not nulhomotopic.

18. State and establish the Seifert-Van Kampen theorem for classical version.

19. Let w be a proper scheme of the form $w = w_0 w_1$, where w_1 is a scheme of torus type that does not contain two adjacent terms having the same label? Prove that w is equivalent to a scheme of the form $w_0\, w_2$, where w_2 has the same length as w_1 and has the form $w_2 = aba^{-1}b^{-1}w_3$, where w_3 is of torus type or is empty.

20. State and prove the general lifting lemma.
