D–1508

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2021.

First Semester

Mathematics

ALGEBRA – I

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define relatively prime integers.

2. Define normalizer of an element in G.

3. Define an internal direct product of a group.

4. Give an example of a finite non-commutative ring.

5. Define a division ring. Give an example.

6. Define an ideal of R. Give an example.

7. If R is a ring and $a \in R$, let $\gamma(a) = \{x \in R / ax = 0\}$. Prove that $\gamma(a)$ is a right-ideal of R.

8. When will you say a polynomial is integermonic?

9. Define an unique factorization domain.

10. Prove that an Euclidean ring possess unit element.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any three sets A, B and C, Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Or

(b) Prove that if H and K are finite subgroups of G of order $o(H)$ and $o(K)$ respectively, then

$$o(HK) = \frac{o(H) \cdot o(K)}{o(H \cap K)}.$$

12. (a) State and prove Euler's theorem.

Or

(b) Prove that the homomorphism φ of R into R' is an isomorphism if and only if $I(\varphi) = 0$.

13. (a) State and prove Fermat's theorem.

Or

(b) If $o(G) = p^2$, where p is a prime number, then prove that G is abelian.

14. (a) Prove that any finite integral domain is a field.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (o) and R itself. Prove that R is a field.

15. (a) State and prove division algorithm.

Or

(b) If p is a prime number of the form $4n+1$, then prove that $p = a^2 + b^2$ for some integers a and b.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

17. State and prove the third part of the Sylow's theorem.

18. (a) If $\varphi : R \to R'$ is a ring homomorphism with Kernel $I(\phi)$, then prove that if $a \in I(\varphi)$ $\gamma \in R$, then both $a\gamma$ and γa are in $I(\varphi)$.

(b) If $a, b \in R$, where R is a ring then prove that $(a + b^2) = a^2 + ab + ba + b^2$ where $x^2 = x - x$.

19. Prove that every integral domain can be imbedded in a field.

20. (a) State and prove the Gauss lemma.

(b) If R is a unique factorization domain, then prove that $R[x]$ is also an unique factorization domain.

D–1509

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2021.

First Semester

ANALYSIS–I

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define the cantor set. Give an example.

2. Prove that every neighborhood is an open set.

3. Define connected set. Give an example.

4. Define Cauchy sequence. Give an example.

5. Define bounded mapping.

6. If (a_n) converges then show that $\lim_{n\to\infty} a_n = 0$.

7. Define product of two series.

8. Define accumulation point.

9. State the generalized mean value theorem.

10. Define local maximum.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $x,\ y \in R^k$, Prove that $|x + y| \leq |x| + |y|$.

Or

(b) Prove that every infinite subset of a countable set A is countable.

12. (a) Let $\{S_n\}$ be a monotonic sequence. Prove that $\{S_n\}$ converges if and only if it is bounded.

Or

(b) Prove that e is an irrational number.

13. (a) Show that the Cauchy product of two absolutely convergent series converges absolutely.

Or

(b) Let f be monotonic on (a, b). Prove that the set of points of (a, b) at which f discontinuous is at most countable.

14. (a) Let f be a continuous real valued function on a metric space X. Let $z(f)$ be the set of all $p \in X$ at which $f(p) = 0$. Prove that $z(f)$ is closed.

Or

(b) Prove that the union of any collection of open sets is an open set.

15. (a) State and prove intermediate–value theorem for derivatives.

Or

(b) State and prove Rolle's theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that for every positive real x and every positive integer n, there is one and only one real y such that $y^n = x$.

17. Let E be a non compact set in R'. Then prove that:

(a) There exists a continuous function on E which is not bounded.

(b) There exists a continuous and bounded function on E which has no maximum.

(c) If E is bounded, there exists a continuous function on E which is not uniformly continuous.

18. Prove that every bounded infinite subset of R^K has a limit point in R^K.

19. State and prove the Bolzano–Weierstrass theorem.

20. State and prove implicit function theorem.

D–1510

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2021.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State the uniqueness theorem for linear equation with constant coefficients $L(y)=0$.

2. Compute the Wronskian of $\phi_1(x)=x^2, \phi_2(x)=5x^2$.

3. One solution of $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$ for $x>0$ is $\phi_1(x)$. Find a basis for the solutions for $x>0$.

4. Define singular point of $L(y)=0$.

5. Compute the indicial polynomial of the equation $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$.

6. Define indicial polynomial.

7. Prove that $P_n(-1)=(-1)^n$.

8. Find an integrating factor for the equation $\cos x \cos y \, dx - 2\sin x \sin y \, dy = 0$.

9. Let $f(x, y) = \frac{\cos y}{1 - x^2}, (|x| < 1)$, show that the initial value problem $y' = f(x, y)$, $y(0) = y_0$, $(|y_o| < \infty)$ has a solution which exists for $|x| < 1$.

10. Verify whether the equation $2xy \, dx + (x^2 + 3y^2)dy = 0$ is exact or not.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\phi_1, \phi_2, \ldots, \phi_n$ are n solutions of $L(y) = 0$ on an interval I, they are linearly independent there if and only if $W(\phi_1, \phi_2, \ldots, \phi_n)(x) \neq 0$ for all x in I – Prove.

Or

(b) Find all the solutions of the equation $y'' - \frac{2}{x^2} y = x$, $0 < x < \infty$.

12. (a) Show that the coefficient of x^n in $p_n(x)$ is $\frac{(2n)!}{2^n (n!)^2}$.

Or

(b) One solution of $x^2 y'' - 2y = 0$ on $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solutions of $x^2 y'' - 2y = 2x - 1$ on $0 < x < \infty$.

13. (a) Are the solutions e^{2x} and xe^{2x} of $y'' - 4' + 4y = 0$ linearly independent on any interval? Justify.

Or

(b) Show that $\int_{-1}^{1} p_n(x) p_m(x)\, dx = 0, m \neq n.$

14. (a) Compute the indicial polynomial and their roots for the equation $x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$.

Or

(b) Show that –1 and 1 are regular singular points for the Legendre equation $(1 - x^2) y'' - 2xy' + \alpha(\alpha + 1) y = 0$.

15. (a) Find all real-valued solutions of $y' = x^2 y$.

Or

(b) By computing appropriate Lipschitz constants, show that $f(x, y) = x^2 \cos^2 y + y \sin^2 x$, on $S : |x| \leq 1, |y| < \infty$ satisfy Lipschitz conditions on S.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let ϕ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an internal I containing a point x_0. Prove that for all x in I.

$\|\phi(x_0)\| e^{-K|x - x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\| e^{K|x - x_0|}$ where $\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2\right]^{\frac{1}{2}}$, $K = 1 + |a_1| + |a_2|$.

17. Find all solutions of

(a) $y''' - 8y = 0$

(b) $y^{(4)} - 16y = 0$

18. Find all solutions of

(a) $y''' - y' = x$ and

(b) $y^{(4)} - y = \cos x$

19. Derive the n[th] Legendre polynomial.

20. Let f be a real-valued continuous function on the strip $S : |x - x_0| \le a, |y| < \infty$, $a > 0$, and suppose that f satisfies on S a Lipschitz condition with constant $K > 0$. Prove that the successive approximations $\{\phi_k\}$ for the problem $y' = f(x, y), y(x_0) = y_0$ exit on the entire interval $|x - x_0| \le a$ and converge there to a solution ϕ of $y' = f(x, y)$.

D–1511

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2021.

First Semester

TOPOLOGY–I

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define bijective function and give an example.

2. Define the discrete topology. Give an example.

3. Is in the plane $\mathbb{R}^2$, the set $\{x \times y / x \geq 0,\ y \geq 0\}$ closed? Justify your answer.

4. What is meant by the product topology?

5. State the pasting lemma.

6. Define path components of X.

7. Define compact space. Give an example.

8. Define first countability axiom.

9. Define Hausdorff space.

10. Define completely regular space.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any three sets A, B, C. Prove that $A-(B \cup C)=(A-B) \cap(A-C)$.

Or

(b) Prove that every non–empty subset of Z_+ has a smallest element.

12. (a) Prove that the collection $S=\{\pi_1^{-1}(U) / U \text{ open in } X\} \cup \{\pi_2^{-1}(V) / V \text{ open in } Y\}$ is a subbasis for the product topology on $X \times Y$

Or

(b) State and prove uniform limit theorem.

13. (a) State and prove sequence lemma.

Or

(b) Prove that the image of a connected space under a continuous map is connected.

14. (a) Prove that a space X is locally connected if and only if for every open set U of X, each components of U is open in X.

Or

(b) Prove that compactness implies limit point compactness, but not conversely.

15. (a) Show that a subspace of a completely regular space is completely regular.

Or

(b) Prove that every metrizable space is normal.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that a finite product of countable set is countable.

17. Let X be a topological space. Prove the following:

(a) ϕ and X are closed.

(b) Arbitrary intersections of closed sets are closed.

(c) Finite unions of closed sets are closed.

18. Prove that the cartesian product of connected space is connected.

19. State and prove the tube lemma.

20. State and prove Urysohn lemma.

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D–1512

Sub. Code
31121

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2021.

Second Semester

ALGEBRA–II

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. If V is a vector space over F, then prove that $(-\alpha)v = -(\alpha v)$ for $\alpha \in F$, $v \in V$.

2. Define dual space.

3. Define vector space over a field F. Give an example.

4. What is meant by an inner product space?

5. Define an orthonormal set.

6. Find the Galois group $G(\mathbb{C} : R)$

7. When will you say that an element in $A(V)$ is singular?

8. Define characteristics root.

9. Define Hermitian and skew–Hermitian.

10. Define companion matrix of $f(x)$ in $F[x]$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) With the usual notations, prove that $F^{(n)}$ is isomorphic with F if and only if $n = m$.

Or

(b) Prove that $A(w)$ is a subspace of $\hat{V}$.

12. (a) If V is a finite dimensional and $v \neq 0 \in V$, then show that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

Or

(b) State and prove Bessel's inequality.

13. (a) Prove that $G(K, F)$ is a subgroup of the group of all automorphism of K.

Or

(b) State and prove remainder theorem.

14. (a) Let F be the field of rational numbers. Determine the degree of the splitting field of $x^6 + 1$ over F.

Or

(b) Define normal extension and show that if K is a normal extension of F then K is a splitting field of some polynomial over F.

15. (a) Show that the polynomial $f(x) \in F[x]$ has multiple roots if and only if $f(x)$ and $f'(x)$ have a non trivial common factor.

Or

(b) (i) If $T \in A(V)$ is Hermitian, then prove that all its characteristic roots are real.

(ii) If T is a Hermitian and $vT^K = 0$ for $K \geq 1$, then prove that $vT = 0$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If V is finite – dimensional and if W is a subspace of V, then prove that W is finite dimensional, $\dim W \leq \dim V$ and $\dim \frac{V}{W} = \dim V - \dim W$

17. (a) If V is finite dimensional, then prove that ψ is an isomorphism of V onto V.

(b) With usual notations, prove that $\|u + v\|^2 + \|u - v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$.

18. State and prove the fundamental theorem of Galois theory.

19. Prove that an element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

20. If V is n-dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, v_2, ..., v_n$ and the matrix $m_2(T)$ in the basis $w_1, w_2, ..., w_n$ of V over F, then prove that there is an element $\mathcal{C} \in F_n$ such that $m_2(T) = \mathcal{C}\, m_1(T)\, \mathcal{C}^{-1}$

D–1513

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2021.

Second Semester

Mathematics

ANALYSIS -II

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define partition of an interval.

2. For fixed $m, n \in z_+$, Show that $\lim_{\substack{m \to \infty \\ n \to \infty}} S_{m,n} = 0$ where $S_{m,n} = \frac{m}{m+n}$.

3. Define refinement.

4. Define equicontinuity of a function.

5. Define orthogonal system of function.

6. Define Fourier series.

7. Define Borel set.

8. Let f be integrable over E. If A and B are disjoint measurable sets contained in E. Show that

$$\int_{A\cup B} f = \int_A f + \int_B f$$

9. Define step function.

10. Define simple function. Give an example.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If f is monotonic on $[a,b]$ and if α is continuous on $[a,b]$, then prove that $f \in \Re(\alpha)$.

Or

(b) If f maps $[a,b]$ into $\mathbb{R}^K$ and if $f \in \mathbb{R}(\alpha)$ for some monotonically increasing function α on $[a,b]$, then prove that $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$.

12. (a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

Or

(b) Prove that every informally convergent sequence of bounded functions is informally bounded.

13. (a) Prove that $\lim_{x \to 0} (1 + x)^{1/x} = e$.

Or

(b) Let f be measurable and B a Borel set. Prove that $\overline{f}^{'}[B]$ is a measurable set.

14. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Prove that $m^*(U A_n) \leq \sum m^* A_n$.

Or

(b) Let $< E_n >$ be an infinite decreasing sequence of measurable sets for each n. Let mE, be infinite. Prove that $m\left(\bigcap_{i=1}^{\infty} Ei\right) = \lim_{n \to \infty} mE_n$

15. (a) State and prove bounded convergence theorem.

Or

(b) State and prove Lebesgue convergence theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Assume α increases monotonically and $\alpha' \in \mathbb{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Prove that $f \in \mathbb{R}(\alpha)$ if and only if $f\alpha' \in \mathbb{R}$ in that case $\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)\, dx$.

17. State and prove Stirling's formula.

18. Prove that:

(a) The functional equation $\sqrt{(x+1)} = x\sqrt{(x)}$ holds if $0 < x < \infty$.

(b) $\sqrt{n+1} = n!$ for $n = 1,2,3,......$

(c) $\log \sqrt{}$ is convex on $(0, \infty)$

19. Prove that the outer measure of an interval is its length.

20. Let f be bounded on a measurable set E with mE is finite. Prove that for all simple functions Q and ψ $\inf_{f \leq \psi} \int_E \psi(x)\, dx = \sup_{f \geq Q} \int_E Q(x)\, dx$ if and only if f is measurable.

D–1514

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2021.

Second Semester

TOPOLOGY–II

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. What is meant by completely regular space?

2. Is the space $\mathbb{R}^n$ locally compact? Justify.

3. Define G_δ-set.

4. When will you say that two compactifications are equivalent?

5. Give an example of a Cauchy sequence in Q that is not convergent in it.

6. Define closed refinement.

7. Define Peano space.

8. State the Negata–Smirnov metrization theorem.

9. Define compact open topology.

10. Define Baire space–Give an example.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define locally compact with an example. Show that the rationals Q are not locally compact.

Or

(b) Prove that product of completely regular space is completely regular.

12. (a) Prove that every para compact Hausdroft space X is normal.

Or

(b) State and prove the extension property to stone cech compactification.

13. (a) Let A be the collection of subsets of R:

$A = \{(n, n+2) / n \in z\}$. Which of the following collections refines A?

$B = \{(x, x+1)(x \in R)\}$, $\mathcal{C} = \left\{\left(n, n+\frac{3}{2}\right) / n \in \mathbb{Z}\right\}$ and

$D = \left\{\left(x, x+\frac{3}{2}\right) / x \in \mathbb{R}\right\}$.

Or

(b) Let X be a regular space with a basis $\mathcal{B}$ that is countably locally finite. Show that X is normal and every closed set in X is a G_δ set in X.

14. (a) Prove that a metric space X is complete is every Cauchy sequence in X has a convergent subsequence.

Or

(b) If the space Y is complete in the metric d, then show that the space Y^J is complete in the uniform metric $\bar{\delta}$ corresponding to d.

15. (a) If X is a compact Hausdorff space or a complete metric space, then prove that X is a Baire space.

Or

(b) Show that any connected Hausdorff space having more than one point has dimension at least 1.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Tietze–extension theorem.

17. State and prove the sufficient part of Negata–Smirnov metrization theorem.

18. Let $I = [0, 1]$. Prove that there exists a continuous function $f : I \to I^2$ whose image fills up the entire square I^2.

19. Let $h : [0,1] \to R$ be a continuous function. Prove that, given $\in > 0$, there is a function $g : [0,1] \to R$ with $|h(x) - g(x)| < \in$ for all x, such that g is continuous and nowhere differentiable.

20. Prove that every compact subset of $\mathbb{R}^n$ has topological dimension at most N.

D–1515

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2021.

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. What is meant by orthogonal trajectories?

2. Write down the necessary and sufficient condition for the Pfaffian differential equation $x.dr = 0$ is integrable.

3. Eliminate the constants a and c from the equation $x^2 + y^2 + (z-c)^2 = a^2$.

4. Define the general integral of $F(x, y, z, p, q) = 0$.

5. Form the partial differential equation by eliminating arbitrary function f from $z = f\left(\frac{y}{x}\right)$.

6. Find the complete integral of $(p+q)(z - xp - yq) = 1$.

7. When we say that two surface are circumscribe to each other?

8. If $u = f(x+iy) + g(x-iy)$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

9. Write down the exterior Dirichlet boundary value problem for Laplace's equations.

10. Write down the interior Neumann boundary value problem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the integral curves of the sets of equations

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}.$$

Or

(b) Prove that a Pfaffian differential equation in two variables always possesses an integrating factor.

12. (a) Eliminate the arbitrary function f from $f(x^2+y^2+z^2, z^2-2xy) = 0$.

Or

(b) Find the general solution of the partial differential equation $x^2p + y^2q = (x+y)z$.

13. (a) Find the equation of the integral surface of the partial differential equation $2y(z-3)p + (2x-z)q = y(2x-3)$.

Or

(b) Using Jacobi's method, solve $2(y+zq) = q(xp+yq)$.

14. (a) Verify that the partial differential equation $t = a^2 r$ satisfied by $z = f(x + ay) + g(x - ay)$, where f and g are arbitrary functions.

Or

(b) Solve $r - s + 2q - z = x^2 y^2$.

15. (a) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

Or

(b) Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r, \theta)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Verify that the equation

$yz\,dx + (x^2 y - zx)dy + (x^2 z - xy)dz = 0$ is integrable and find its primitive.

17. Find the integral surface of the equation $xp + yq = z$ passing through $x + y = 1$ and $x^2 + y^2 + z^2 = 4$.

18. Show that the equation $xpq + yq^2 = 1$ has integrals

(a) $(z+b)^2 = 4(ax+y)$

(b) $kx(z+h) = k^2y + x^2$, and deduce (b) from (a).

19. Find the canonical form of the one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$.

20. By separating the variables, solve the diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

D–1516

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2021.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 19 Academic Year)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define the normal plane.

2. Define involute of a space curve.

3. What is meant by direction coefficients?

4. Define the pitch of the helix.

5. Define the centre of spherical curvature.

6. State the Meusnier's theorem.

7. Define Gaussian curvature.

8. Define pseudo–sphere.

9. When will you say the conic is Dupin's indicatrix?

10. Define the polar developable.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If a curve lies on sphere, show that ρ and σ are related by $\frac{d}{ds}(\sigma\,\rho')+\frac{\rho}{\sigma}=0$.

Or

(b) Find the length of the curve given by the intersection of the surfaces $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $x=\alpha\cos h\left(\frac{Z}{\alpha}\right)$, from the point $(\alpha, 0,\ 0)$ to the point (x, y, z).

12. (a) Find the curvature and the torsion of the curve $\vec{r}=\{\alpha(3u-u^3),\ 3\,\alpha\,u^2,\ \alpha(3u+u^3)\}$.

Or

(b) Obtain the necessary and sufficient condition for the direction at (u,v) of $pdu^2+2Q\,dudv+Rdv^2=0$ to be orthogonal.

13. (a) Show that the involutes of a circular helix are plane curves.

Or

(b) Enumerate the normal property of geodesic.

14. (a) Find the Gaussian curvature at the point (u,v) of the anchor ring and show that the total curvature of the whole surface is zero.

Or

(b) Prove that every helix on a cylinder is a geodesic.

15. (a) State and prove Rodrigue's theorem.

Or

(b) Prove that the edge of regression of the rectifying developable has equation $\vec{R} = \vec{r} + k\left(\frac{\tau\vec{t} + k\vec{b}}{k'\tau - k\tau'}\right)$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces. $ax^2 + by^2 + cz^2 = 1$ and $a'x^2 + b'y^2 + c'z^2 = 1$.

17. (a) Show that the metric is invariant under a parameter transformation.

(b) Prove that the curves bisecting the angle between the parametric curves are given by $E\,du^2 - G\,dv^2 = 0$.

18. Find the surface of revolution which is isometric with a region of the right helicoid.

19. Prove that if (λ, μ) is the geodesic curvature vector, then $Kg = \frac{-H\lambda}{Fu' + Gv'} = \frac{H\mu}{Eu' + Fv'}$.

20. State and prove the Minding's theorem.

D–1517

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2021.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define oriented, connected network.

2. Define cut and dummy activity.

3. Define critical and CPM.

4. Define basis and extreme point.

5. What is the feasibility condition of the revised simplex method?

6. Define pure saddle point.

7. Define relative maxima.

8. Define Lagrangean function.

9. Define Jacobian matrix.

10. Define sensitivity analysis.

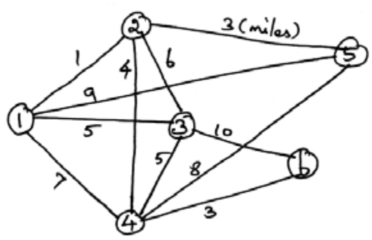
PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write down the Dijkstra's algorithm.

Or

(b) Mid west TV cable company providing cable service to five new housing developments. The following figure depicts possible TV connections to five areas with cable miles affixed on each arc. Determine the most economical cable network.



12. (a) Determine all basic feasible solutions of the following system of equations $\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

Or

(b) Write down the Bounded variable Algorithm.

13. (a) Write down the revised simplex algorithm.

Or

(b) Determine the extreme point of the following function $f(x) = x_1^2 + x_2^3 - 3x_1x_2$.

14. (a) Write down the Lagrangean method.

Or

(b) Find the maximum of the following function by dichotomous search. Assume that $\Delta = 0.05$. $f(x) = -(x-3)^2, 2 \le x \le 4$.

15. (a) Construct the project network.

Activity	Predecessor (S)	Duration
A	-	2
B	A	14
C	A	14
D	B,C	3
E	D	70
F	D	14
G	F	1
H	D	1
I	D	7
J	D	7
K	J	14
L	K	1
M	E, G,L	1
N	H, L,M	1
O	I, N	1

Or

(b) Explain PERT network.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

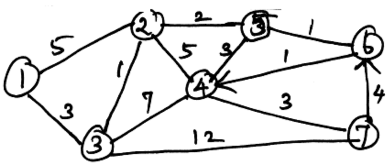
16. Determine the shortest route between each of the following pairs of nodes

(a) From node 5 to node 1

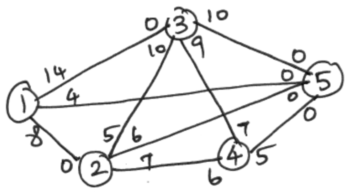
(b) From node 3 to node 5

(c) From node 1 to node 4

(d) From node 3 to node 2



17. Determine the maximum flow and the optimum flow in each are for the network.



18. Solve the game by Linear programming technique.

$$\text{Player A} \overset{\text{Player B}}{\begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{pmatrix}}$$

19. Solve the function $f(x) = 4x^4 - x^2 + 5$ by Newton Raphson method.

20. Solve the following Linear programming problem by Lagrangean method.

Maximize $f(x) = 5x_1 + 3x_2$

Subject to $g_1(x) = x_1 + 3x_2 + x_3 - 6 = 0$

$g_2(x) = 3x_1 + x_2 + x_4 - 9 = 0$

$x_1, x_2, x_3, x_4 \geq 0$

D–1518

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2021.

Third Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. When will you say an integer is square free?

2. Define a reduced fraction.

3. If a prime p does not divide a, then prove that $(p,a)=1$.

4. When will you say that the functions are division functions?

5. Define Mobius function $\mu(n)$.

6. Prove that $[-x]=\begin{cases}-[x] & if \quad x=[x] \\ -[x]-1 & if \quad x\neq[x]\end{cases}$

7. What are the solutions of the congruence $x^2 \equiv 1 (\bmod 8)$?

8. Write down the Euler's summation formula.

9. Determine the quadratic residue modulo II.

10. State the Little Fermat theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $(a,m)=(b,m)=1$, then prove that $(ab,m)=1$.

Or

(b) Prove that $d(n)$ is odd if and only if n is a square.

12. (a) For $n \geq 1$, prove that $\phi(n)=n\prod_{p/n}\left[1-\frac{1}{p}\right]$

Or

(b) Derive the Selberg identity.

13. (a) If $x \geq 1$, prove that $\sum_{n>x}\frac{1}{n^s}=0\left(x^{1-s}\right)$ if $s>1$.

Or

(b) Prove that $\sum_{n\leq x}\lambda(n)\left[\frac{x}{n}\right]=\left[\sqrt{x}\right]$

14. (a) Find all n for which $\phi(n)\equiv 2(\text{mod } 4)$.

Or

(b) State and prove Euler-Fermat theorem.

15. (a) Determine whether 219 is a quadratic residue or non-residue mod 383.

Or

(b) State and prove reciprocity law for Jacobi symbol.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove division algorithm.

17. Define Liouville's function $\lambda(n)$. For every $n \geq 1$, prove that $\sum_{d/n} \lambda(d) = \begin{cases} 1; & \text{if } n \text{ is a square} \\ 0; & \text{otherwise} \end{cases}$.

Also prove that $\lambda^{-1}(n) = |\mu(n)|$ for all n.

18. For all $x \geq 1$, prove that $\left| \sum_{n \leq x} \frac{\mu(n)}{n} \right| \leq 1$ with equality holding only if $x < 2$.

19. State and prove Lagrange theorem.

20. State and prove Gauss lemma.

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D–1519

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2021.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define denumerable Markov chain. Give an example.
2. Define null persistent.
3. Define the transition probability matrix.
4. Define the drift coefficient.
5. Is wiener process, a Markov process? Justify your answer.
6. Write the Fokker–Planck equation.
7. State the distribution of the total number of progeny.
8. When does a Markov branching process is a continuous–time Markov chain?
9. Write the equilibrium equations.
10. Define virtual waiting time.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove first Entrance theorem.

Or

(b) Prove: If state K is persistent null, then for every $j \lim_{n\to\infty} P_{jk}^{(n)} \to 0$ and if state K is aperiodic, persistent non–null then $\lim_{n\to\infty} P_{jk}^{(n)} \to \frac{F\,jk}{\mu_{kk}}$.

12. (a) The t.p.m of a Markov chain $\{X_n, n = 1, 2, ...\}$ having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $\pi_o = (0.7, 0.2, 0.1)$. Find

(i) $P_r(X_2 = 3)$

(ii) $P_r(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

Or

(b) If $M(t)$ is the number of occurrences recorded in an interval of length t, then prove that $M(t)$ is a poisson process with parameter λp.

13. (a) Let $\{X(t)\}$ be Wiener process with $X(0) = 0$. Its mean function is μt and variance function $\sigma^2 t$. Find the covariance function $C(s,t)$ in $0 < s < t$.

Or

(b) If $X(t)$ with $X(0)$ and $\mu = 0$ is a Wiener process and $0 < s < t$, show that for at least one τ satisfying $s \le \tau \le t$, $\Pr\{X(\tau) = 0\} = \left(\frac{2}{\pi}\right)\cos^{-1}\left((s/t)^{1/2}\right)$.

14. (a) Obtain the expected waiting time in the system for $M / M / S$ model.

Or

(b) Prove that the probability of extinction q is the smallest root in $[0,1]$ of the equation $u(s) = 0$, further $q = 1.(q < 1)$ if and only if $u'(1) \leq (>) 0$.

15. (a) Show that the p.d.f $W_s(x)$ of W_s has an exponential distribution with variance $\frac{1}{\{\mu(1-s)\}^2}$.

Or

(b) Show that the expected busy period $E(B) = \frac{1}{\mu(1-r)}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. A particle starting from the origin moves from position j to $(j+1)$ with probability a_j and returns to origin with probability $(1 - a_j)$. Suppose that the states, after n moves 0, 1, 2, 3,...... Show that the state 0 is persistent if and only if $\lim \prod_{i=1}^{n} a_i \to 0$ as $n \to \infty$.

17. Derive the Chapman–Kolmogorov equations.

18. For a G.W. process with $M = 1$ and $\sigma^2 < \infty$, prove that $\lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{1-P_n(s)} - \frac{1}{1-s}\right\} \to \frac{\sigma^2}{2}$ uniformly in $0 \le s \le 1$.

19. State and prove Yaglom's theorem.

20. Derive Pollaczek–Khinchine formula.

D–1520

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2021.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 19 Academic Year Onwards)

Time : 3 hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Let G be a graph in which all vertices have degree at least two. Prove that G contains a cycle.

2. Show that every column sum of the incidence matrix is 2.

3. Draw a graph which is both Eulerian and Hamiltonian.

4. Determine the edge chromatic number of a Petersen graph.

5. Define maximal matching and perfect matching.

6. Prove that $\alpha + \beta = \gamma$.

7. Define independent set, maximum independent set.

8. Show that $\gamma(k, 2) = K$.

9. Define Eulerian trail in a directed graph.

10. Define directed walk and directed cycle.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $G \cong H$, then show that $\gamma(G) = \gamma(H)$ and $\varepsilon(G) = \varepsilon(H)$. Is the converse true?

Or

(b) Prove that every connected graph contains a spanning tree.

12. (a) Prove that every Hamiltonian graph is 2– connected.

Or

(b) Prove that $\gamma(m,n) = \gamma(n,m)$.

13. (a) Prove that $\lambda^4 - 3\lambda^2 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

Or

(b) If G is bipartite, then prove that $\chi' = \Delta$.

14. (a) Prove that every polyhedron has atleast two faces with the same number of edges on the boundary.

Or

(b) Prove that $f(K_n, \lambda) = \lambda(\lambda - 1)......(\lambda - N + 1)$

15. (a) Prove that if two digraphs are isomorphic then the corresponding vertices have the same degree pairs.

Or

(b) Prove that a connected graph G is strongly orientable if and only if G has no cut edges.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Show that a graph G is bipartite if and only if it contains no odd cycle.

17. State and prove Cayley's theorem.

18. State and prove Vizing's theorem.

19. State and prove five colour theorem.

20. Prove that a weak digraph D is Eulerian if and only if every vertex of D has equal in degree and outdegree.

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D–1521

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2021.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a normed space. Give an example.

2. Define convex set.

3. Define bounded linear map.

4. Define bounded linear operator.

5. Define normed dual.

6. Define orthonormal basis.

7. Define self-adjoint operator.

8. Does the Riesz representation theorem hold for incomplete inner product space? Justify.

9. State Pythagoras theorem.

10. State the closed graph theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\mathbb{R}^n$ is a normed linear space with the norm $\|x\|_2 = \left(\sum_{i=1}^{n} |x_i|^2 \right)^{\frac{1}{2}}$.

Or

(b) By an example, show that an infinite dimensional subspace of a normed space X may not be closed in X.

12. (a) Prove that the linear functional f defined on the normed linear space X is bounded if and only if it is continuous.

Or

(b) Let Y be a closed linear subspace of a normed linear space X and let ϕ be the natural mapping of $X \rightarrow X / Y$ defined by $\phi(x) = x + y$. Show that ϕ is a continuous linear transformation for which $\|\phi\| \leq 1$.

13. (a) Let $< . >$ be an inner product on a linear space X. Prove that $|< x, y >|^2 \leq < x, x > . < y, y >$ for all $x, y \in X$.

Or

(b) Let A be a linear transformation on the finite–dimensional space X. Prove that A is completely continuous.

14. (a) Let f be a linear functional on the Hilbert space X; let N be the null space of f. Show that, if f is not continuous, then $\widetilde{N} = X$.

Or

(b) Prove that, U is an isometry if and only if $U^*U = 1$.

15. (a) State and prove Baire category theorem.

Or

(b) Suppose A is a symmetric operator and suppose that $R(A) = X$, that is A is an onto mapping. Prove that A is self–adjoint.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let M be a closed proper subspace of the normed linear space X, and let a be a real number such that $0 < a < 1$. Prove that there exists a vector $x_a \in X$ such that $\|x_a\| = 1$ and $\|x - x_a\| \geq a$ for all $x \in M$.

17. Let $\widetilde{X}$ denote the normed linear space of all bounded linear functional over the normed linear space X. Prove that $\widetilde{X}$ is a Banach space.

18. State and prove Bessel's inequality.

19. State and prove uniform boundedness theorem.

20. State and prove closed graph theorem.

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D–1522

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2021.

Fourth Semester

Mathematics

NUMERICAL ANALYSIS

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the Sturm's theorem.

2. Define the efficiency index of an iterative method.

3. Find the spectral radius of the matrix $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$.

4. Define eigen values of a matrix.

5. State the condition to Hermite interpolating polynomial.

6. State the Newton's bivariate interpolating polynomial.

7. Define order of a numerical differentiation method.

8. State the Legendre polynomials in the interval $[-1, 1]$.

9. Derive periodic stability.

10. State the initial value problem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Obtain the complex root of the equation $f(z) = z^3 + 1 = 0$ by using Newton-Raphson method.

Or

(b) Perform one iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$.

12. (a) Solve the following systems of equations by the decomposition method.

$x_1 + x_2 - x_3 = 2$

$2x_1 + 3x_2 + 5x_3 = -3$

$3x_1 + 2x_2 - 3x_3 = 6$.

Or

(b) Find the inverse of the matrix

$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, by Gauss-Jordan method.

13. (a) Prove that the eigen values of the tridiagonal matrix

$$A = \begin{bmatrix} a & b & & 0 \\ c_1 & a & b_2 & \\ \ddots & c_2 & \ddots & b_3 \\ 0 & & c_{n-1} & a \end{bmatrix}$$

satisfy the inequality $|\lambda - a| < 2\sqrt{\max_i |b_i| \max_i |c_i|}$.

Or

(b) Determine the polynomial of second degree, which is the best approximation in maximum norm to $\sqrt{x}$ on the point set $\left\{.0, \frac{1}{9}, \frac{4}{9}, 1\right\}$.

14. (a) Using linear interpolation, find $f(0.25, 0.75)$ from the following data of function $f(x, y)$.

$y \backslash x$	0	1
0	1	1.414214
1	1.732051	2

Or

(b) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss-Legendre three point formula.

15. (a) By using backward Euler method, solve the initial value problem $u^1 = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval $[0,1]$.

Or

(b) Apply the Taylor's series second order method to integrate $y' = 2t + 3, y(0) = 1, t \in (0, 0.4)$ with $h = 0.1$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find all the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$, using the Graeffe's root squaring method.

17. Find all the eigen values and eigen vectors of the matrix.

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}.$$

18. Given the following values of $f(x)$ and $f^1(x)$

x	$f(x)$	$f^1(x)$
–1	1	–5
0	1	1
1	3	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

19. Compute $\int_0^{\pi/2} \left(\frac{1}{\sin x}\right)^{1/4} dx$ to six decimal places.

20. Solve the initial value problem $u^1 = -2t\,u^2, u(0) = 1$ with $h = 0.2$ on the interval [0,1], by using fourth order classical Runge-Kutta method.

D–1523

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, DECEMBER 2021.

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours　　　　　　　　Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Prove that the probability of the null set is zero.

2. Find the mode of the distribution $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, 3\ldots \\ 0, & elsewhere \end{cases}$.

3. Define independent random variable.

4. Let $f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1;\ 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$ find the marginal p.d.f. of x_1 and x_2.

5. If the m.g.f. of a random variable X is $M(t) = \left(\frac{2}{3} + \frac{1}{3}e^t\right)^5$ then find the p.d.f.

6. Let X be $N(2,\ 25)$ find $P_r(0 < x < 10)$.

7. Let X have the p.d.f. $f(x) = \begin{cases} \frac{x^2}{9}, & 0 < x < 3 \\ 0, & elsewhere \end{cases}$ find the p.d.f. of $Y = x^3$.

8. Write down the mean and covariance of the Beta distribution.

9. Define convergence in distribution.

10. Let Z_n be $X^2(n)$. Find the mean and variance of Z_n.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let C_1 and C_2 be subsets of $\mathcal{C}$. Prove that $P(C_1 \cup C_2) = P(C_1) + P(C_2) - P(C_1 \cap C_2)$.

Or

(b) Let X have the p.d.f. $f(x) = \begin{cases} \frac{x+1}{2}, & -1 < x < 1 \\ 0, & elsewhere \end{cases}$.

Find the mean and variance.

12. (a) Let $f(x_1,\ x_2) = \begin{cases} 2e^{-x_1 - x_2}. & 0 < x_1 < x_2,\ 0 < x_2 < \infty \\ 0, & elsewhere \end{cases}$

show that x_1 and x_2 are independent.

Or

(b) Prove that

(i) $E[E(X_2/X_1)] = E(X_2)$ and

(ii) $\text{var}\,[E(X_2/X_1)] \le \text{var}\,(X_2)$.

13. (a) Find the mean and variance of Poisson distribution.

Or

(b) Find the value of μ and σ^2 for the normal distribution.

14. (a) If X is $N(75,100)$, find $P_r(x < 60)$ and $P_r(70 < x < 100)$.

Or

(b) Determine the constant c so that $f(x) = \begin{cases} cx(3-x)^4, & 0 < x < 3 \\ 0, & elsewhere \end{cases}$ is a p.d.f.

15. (a) Let y_n denote the n[th] order statistic of a random sample from the uniform distribution that has the p.d.f. $h_n(z) = \begin{cases} \dfrac{(\theta - z/n)^{n-1}}{\theta^n} & 0 < z < n\theta \\ 0, & elsewhere \end{cases}$. Let $z_n = n(\theta - y_n)$. Investigate the limiting distribution of z_n.

Or

(b) Let the p.d.f. y_n be $f_n(y) = 1,\ y = n$; zero elsewhere show that y_n does not have a limiting distribution.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions

16. Let X have the p.d.f. $f(x) = \begin{cases} \dfrac{x+2}{18} & -2 < x < 4 \\ 0, & elsewhere \end{cases}$ find $E(X)$, $E\left[(X+2)^3\right]$ and $E\left[6x - 2(X+2)^3\right]$.

17. Let the random variable X and Y have the joint p.d.f. $f(x, y) = \begin{cases} 2, & 0 \le x \le y \le 1 \\ 0, & elsewhere \end{cases}$ find the correlation coefficient ρ_{xy}.

18. If the random variable X is $N(\mu, \sigma^2)$, $\sigma^2 > 0$, then prove that the random variable $V = \frac{X - \mu^2}{\sigma^2}$ is $\psi^2(1)$.

19. Derive the p.d.f of the Beta– distribution with parameters α and β.

20. State and prove the central limit theorem.
