D–5026

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

First Semester

ALGEBRA – I

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define equivalence relation.

2. Define normal subgroup. Give an example.

3. Show that a group of order 21 is not simple.

4. Define isomorphism of groups.

5. Show that $a \in z$ if and only if $N(a) = G$.

6. Define commutative ring with an example.

7. Define two sided ideal of a ring.

8. When will you say a polynomial is integer monic?

9. Prove that an Euclidean ring posses an unit element.

10. Find the greatest common divisor of the polynomials.
$x^2 + 1$ and $x^6 + x^3 + x + 1$ in $Q[x]$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any three sets $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Or

(b) If a is relatively prime to b but a / bc, then prove that a / c.

12. (a) Prove that there is a one-to-one correspondence between any two right cosets of H in G.

Or

(b) If H is a non-empty finite subset of a group G and H is closed under multiplication, then prove that H is a subgroup of G.

13. (a) Prove that any two p-Sylow subgroups of a group G are conjugate.

Or

(b) Prove that a finite integral domain is a field.

14. (a) If F is a field, then prove that its only ideals are (0) and F itself.

Or

(b) Let R be an Euclidean ring. Prove that every element in R is either a unit in R or can be written as the product of finite number of prime elements of R.

15. (a) State and prove that division algorithm for polynomials.

Or

(b) If R is an integral domain, then prove that $R[x_1, x_2, \ldots x_n]$ is also an integral domain.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Cayle's theorem.

17. Let Q be a homomorphism of G onto $\overline{G}$ with Kernel K. Prove that $G | K \cong \overline{G}$.

18. State and prove the first part of Sylow's theorem.

19. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

20. Prove that $J(i)$ is an Euclidean ring with usual notations.

D–5027

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

First Semester

ANALYSIS – I

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Is Q an ordered set? Justify.

2. If a and b are real, then show that $(a,b) = a + bi$.

3. Define compact set. Give an example.

4. Define metric space.

5. Let $E = \{x / x^2 < 2\}$. Find $\sup E$.

6. Define monotonically increasing and decreasing functions.

7. Is the convergence of $\{S_n\}$ implies convergence of $\{|S_n|\}$? Justify.

8. Define uniformly continuous function. Give an example.

9. State chain rule for differentiation.

10. State the intermediate value theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the rational numbers are dense in R.

Or

(b) Prove that a subset of a countable set is countable.

12. (a) If $p > 0$, then prove that $\lim_{n \to \infty} \frac{1}{n^p} = 0$.

Or

(b) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

13. (a) Let E be a non-compact set in R^1. Prove that there exists a continuous function on E which is not bounded.

Or

(b) Let f be monotonic on (a,b). Prove that the set of points of (a,b) at which f is discontinuous is at most countable.

14. (a) Let f be defined on $[a,b]$, if f has a local maximum at a point $x \in (a,b)$, and if $f'(x)$ exists, then prove that $f'(x) = 0$.

Or

(b) Let f be defined for all real x, and $|f(x) - f(y)| \leq (x - y)^2$ for all real x and y. Prove that f is constant.

15. (a) Suppose of maps an open set $E \subset R^n$ into R^m. For $1 \le i \le m$, $1 \le j \le n$, then partial derivatives $D_j f_i$ exist and are continuous on E, prove that $f \in \zeta'(E)$.

Or

(b) If X is a complete metric space, and if φ is a contraction of X into X, then prove that there exists one and only one $x \in X$ such that $\varphi(x) = x$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that every k-cell is compact.

17. Prove that every bounded infinite subset of R^k has a limit point in R^k.

18. Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$.

19. State and prove Bolzano – Weierstrass theorem.

20. State and prove Taylors theorem.

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D–5028

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Find all real valued solutions of the equation $y'' - y = 0$.

2. State the existence theorem for analytic coefficients.

3. Find the regular singular points of $xy'' + 4y = 0$.

4. Suppose $\varphi_1(x) = \cos x$ and $\varphi_2(x) = \sin x$ exist in $-\infty < x < \infty$. Determine whether they are linearly dependent or linearly independent.

5. Show that $P_n(-x) = (-1)^n P_n(x)$.

6. Solve : $y'' + 4y = \cos x$.

7. Define singular points.

8. Solve $y' = 3y^{2/3}$.

9. State the local existence theorem for the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ on z.

10. State the interior Churchill problem.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that there exits n linearly independent solutions of $L(y) = 0$ on an interval I.

Or

(b) Find all solutions of $y'' + y = 2\sin x \sin 2x$.

12. (a) Find the indicial polynomials and their roots of $x^2 y'' + (x^2 + x)y' - y = 0$.

Or

(b) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\varphi_1(x) = x^2$. Find all solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

13. (a) Find two linearly independent power series solutions of $y'' + 3x^2 y' - xy = 0$.

Or

(b) Show that $x^{\frac{1}{2}} J_{\frac{1}{2}}(x) = \frac{\sqrt{2}}{\Gamma(\frac{1}{2})} \sin x$ and $x^{\frac{1}{2}} J_{-\frac{1}{2}}(x) = \frac{\sqrt{2}}{\Gamma(\frac{1}{2})} \cos x$.

14. (a) By computing appropriate Lipschitz constants, show that $f(x,y) = 4x^2 + y^2$ on $S : |x| \le 1, \quad |y| \le 1$ satisfy Lipschitz conditions.

Or

(b) Verify whether the equation $(x^2 + xy)dx + xydy = 0$ is exact or not, if exact solve.

15. (a) Let f be continuous and satisfy Lipschitz condition on R. If φ and ψ are two solutions of $y' = f(x,y)$, $y(x_0) = y_0$ on an interval I containing x_0, then prove that $\varphi(x) = \psi(x)$ for all x in I.

Or

(b) Establish a necessary condition for the existence of the solution of the interior Neumann problem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that $\int_{-1}^{1} P_n(x) P_m(x) dx = 0$, $n \ne m$.

17. Find all solutions of

(a) $y'' - 4y' + 5y = 3e^{-x}$

(b) $y'' - 7y' + 6y = \sin x$.

18. If $\varphi_1, \varphi_2, \ldots \varphi_n$ are n solutions of $L(y) = 0$ on an interval I, they are linearly independent there if and only if $W(\varphi_1, \varphi_2, \ldots, \varphi_n)(x) \ne 0$ for all x in I – Prove.

19. Let M and N be two real valued functions which have continuous first order partial derivatives on some rectangle $R: |x - x_0| \le a$, $|y - y_0| \le b$. Prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

20. Show that all real-valued solutions of the equation $y'' + \sin y = b(x)$ which b is continuous for $-\infty < x < \infty$ exist for all real x.

D–5029

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

First Semester

TOPOLOGY – I

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define injective function.

2. State the maximum principle.

3. Define countably infinite set, give an example.

4. Define indiscrete topology with an example.

5. Define projection mapping.

6. Define compact space, give an example.

7. State the intermediate value theorem.

8. Show that (0, 1) is not compact.

9. Prove that a subspace of Lindelof space need not be Lindelof.

10. Define completely regular space.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove well-ordering property.

Or

(b) Prove that a countable union of countable set is countable.

12. (a) Show that the topologies R_l and R_k are not comparable.

Or

(b) Let Y be a subspace of X. If U is open in Y and Y is open in X, then prove that U is open in X.

13. (a) Prove that the uniform topology $\mathbb{R}^J$ is finer then the product topology.

Or

(b) State and prove sequence lemma.

14. (a) If L is a linear continuum in the order topology, then prove that L is connected and so are intervals and rays in L.

Or

(b) Show that compactness implies limit point compactness, but not conversely.

15. (a) Prove that a subspace of a regular space is regular.

Or

(b) Define Lindelof space. Is a subspace of a Lindelof space Lindelof? Justify.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that finite unions and finite Cartesian products of finite sets are finite.

17. If B is a basis for the topology of X and $\mathcal{C}$ a basis for the topology of Y, then prove that the collection

$D = \{B \times C / B \in B$ and $c \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

18. Prove that a space X is locally connected if and only if for every open set U of x each component of U is open in X.

19. State and prove Lebesgue number lemma.

20. State and prove Urysohn lemma.

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D–5030

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

Second Semester

ALGEBRA – II

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define subspace of a vector space.

2. Define homomorphism, $Hom(u,v)$.

3. If $\dim_F V = m$ then find $\dim_F Hom(V,F)$.

4. Define orthonormal set.

5. Define algebraic number.

6. Find the degree of the splitting field $x^4 + 1$ over F.

7. Define normal extension of F.

8. What is $F[(x_1, x_2, \ldots, x_n) : s]$?

9. Define characteristic vector of T.

10. If $TT^* = 1$, $T \in A(V)$, then prove that T is unitary.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the Kernal of a homomorphism is a subspace.

Or

(b) If V is the internal direct sum of $U_1, U_2, \ldots, U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2, \ldots, U_n$.

12. (a) If F is a field of characteristic $p \neq 0$, then prove that the polynomial $x^{p^n} - x \in F[x]$, for $n \geq 1$, has distinct roots.

Or

(b) If F is the field of real numbers, prove that the vectors (1, 1, 0, 0), (0, 1, –1, 0), (0, 0, 0, 3) in $F^{(4)}$ are linearly independent over F.

13. (a) State and prove Schwarz inequality.

Or

(b) Prove that if L is an algebra extension of K and K is an algebraic extension of F, then L is an algebraic extension of F.

14. (a) Prove that the elements in K which are algebraic over F form a subfield of K.

Or

(b) If $T \in A(V)$ is nilpotent, then prove that $\alpha_0 + \alpha_1 T + \cdots + \alpha_m T^m$, where the $\alpha_i \in F$, is invertble if $\alpha_0 \neq 0$.

15. (a) Prove that the linear transformation T on V is unitary if and only if takes an orthonormal basis of V into an orthonormal basis of V.

Or

(b) Prove that the normal transformation N is unitary if and only if its characteristic roots are all of absolute value 1.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If V is finite dimensional and if W is a subspace of V, then prove that W is finite dimensional, $\dim W \leq \dim V$ and $\dim V/W = \dim V - \dim W$.

17. Prove that the number e is transcendental.

18. Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

19. State and prove Cayley Hamilton theorem.

20. Prove that, for every prime number p and every positive integer m there is a unique field having p^m elements.

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D–5031

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

Second Semester

ANALYSIS – II

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours | Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define rectifiable curve.

2. Define uniform convergence.

3. Define pointwise bounded sequence. Give an example.

4. Define algebra. Give an example.

5. Define orthogonal system.

6. Show that $\lim_{x \to \infty} x^n - e^{-x} = 0$ for every n.

7. If $0 < t < 2\pi$, show that $E(it) \neq 1$.

8. If A is countable, find $m^* A$.

9. State the Fatou's theorem.

10. Let f be non negative measurable function. Show that $\int f = 0 \Rightarrow f = 0$ almost everywhere.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If f is monotonic on $[a,b]$, and if α is continuous on $[a,b]$ then prove that $f \in R(\alpha)$.

Or

(b) State and prove the fundamental theorem of calculus.

12. (a) State and prove uniform convergence theorem.

Or

(b) Is the limit of the integral equal to the integral of the limit? Justify.

13. (a) If $x > 0$ and $y > 0$, then prove that $\int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma x \,\Gamma y}{\Gamma x+y}$.

Or

(b) Prove that $\lim_{x \to 0}(1+x)^{1/x} = e$.

14. (a) Let $\{f_n\}$ be a sequence of measurable functions. For $x \in X$, $g(x) = \sup f_n(x)$, $n = 1,2,\ldots$

$$h(x) = \lim_{x \to \infty} \sup f_n(x)$$

Prove that g and h are measurable.

Or

(b) Suppose f is measurable on E, $|f| \le g$ and $g \in L(\mu)$ on E. Prove that $f \in L(\mu)$ on E.

15. (a) If E_1 and E_2 are measurable, then show that $E_1 \cup E_2$ is measurable.

Or

(b) If $f \in L^2(\mu)$ and $g \in L^2(\mu)$ then prove that $f + g \in L^2(\mu)$ and $\|f + g\| \leq \|f\| + \|g\|$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If γ' is continuous on $[a,b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)| dt$.

17. State and prove Stone Weierstrass theorem.

18. State and prove Egoroff's theorem.

19. State and prove Little Wood's third principle.

20. State and prove Lebesgue monotone convergence theorem.

D–5032

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

Second Semester

TOPOLOGY – II

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define one point compactification, give an example.
2. Give an example of a collection of sets A that is not locally finite, such that the collection $B = \{\overline{A} / A \in A\}$ is locally finite.
3. Define completely regular space.
4. State locally finite property in topological space.
5. Define an open refinement, give an example.
6. When will you say Cauchy sequence is complete?
7. Is the interval (–1, 1) in R with metric $d(x, y) = |x - y|$, complete? Justify.
8. When will you say a space X is topologically complete?
9. State totally bounded metric space.
10. Define Baire space, give an example.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a space. Let D be a collection of subsets of X that is maximal with respect to the finite intersection property. Let $D \in D$. Show that if $D \subset A$, then $A \in D$.

Or

(b) Define locally compact with an example. Show that the set of all rationals Q are not locally compact.

12. (a) Prove that a product of completely regular space is completely regular.

Or

(b) Let X be a completely regular space. If Y_1 and Y_2 are two compactifications of X satisfying the extension property that every bounded continuous map $f : X \to R$ extends uniquely to a continuous map of Y into R, then prove that Y_1 and Y_2 are equivalent.

13. (a) Prove that every closed subspace of a para compact space is para compact.

Or

(b) Let X be a normal space and let A be a closed G_δ set in X prove that there is a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) Prove that a metric space X is complete if every Cauchy sequence in X has a convergent subsequence.

Or

(b) Let X be the product space $X = \prod_{\alpha} X_{\alpha}$ and let x_n be a sequence of points of X. Prove that $X_n \to x$ if and only $\prod_{\alpha}(x_n) \to \prod_{\alpha}(x)$ for each α.

15. (a) Prove that any open surface Y of a Baire Space X is itself a Baire space.

Or

(b) Show that in the compact open topology, $\zeta(X,Y)$ is regular if Y is regular.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Tietze extension theorem.

17. State and prove Negta – Smirnov metrization theorem.

18. Let (X,d) be a metric space. Prove that there is an isometric imbedding of X into a complete metric space.

19. State and prove Ascoli's theorem.

20. Let $X = Y \cup Z$, where Y and Z are closed subspace of X having finite topological dimension. Prove that $\dim X = \max\{\dim Y, \dim Z\}$.

D–5033

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Find the integral curves of the sets of equations $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

2. Define orthogonal trajectories.

3. Form the partial differential equation by eliminating the arbitrary constants a and b from $ax^2 + by^2 + z^2 = 1$.

4. Eliminate the arbitrary function f from $z = f(x-y)$

5. Solve : $(D^4 - D'^4 - 2D^2D'^2)z = 0$

6. Show that $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is hyperbolic.

7. State the exterior Churchill problem.

8. Give an example of first order non-linear partial differential equation.

9. Find a complete integral of $p+q=pq$.

10. If $u=f(x+iy)+g(x-iy)$, where f and g are arbitrary functions, then show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Determine the integrability and then solve the equation $ydx+xdy+2zdz=0$.

Or

(b) Solve : $a^2y^2z^2dx+b^2z^2x^2dy+c^2x^2y^2dz=0$.

12. (a) Form the partial differential equation by eliminating the arbitrary constants from $x^2+y^2=(z-c)^2\tan^2\alpha$.

Or

(b) Find the general solution of $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$.

13. (a) Show that the equations $xp-yq=x$ and $x^2p+q=xz$ are compatible and find their solution.

Or

(b) Find a complete integral of $p^2x+q^2y=z$, by Jacobi's method.

14. (a) Find the D'Alembert's solution of the one dimensional wave equation $\frac{\partial^2 y}{\partial x^2} + \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$.

Or

(b) Solve $\left(D^2 - D^1\right)z = 2y - x^2$.

15. (a) Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(\gamma, \theta)$,

Or

(b) Solve the equation $\frac{\partial^2 \theta}{\partial x^2} + \frac{1}{k}\frac{\partial \theta}{\partial t}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the orthogonal trajectories on the curve $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = 0$.

17. Verify that the equation

$z\left(z + y^2\right)dx + z\left(z + x^2\right)dy - xy(x + y)dz = 0$ is integrable and find its primitive.

18. Find the equation of the integral surface of the partial differential equation $2y(z-3)p + (2x - z)q = y(2x - 3)$ which passes through the circle $z = 0$ and $x^2 + y^2 = 2x$.

19. The point of trisection of a string are pulled aside through a distance $\in$ on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the midpoint of the string always remains at rest.

20. Determine the temperature $\theta(\rho,t)$ in the infinite cylinder $0 \le \rho \le a$ when the initial temperature is $\theta(\rho,0) = f(\rho)$ and the surface $\rho = a$ is maintained at zero temperature.

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D–5034

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Write down the equation of the osculating plane at a point of inflexion.

2. What do you mean by Torsion?

3. Write a short note on the general helicoids.

4. What is meant by pitch of the helicoids?

5. Explain geodesic parallel.

6. State a characteristic property of a geodesic.

7. Define Geodesic curvature.

8. Prove that if (λ, μ) is the geodesic curvature vector then $kg = \dfrac{-\lambda H}{Fu' + Gv'}$.

9. Define principal curvature.

10. State the Meusnier's theorem.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the equation of the osculating plane of the curve given by $\bar{r} = [a \sin u + b \cos u, a \cos u + b \sin u, c \sin 2u]$

Or

(b) Prove that $[\dot{r}, \ddot{\bar{r}}, \dddot{\bar{r}}] = 0$ is a necessary and sufficient condition that the curve be plane.

12. (a) Find the area of the anchor ring.

Or

(b) Prove that the curves bisecting the angle between the parametric curves are given by $Edu^2 - Gdv^2 = 0$.

13. (a) Show that every helix on a cylinder is a geodesic.

Or

(b) Prove that on a general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$, where $v = c$, for all values of u.

14. (a) Derive an expression by k_g.

Or

(b) Find the Gaussian curvature at (u,v) of the anchor ring.

15. (a) State and prove Euler's theorem.

Or

(b) State and prove Rodrigue's formula.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Obtain the curvature and Torsion of the curve of intersection of two quadratic surfaces $ax^2 + by^2 + cz^2 = 1$, $a^1x^2 + b^1y^2 + c^1z^2 = 1$.

17. Show that the intrinsic equations of the curve given by $x = ae^u \cos u$, $y = ae^u \sin u$, $z = be^u$ are $k = \frac{a\sqrt{2}}{(2a^2 + b^2)^{1/2}} \cdot \frac{1}{s}$; $I = \frac{b}{(2a^2 + b^2)^{1/2}} \cdot \frac{1}{s}$.

18. Derive the differential equations for a geodesic using the normal property.

19. State and prove Gauss – Bonnet theorem.

20. Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normals along the curve form a developable.

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D–5035

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is Three-Jug problem?
2. Write down the Dijikstra's algorithm.
3. Define convex set and extreme point.
4. What is bounded variables algorithm?
5. Define critical and non critical activities.
6. What is necessary and sufficient condition for x_0 to be an extreme point?
7. Define Legrangian function and Legrangian multipliers.
8. Write down Karush – Kuhn – Tucka condition.
9. What is direct search method?
10. Write down the Quadratic Programming model.

PART B — (5 × 5 = 25 marks)

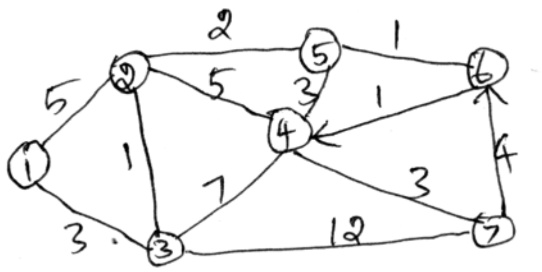
Answer ALL questions, choosing either (a) or (b).

11. (a) Apply Floyd's algorithm for the following network, to determine the shortest route between the following pairs of nodes.

(i) From node 1 to node 7

(ii) From node 7 to node 1

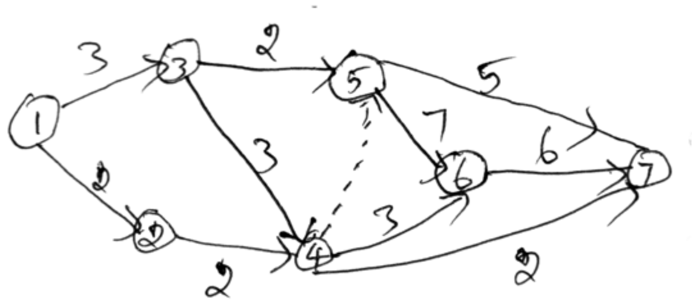
(iii) From node 6 to node 7.



Or

(b) Explain Forward Pass and Backward Pass method.

12. (a) Determine the critical path for the following project network.



Or

(b) Optimize the following LP

Maximize $z = x_1 + 4x_2 + 7x_3 + 5x_4$

Subject to : $2x_1 + x_2 + 2x_3 + 4x_4 = 10$

$3x_1 - x_2 - 2x_3 + 6x_4 = 5$

$x_1, x_2, x_3, x_4 \geq 0.$

13. (a) Explain Revised simplex algorithm.

Or

(b) Solve by bounded algorithm

Maximize $z = 3x_1 + 5x_2 + 2x_3$

Subject to $x_1 + 2x_2 + 2x_3 \leq 10$

$2x_1 + 4x_2 + 3x_3 \leq 15$

$0 \leq x_1 \leq 4$, $0 \leq x_2 \leq 3$, $0 \leq x_3 \leq 3$.

14. (a) Solve the following 2×4 game

	B_1	B_2	B_3	B_4
A_1	2	2	3	–1
A_2	4	3	2	6

Or

(b) Explain Newton – Raphson method.

15. (a) Solve the following problem using KKT condition

Minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

Subject to $2x_1 + x_2 - 5 \leq 0$

$x_1 + x_3 - 2 \leq 0$

$2 - x_2 \leq 0$

$-x_3 \leq 0.$

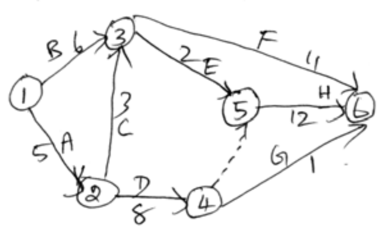
Or

(b) Explain the gradient method.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Determine the critical path for the project network given below.



17. Solve the following LP by upper bound algorithm

Maximize $z = 3x_1 + 5y + 2x_3$

Subject to $x_1 + y + 2x_3 \leq 14$

$2x_1 + 4y + 3x_3 \leq 43$

$0 \leq x_1 \leq 4$, $7 \leq y \leq 10$, $0 \leq x_3 \leq 3$.

18. Solve the following game by linear programming

	B_1	B_2	B_3
A_1	3	–1	–3
A_2	–2	4	–1
A_3	–5	–6	2

19. Solve the following problem by separable programming.

Maximize $Z = x_1 + x_2^4$

Subject to $3x_1 + 2x_2^2 \leq 9$

$x_1, x_2 \geq 0.$

20. Solve the following by quadratic programming method

Maximize $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to $x_1 + 2x_2 \leq 2$

$x_1, x_2 \geq 0$.

D–5036

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. If $(a,b)=1$, then prove that $(a+b, a-b)$ is either 1 or 2.

2. State the Euler totient function $\varphi(n)$

3. Define divisibility. Also write any two properties of divisibility.

4. Define the Bell-series of f module p.

5. Write down the Dirichlet's asymptotic formula.

6. Solve the congruence $x^2 \equiv 1 \pmod 8$.

7. Write down the Legendre's symbol $\left(n/p\right)$.

8. Write down the value of $\left(\frac{7}{11}\right)$ and $\left(\frac{22}{11}\right)$.

9. Prove the converse of Wilson's theorem.

10. Determine whether 888 is a quadratic residue or non-residue of the prime 1999.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that theme are infinitely many prime numbers.

Or

(b) If $n \geq 1$, prove that $\varphi(n) = \sum_{d/n} \mu(d) \frac{n}{d}$.

12. (a) Let f be multiplicative. Then prove that f is completely multiplicative iff $f^{-1}(n) = \mu(n) \cdot f(n)$, $\forall n \geq 1$.

Or

(b) Let $f(n) = \lfloor \sqrt{n} \rfloor - \lfloor \sqrt{n-1} \rfloor$. Prove that f is multiplicative but not completely multiplicative.

13. (a) State and prove Euler's summation formula.

Or

(b) For $x > 1$, we have $\sum_{n \leq x} \varphi(n) = \frac{3}{\pi^2} x^2 + O(x \log x)$, show that the average order of $\varphi(n)$ is $3n / \pi^2$.

14. (a) State and prove Wolstersholune's theorem.

Or

(b) Let p be an odd prime. Show that for all n, $\left(\frac{n}{p}\right) = n^{(p-1)/2} (\text{mod } p)$.

15. (a) State and prove the reciprocity law for Jacobi symbols.

Or

(b) Prove that the set of Lattice points visible from the origin has density $6/\pi^2$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) Prove that if $2^n - 1$ is prime, then n is prime.

(b) State and prove the Euclidean algorithm.

17. Define Liouville's function $\lambda(n)$. For every $n \geq 1$, prove that $\sum_{d/n} \lambda(d) \begin{cases} 1 & \text{if n is a square} \\ 0 & \text{otherwise} \end{cases}$. Also prove $\lambda^{-1}(n) = |\mu(n)|$, for all n.

18. For all $x \geq 1$, prove that $\sum_{n \leq x} d(n) = x \log x + (2c-1)x + 0(\sqrt{x})$, where c is Euler's constant.

19. (a) If $x \geq 1$ and $\alpha > 0$, $\alpha \neq 1$, prove that $\sum_{n \leq x} \sigma_\alpha(n) = \frac{\zeta(\alpha+1)}{\alpha+1} x^{\alpha+1} + O(x^\beta)$, where $\beta = \max\{1, \alpha\}$.

(b) For all $x \geq 1$, show that $\left| \sum_{n \leq x} \frac{\mu(n)}{n} \right| \leq 1$, with equality holding only if $x < 2$.

20. Show that $\varphi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$.

D–5037

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DEC 2020.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018-2019 Academic Year onwards)

Time : 3 hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Write the Chapman – Homogorov equation.

2. Define irreducible chain.

3. Define Brawnian motion process.

4. Write the forward diffusion equation of the Wierner process.

5. State Yaglom's theorem.

6. Define Markov branching process.

7. Write the Fokker – Planck equation.

8. Define utilization factor.

9. Write the Erlang's second formula.

10. Define Galtar – Watson process.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix $\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and initial distribution $\Pr(X_0 = i) = 1/3$, $i = 0,1,2$. Find $\Pr(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$.

Or

(b) Classify the nature of the states for the transition probability matrix $\begin{pmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$.

12. (a) Show that if $y(t)$ is a O-U.P with mean value 0 and covariance function $c(y(s), y(t)) = ae^{-\beta|t-s|}$, then $x(t) = (a/t)^{-1/2} y\left(\frac{1}{2}\beta \log(t/a)\right)$.

Or

(b) If $m = E(X_1) = \sum_{k=0}^{\infty} kP_k$ and $\sigma^2 = \mathrm{var}(X_1)$, prove that $E(X_n) = m^n$ and

$$\mathrm{var}(X_n) = \begin{cases} \dfrac{m^{n-1}(m^n - 1)}{m-1}\sigma^2, & \text{if } m \neq 1 \\ n\sigma^2, & \text{if } m = 1 \end{cases}.$$

13. (a) Prove that the p.g.f. $R_n(s)$ of y_n satisfies the reassurance relation $R_n(s) = sp(R_{n-1}(s))$, $P(s)$ being the p.g.f. of the offspring distribution.

Or

(b) If $m = 1$, $\sigma^2 < \infty$, find $\lim_{n \to \infty} \Pr\left(\frac{X_n}{n} > n / X_n > 0\right)$.

14. (a) Patients arrive at the outpatient department of a hospital in accordance with a Poisson process at the mean rate of 12 per hour and the distribution of time for examination by an attending physician is exponential with a mean of 10 minutes. What is the minimum number of physicians to be posted for ensuring a steady state distribution? For this number, find (i) The expected waiting time of a patient prior to being examined and (ii) The expected number of patients in the out-patient department. How many physicians, on an average, will remain idle?

Or

(b) Obtain the average waiting time in queue for the model $M / M(a,b)/1$.

15. (a) Obtain an expression for $v(t)$ and $E(T)$ for $M / M(a,\infty)/1$ model.

Or

(b) Suppose that $\{X(t), o < t\}$ is a Wierner process with $X(0) = 0$ and $N = 0$. Prove that $\Pr(X(t) \le x) = \Pr\left(X(t)/\sigma\sqrt{t} \le x / \sigma\sqrt{t}\right) = \phi\left(x / \sigma\sqrt{t}\right)$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove ergodic theorem.

17. Let $\{X(t), 0 \le t \le T\}$ be a Wierner process with $X(0) = 0$ and $\mu = 0$. Let $M(T)$ be the maximum of $X(t)$ in $0 \le t \le T$. Prove that for any $a > 0$, $\Pr(M(T) \ge a) = 2\Pr(X(T) \ge a)$.

18. If $\{X_n, n = 0,1,2,\ldots\}$, $X_0 = 1$ is a branching process with offspring distribution p.g.f. $P(s)$. Show that $\{X_{rk}, r = 0,1,2,\ldots\}$, where k is a fixed positive integer, is also a branching process.

19. Consider an $M/M/1$ queue and let $N(t)$ denote the system size at time t. Find the transition probabilities of the Markov process $\{N(t), t \ge 0\}: P_{ij}(\Delta t) = \Pr\{N(t + \Delta t) = j \mid N(t) = i\}$. Write down the rate matrix and hence derive the Kolmogorov equations.

20. (a) State and prove the additive property of a Poisson process.

    (b) If $N(t)$ is a Poisson process. Then prove that the auto-correlation coefficient between $N(t)$ and $N(t+s)$ is $\left[t/(t+s) \right]^{1/2}$.

D–6478

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2020.

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS 2018-2019 Academic year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define isomorphism of graphs.

2. Define bipartite graph with an example.

3. Prove that in a tree, any two vertices are connected by a unique path.

4. Draw all the trees with 6 vertices.

5. Determine the edge chromatic number of a Peterson graph.

6. Find the number of different perfect matchings in Kn.

7. Define $\gamma(k,l)$.

8. When will you say a graph is critical graph?

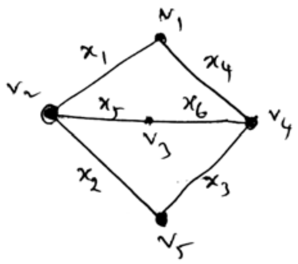
9. Embed k_5 on forms.

10. Define directed cycle.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Find the incidence matrix of the graph:



Or

(b) If G is a tree, prove that $\varepsilon = \gamma - 1$.

12. (a) Prove that a connected graph has an Euler trail if and only if has atmost two vertices of odd degree.

Or

(b) Prove that every connected graph has a spanning tree.

13. (a) If G is bipartite, then prove that $\chi' = \Delta$.

Or

(b) Suppose G is a k-regular bipartite graph, $k > 0$. Prove that G has a perfect matching.

14. (a) Show that in a critical graph no vertex cut is a clique.

Or

(b) With usual notations, prove that $\gamma(k,k) \geq 2^{k/2}$.

15. (a) Show that $K_{3,3}$ is non-planar.

Or

(b) Define self-dual graph. Also prove that: if g is self-dual then $\varepsilon = 2$,v = 2 .

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

17. Prove that a graph G with at least two vertices is bipartite if and only if all its cycles are of even length.

18. State and prove Chrital theorem.

19. State and prove the Dirac theorem.

20. Prove every planar graph is 5-vertex colourable.

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D–6479

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2020.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define normed linear space with an example.

2. Define quotient space.

3. What do you say a normed space is a Banach space? Give an example.

4. Define inner product space.

5. Define orthogonal projection.

6. State the Bessel's inequality.

7. Does Riesz representation theorem hold for incomplete inner product space? Justify.

8. Define adjoint operators.

9. Define orthonormal basis.

10. Is every closed map is continuous? Justify.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either, (a) or (b).

11. (a) Let X be a normed space and Y be a subspace of X. Prove that Y is complete if Y is finite dimensional.

Or

(b) If (X,d) and (Y,d') are metric space where $f: X \to Y$, then Prove that f is continuous at the point x if and only if for every sequence $\{x_n\}$ converging to x, $f(x_n) \to f(x)$.

12. (a) Prove that the convex hull of the subset S of the vector space X consists of all vectors of the form $\alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_n x_n$, where $x_i \in S$, $\alpha_i \geq 0$, for $i = 1,2,...,n$ and $\sum_{n=1}^{n} \alpha_i = 1$.

Or

(b) Prove that the linear functional of defined on the normed linear space X is bounded if and only if it is continuous.

13. (a) Let X be a Hilbert space and Let $A: x_1, x_2....$ be an orthonormal set in X. If $\alpha_n \in F$, $n = 1,2,....$; prove that $\sum_{n=1}^{\infty} \alpha_n x_n$ converges if and only if $\sum_{n=1}^{\infty} |\alpha_n|^2 < \infty$.

Or

(b) Let X be an inner product space and S a subset of X. Show that $[S]^{\perp} = S^{\perp}$.

14. (a) Prove that the eigen vectors associated with distinct eigen values of a self-adjoint linear transformation are orthogonal.

Or

(b) If X is a Hilbert space, then prove that X is reflexive.

15. (a) Let X and Y be normed linear space and let $A : X \rightarrow Y$ be linear transformation. If X is finite dimensional, then prove that A is bounded.

Or

(b) Prove that if X is an inner product space and if $x_n \underset{\rightarrow}{w} x$ then $(x_n y) \rightarrow (x, y)$ for each $y \in X$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that all compact sets are countably compact.

17. State and prove Minkowski inequality.

18. State and prove uniform boundedness theorem.

19. State and prove Hahn Banach theorem.

20. State and prove closed graph theorem.

D–6480

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2020.

Fourth Semester

Mathematics

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : 3 hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define asymptotic error constant from rate of convergence.

2. Define strum functions.

3. Define eigen vectors of a matrix A.

4. Define spectral norm.

5. State the Hermite interpolating polynomial

6. What is the error approximation in the r^{th} order derivative?

7. What is the trapezoidal rule?

8. Write the form of the Lobatto integration method.

9. What is meant by the order of a difference equation.

10. When the linear multistep method become unstable?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Obtain the complex root of the equation $f(z) = z^3 + 1 = 0$.

Or

(b) Find all the roots of the polynomial $x^3 - 6x^2 + 11x - 6 = 0$ using the Graeffe's root squaring method.

12. (a) Solve the equations

$$x_1 + 2x_2 - x_3 = 2$$
$$3x_1 + 6x_2 + x_3 = 1$$
$$3x_1 + 3x_2 + 2x_3 = 3$$

by using Cramer's rule

Or

(b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ using the iterative method, given that its approximate inverse is $B = \begin{bmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}$

13. (a) Derive the Hermite interpolating polynomial.

Or

(b) Obtain the least square polynomial approximation of degree one for $f(x) = x^{1/2}$ on $[0,1]$.

14. (a) Using the formula $f'(x_1) = (f(x_2) - f(x_0)/2h)$ and the Richardson extrapolation, find the value of $f'(3)$ from the following data:

x	–1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

Or

(b) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss-Legendre three point formula.

15. (a) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ using Euler method with $h = 0.2$ over the interval $[0,1]$.

Or

(b) Use the Numerov method to solve the initial value problem $u'' = (1+t^2)u$, $u(0)=1$, $u'(0)=0$, $t \in [0.1]$ with $h = 0.2$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find all the roots of the polynomial $x^4 - x^3 + 3x^2 + x - 4 = 0$ using the Graeffe's root squaring method.

17. Find the inverse of the coefficient matrix of the system $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$ by the Gauss Jordan method with partial pivoting and hence solve the system.

18. From the following valves of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

19. Evaluate the integral $I = \int_{-1}^{1} (1 - x^2)^{3/2} \cos x \, dx$ using Gauss-Chebyshev three point formula and Gauss-Legendre three point formula.

20. Solve the system of equations.

$u' = 3u + 2v, u(0) = 0$
$v' = 3u - 4v, u(0) = 0.5$

for $h = 0.2$ on the interval $[0,1]$ use the Euler- cauchy method.

D–6481

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2020.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Prove that the probability of a null set is zero.

2. Let X have the p.d.f. $f(x) = \begin{cases} 1/4, & 0 < x < 4 \\ 0, & elsewhere \end{cases}$. Determine $E(x)$.

3. Define Covariance of X and Y.

4. Let $f(x_1, x_2) = x_1 + x_2, 0 < x_1, x_2 < 1$, zero elsewhere. Find the marginal p.d.f. of X_1 and X_2.

5. Define geometric distribution.

6. Write the m.g.f of a normal distribution.

7. Define F-distribution.

8. Let X have a p.d.f. $f(x) = \frac{1}{3}$, $x = 1,2,3$, zero elsewhere. Find the p.d.f. of $Y = 2X + 1$.

9. Define the concept of "convergence in distribution".

10. What is degenerate distribution?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If X has the p.d.f $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1,2,3,.... \\ 0, & elsewhere \end{cases}$. Find the m.g.f. mean and variance of X.

Or

(b) Let $f(x) = \frac{1}{x^2}, 1 < x < \infty$, zero elsewhere be the p.d.f of X. $A_1 = \{x : 1 < x < 2\}$, $A_2 = \{x : 4 < x < 5\}$, find $P(A_1 \cup A_2)$ and $P(A_1 \cap A_2)$.

12. (a) Let X_1 and X_2 have the joint p.d.f.

$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1 \\ 0, & elsewhere \end{cases}$. Find the marginal probability density functions.

Or

(b) Let X and Y have joint p.d.f. $f(x, y) = \frac{1}{7}$, zero elsewhere, find the correlation coefficient ρ, where $(x, y) = (0,0), (1,0), (0,1), (1,1), (2,1), (1,2), (2,2)$.

13. (a) Suppose that X has a poisson distribution with $\mu = 2$. Then the p.d.f of X is $f(x) = \frac{2^x e^{-2}}{x!}, x = 0,1,2 \ldots.$ Find $P_r(3 \le x)$.

Or

(b) Let X be $n(\mu, \sigma^2)$, so that $P_r(X < 89) = 0.90$ and $P_r(X < 94) = 0.95$. Find μ and σ^2.

14. (a) Derive the p.d.f of Chi-square distribution.

Or

(b) Let X_1 and X_2 be a random sample from a distribution having the p.d.f $f(x) = \begin{cases} e^{-x}, & 0 < x < \infty \\ 0, & elsewhere \end{cases}$.

Show that $Z = \frac{X_1}{X_2}$ has a F-distribution.

15. (a) Let Z_n be $\chi^2(n)$ and let $W_n = \frac{Z_n}{n^2}$, find limiting distribution of W_n.

Or

(b) Let $F_n(u)$ denote the distribution function of a random variable U_n whose distribution depends upon the positive integer n. Let U_n converges in probability to the constant $c \neq 0$. Prove that the random variance U_n/c converges in probability 1.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) Establish Chebyshev's inequality and give its importance.

(b) If X is a random variable such that $E(X)=3$ and $E(X^2)=13$, determine a lower bound for the probability $P_r(-2<x<8)$ using Chebyshev's inequality.

17. Let X_1 and X_2 have the joint p.d.f. $f(x_1,x_2)=\begin{cases}2, & 0<x_1<x_2<1\\ 0, & \text{otherwise}\end{cases}$. Find the marginal probability density functions and conditional means.

18. Find the moment generating function, mean and variance of the gamma distribution.

19. Let $T=\dfrac{W}{\sqrt{V/r}}$, where W and V are respectively normal with mean 0 and variance 1. Chi-square with r show that T^2 has an F-distribution with parameters $r_1=1$, $r_2=r$.

20. (a) If X_n have a gamma distribution with parameters $\alpha=n$ and β, where β is not a function of n. Find the limiting distribution of $Y_n=\dfrac{X_n}{n}$.

(b) Let S_n^2 denote the variance of a random sample of size 'n' from distribution which is $n_X(\mu,\sigma^2)$. Prove that $\dfrac{ns^2}{n-1}$ converges stochastically to σ^2.

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