D–6871

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

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MAY 2020 ARREAR EXAMINATION

First Semester

ALGEBRA — I

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define onto mapping.

2. If G is a group, show that every $a \in G$ has a unique inverse in G.

3. Define order of an element.

4. Define normalizer of an element in a group.

5. State the pigeonhole principle.

6. Give an example of an integral domain which has an infinite number of elements.

7. Show that (4) is not a prime ideal in $\mathbb{Z}$.

8. Define Euclidean ring.

9. Prove that 5 is not prime element in the ring R of Gaussian integers.

10. Let R be an Euclidean domain. Suppose that $a, b, c, \in R$, a / bc but $(a, b) = 1$ prove that a / c.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any two sets A and B, prove that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

Or

(b) Prove that HK is a subgroup of G if and only if $HK = KH$

12. (a) Show that any two p-sylow subgroups of a group G are conjugate.

Or

(b) State and prove Cauchy's theorem for abelian group.

13. (a) Prove that a ring homorphism $\phi : R \to R'$ is one to one if and only if the Kernal of ϕ is zero submodule.

Or

(b) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R / M is a field.

14. (a) Let R be an Euclidean ring. Suppose that for $a, b, c \in R$, a / bc but $(a, b) = 1$, then prove that a / c.

Or

(b) If R is an Euclidean domain prove that any two elements a and b in R have greatest common divisor.

15. (a) Let $f(x)$, $g(x)$ be two non-zero elements in $F[x]$. Prove that $\deg f(x) \leq \deg (f(x) \cdot g(x))$.

Or

(b) State and prove the division algorithm.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$ respectively, then prove that $o(HK) = \dfrac{o(H)\, o(K)}{o(H \cap K)}$.

17. For a given prime p, show that the number of p-sylow subgroups of G is of the form $1 + kp$.

18. Let $R = \{(a, b) / a, b \in R\}$ and the operator addition and multiplication are defined as $(a,b) + (c,d) = (a + c, b + d)$ and $(a, b) \cdot (c,d) = (ac - bd, ad + bc)$. Show that R is a field.

19. State and prove Gauss lemma.

20. Prove that if R is a unique factorization domain then $R[x]$ is also unique factorization domain.

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D–6872

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DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

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MAY 2020 ARREAR EXAMINATION

First Semester

ANALYSIS – I

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Balls and convex – Justify.
2. Prove that a set E is open if its complement is closed.
3. Define compact set. Give an example.
4. Define convergence of a sequence.
5. Define informally continuous function. Give an example.
6. Write short notes on rearrangement of $\sum a_n$.
7. Define discontinuity of second kind.
8. Define derived set.
9. State the intermediate value theorem.
10. State the Rolle's theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Let z and w be complex numbers. prove that $|z+w| \leq |z| + |w|$.

Or

(b) Prove that the compact subsets of a metric spaces are closed.

12. (a) Prove that the subsequential limits of a sequence $\{p_n\}$ in a metric space X form a closed subset of xX

Or

(b) Calculate $\lim_{n\to\infty}(\sqrt{n^2+n}-n)$

13. (a) Prove that a mapping f of a metric space x into a metric space Y is continuous if and only if $f^{-1}(v)$ is open in X for every open set v in y.

Or

(b) Prove that monotomic functions have no discontinuities of the second kind.

14. (a) If f is a continuous mapping of a metric space X into a metric space Y and if E is connected subset of X prove that $f(E)$ is connected.

Or

(b) Prove that the intersection of a finite collection of open set is open.

15. (a) State and prove generalized mean value theorem.

Or

(b) If f is differentiable at c, then prove that f is continuous at c.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

16. Prove that every K-cell is compact.

17. Prove that $\Sigma \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

18. State and prove the Heine-Borel covering theorem.

19. State and prove Taylor's theorem.

20. State and prove the inverse function theorem.

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D–6873

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DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

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MAY 2020 ARREAR EXAMINATION

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the existence theorem for linear equations with constant coefficients $L(y) = 0$.

2. Define Wronskian.

3. Write any two Legendre polynomials.

4. Find the regular singular points of $(1+x^2)\, y'' - 2xy' + 2y = 0$.

5. Define Indicial polynomial.

6. Determine the equation $2xy\, dx + (x^2 + 3y^2)\, dy = 0$ is exact or not.

7. Show that the solution ϕ of $y' = y^2$ which passes through the point (x_0, y_0) is given by $\phi(x) = \dfrac{y_0}{1 - y_0(x - y_0)}$.

8. Find all real valued solution of the equation $y' = x^2 y$.

9. State the local existence theorem for initial value problem $y' = f(x, y),\ y(x_0) = y_0$.

10. Consider the initial value problem

$$y_1' = y_2^2 + 1$$

$$y_2' = y_1^2$$

$$y_1(0) = 0,\ y_2(0) = 0.$$

Compute the first three successive approximation ϕ_0, ϕ_1, ϕ_2.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that there exist n linearly-independent solutions of $L(y) = 0$ on I.

Or

(b) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0, then prove that

$$W(\phi_1, \phi_2)(x) = e^{-\alpha_1(x - x_0)} W(\phi_1, \phi_2)(x_0).$$

12. (a) Find all solutions of the equation

$$x^2 y''' + 2x^2 y'' - xy' + y = 0, \text{ for } x > 0.$$

Or

(b) Show that $\displaystyle\int_{-1}^{1} P_n^2(x)\,dx = \frac{2}{2n+1}$.

13. (a) One solution of $y'' + \frac{2}{x^2} y = 0$ is $\phi_1(x) = x^2, 0 < x < \infty$.

Find all solutions of $y'' - \frac{2}{x^2} y = x, 0 < x < \infty$.

Or

(b) Find two linearly independent power series solutions of $y'' - xy' + y = 0$.

14. (a) Show that $x^r = e^{i\pi r}|x|^r, x < 0$, where $x^r = e^{r \log x}$, for $x > 0$ and for $x < 0$.

Or

(b) Find the singular points of $x^2 y'' + (x + x^2) y' - y = 0$ and determine those which are regular singular points.

15. (a) Find all real valued solutions of $y' = x^2 y^2 - 4x^2$.

Or

(b) By computing appropriate Lipschitz constants, show that $f(x, y) = 4x^2 + y^2$, on $S : |x| \le 1, |y| \le 1$ satisfy Lipschitz conditions on S.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Solve :

(a) $y'' + 4y = \cos x$

(b) $y'' - 7y' + 6y = \sin x$.

17. Find all solutions of

(a) $y''' - 8y = e^{ix}$

(b) $y^{(4)} + 16y = \cos x$.

18. If ϕ_1 is a solution of $L(y) = y'' + a_1(x)\,y' + a_2(x)\,y = 0$ on an interval I and $\phi_1(x) \neq 0$ on I, then show that a second solution of $L(y) = 0$ is

$$\phi_2(x) = \phi_1(x) \int_{x_0}^{x} \frac{1}{[\phi_1(s)]^2} \exp\left[-\int_{x_0}^{s} a_1(t)\,dt \right] ds .$$

19. Derive the Bessel function of zero order of the first kind.

20. Let M and N be two real valued functions which have continuous partial derivatives on some rectangle.

$R : |x - x_0| \leq a,\ |y - y_0| \leq b$.

Prove that the equation $M(x, y) + N(x, y)\,y' = 0$ is exact in R, if and only if, $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$.

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DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

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MAY 2020 ARREAR EXAMINATION

First Semester

TOPOLOGY – I

(CBCS – 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Linear continuum.
2. Define the lower limit topology with an example.
3. What is meant by the subspace topology?
4. Define Order topology.
5. What is meant by project mapping?
6. Is the rationals Q connected? Justify.
7. Prove that any space X having only finitely many points is compact.
8. State the uniform limit theorem.
9. Define normal space.
10. Show that a subspace of a Lindelöf space need not be Lindelöf.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any three sets A, B, C, Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Or

(b) Prove that two equivalence classes E and E' are either disjoint or equal.

12. (a) If $\mathcal{B}$ is a basis for the topology on X and $\mathcal{C}$ is a basis for the topology of Y, then prove that $\mathcal{D} = \{B \times C / B \in \mathcal{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

Or

(b) State and prove the Pasting lemma.

13. (a) Let $f : A \to X \times Y$ be defined by $f(a) = (f_1(a), f_2(a))$ prove that f is continuous if and only if the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous.

Or

(b) State and prove sequencing lemma.

14. (a) State and prove Intermediate value theorem.

Or

(b) Prove that every compact subspace of a Hausdorff space is closed.

15. (a) Prove that every metrizable space is normal.

Or

(b) Define Hausdorff space. Prove that a product of Hausdorff space is Hausdorff.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that a countable union of a countable sets is countable.

17. Let X and Y be topological spaces ; let $f : X \to Y$. Prove that the following are equivalent :

    (a) f is continuous

    (b) for every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$.

    (c) For every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

18. Prove that the topologies on R^n induced by the Euclidean metric d and square metric ρ are the same as the product topology on R^n .

19. Prove that every regular space with a countable basis is normal.

20. State and prove Urysohn metrization theorem.

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DISTANCE EDUCATION

M.Sc. (Mathematics)DEGREE EXAMINATION.

MAY 2021 EXAMINATION

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MAY 2020 ARREAR EXAMINATION

Second Semester

ALGEBRA – II

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define vector space.

2. Define linear span.

3. Is any two finite dimensional vector space over F of the same dimension are isomorphic? Justify.

4. Define orthogonal complement of w.

5. Show that $w^{\perp}$ is a subspace of v.

6. Define algebraic extension of F.

7. Define trace of a matrix A in f_n.

8. Define unitary transformation.

9. Prove that the fixed field of G is a subfield of K.

10. Define Characteristics root of $T \in A(v)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) In a vector space show that $\alpha(v,w) = \alpha v - \alpha w$.

Or

(b) Prove that $L(s)$ is a subspace of v.

12. (a) If F is of Characteristics 0 and if a,b, are algebraic over F, then prove that there exists an element $c \in F(a,b)$ such that $F(a,b) = F(c)$.

Or

(b) If F is the field of real numbers, find $A(w)$ where w is spanned by (1,2,3) and (0,4,-1).

13. (a) State and prove Schwarz inequality.

Or

(b) Show that a polynomial of degree n over a field can have atmost n roots in any extension field.

14. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) If $u \in V_1$ is such that $uT^{n_1-k} = 0$, where $0 < k \le n$, then prove that $u = u_0 T^k$ for some $u_0 \in V_1$.

15. (a) If $(v^T, v^T) = (v, v)$ for all $v \in V$ then prove that T is unitary.

Or

(b) If N is normal and AN = NA then prove that AN*=N*A.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

16. If V and W are of dimensions m and n respectively, over F, then prove that Hon(V,W) is of dimension mn over F.

17. Let v be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.

18. Prove that the number e is transcendental.

19. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

20. Prove that if V is n-dimensional over F and if $T \in A(v)$ has all its Characteristic roots in F, then T satisfies a polynomial of degree n over F.

D–6876

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31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

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MAY 2020 ARREAR EXAMINATION

Second Semester

ANALYSIS – II

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define upper and lower Riemann integrals of f over $[a,b]$.

2. Define unit step function.

3. Define convergent series of continuous functions having discontinuous sum with example.

4. Define algebra. Give an example.

5. Find M*A if A is countable.

6. Define orthonormal system.

7. Define null space and projection.

8. State Fatou's lemma.

9. If m*E=0 then prove that E is measurable.

10. Define Lebesgue integral.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) If f is continuous on $[a,b]$, then prove that $f \in \mathbb{R}(\alpha)$ on $[a,b]$

Or

(b) If f maps $[a,b]$ onto R^k and if $f \in \mathbb{R}(\alpha)$ for some monotonically increasing function α on $[a,b]$, then prove that $\left|\int_a^b f dx\right| \leq \int_a^b |f| d\alpha$.

12. (a) If K is a compact metric space, if $f_n \in \zeta(k)$ for n=1,2,,... and if $\{f_n\}$ converges informly on k, then prove that $\{f_n\}$ is equicontinuous on K.

Or

(b) Prove that every uniformly converged sequence of bounded functions is uniformly bounded.

13. (a) State and prove localization theorem.

Or

(b) Prove that $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$.

14. (a) If $E = \bigcup_{n=1}^{\infty} E_n$, then prove that $\mu^*(E) \le \sum_{n=1}^{\infty} \mu^*(E_n)$.

Or

(b) Let f and g be measurable real valued functions defined on x, let F be real and continuous on R^2, and take $h(x) = F(f(x), g(x))$, $x \in X$. Prove that h is measurable.

15. (a) State and prove monotone convergence theorem.

Or

(b) Prove that the continuous functions form a dense subset of $\mathcal{L}^2$ on $[a,b]$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that there exists a real continuous function on the real line which is nowhere differentiable.

17. State and prove stone Weierstrass theorem.

18. State and prove Parseval's

19. State and prove Little wood's third principle.

20. If f is a positive function on $(0,\infty)$ such that $f(x+1) = xf(x)$, $f(1) = 1$, logf is curved then prove that $f(x) = \sqrt{(x)}$.

D–6877

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31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

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MAY 2020 ARREAR EXAMINATION

Second Semester

TOPOLOGY — II

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State countable intersection property.

2. Define compactification in a space.

3. Define stone cech compactification.

4. Let $A \subset X$, let $f : X \to Z$ be a continuous map of A into the Hausdorff space Z. Prove that there is atmost one extension of f to a continuous function $g : \overline{A} \to Z$.

5. Prove that the collection $A = \{(n, n+2) / n \in Z\}$ is locally finite.

6. Is the space Q of rational numbers with metric $d(x, y) = |x - y|$, complete? Justify.

7. Define uniform metric on y^J.

8. When will you say a space X is topologically complete?

9. Define point open topology.

10. State Ascoli's theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a set ; let D be a collection of subsets of X that is maximal with respect to the finite intersection property. Show that $x \in \overline{D}$ for every $D \in D$ if and only if every neighborhood of x belongs to D.

Or

(b) Define locally compact space with an example. Show that Q of rationals are not locally compact.

12. (a) Under what conditions does a metrizable space have a metrizable compactification?

Or

(b) Let X be completely regular. Prove that X is connected if and only if $\beta(x)$ is connected.

13. (a) Prove that a product of completely regular space is completely regular.

Or

(b) Prove that every paracompact Hausdorff space X is normal.

14. (a) Prove that the Euclidean space R^k is complete in either of its usual metrics, the Euclidean metric d or the square metric ρ.

Or

(b) Let X be a space and let (y, d) be a metric space. If the subset f of $\zeta(x, y)$ is totally bounded under the uniform metric corresponding to d, then prove that f is equicontinuous.

15. (a) Show that in the compact open topology, $\zeta(x, y)$ is Hausdorff if y is Hausdorff.

Or

(b) Let $(_1)(_2)$.... be a nested sequence of non-empty closed sets in the complete metrics space X. If $\dim c_n \to 0$ then prove that $nc_n \neq \varphi$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Tychonoff theorem.

17. Let X be a metrizable space. If $\mathcal{A}$ is an open covering of X, then prove that there is an open covering ξ of X refining $\mathcal{A}$ that is countably locally finite.

18. Prove that a space X is metrizable if and only if it is a paracompact Hausdorff space that is locally metrizable.

19. State and prove Ascoli's theorem.

20. State and prove Baire category theorem.

D–6878

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DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

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MAY 2020 ARREAR EXAMINATION

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$ are proportional to $(by - cz), cz - ax, ax - by)$.

2. Solve $yz\,dx + xz\,dy + xy\,dz = 0$.

3. Eliminate the arbitrary constants a and b from $z = (x + a)(y + b)$.

4. Form the partial differential equation by eliminating the arbitrary function from $z = x + y + f(xy)$.

5. Find a particular integral of $(D^2 - D^1)z = e^{2x+y}$.

6. Show that $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial z}{\partial y} = 0$ is parabolic.

7. State the interior Neumann problem.

8. Find a complete integral of $pq = 1$.

9. Show that the equation $(x^2 z - y^3)dx + 3xy^2 dy + x^3 dz = 0$ is integrable.

10. When we say that the equation
$Rr + Ss + Tt + f(x, y, z, p, q) = 0$ is elliptic.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Find the integral curves of the sets of equations

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}.$$

Or

(b) Solve: $(y+z)\,dx + (z+x)\,dy + (x+y)\,dz = 0$.

12. (a) Form the partial differential equation by eliminating the arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = 1$.

Or

(b) Find the general integral of $z(xp - yq) = y^2 - x^2$.

13. (a) Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve them.

Or

(b) Find a complete integral of $p^2 x + q^2 y = z$, by Jacobi's method.

14. (a) Solve the equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}.$$

Or

(b) Find the D'Alembert's solution of the one dimensional wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$.

15. (a) Find the solution of one dimensional diffusion equation $\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{k}\frac{\partial \theta}{\partial t}$.

Or

(b) Prove that if ψ is continuous within and on the circumference of a circle and is harmonic in the interior, then the value of ψ at the centre is equal to the mean value on the boundary.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Verify that the differential equation

$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and find its primitive.

17. From

(a) $x^2 + y^2 + (z-c)^2 = a^2$,

(b) $x^2 + y^2 = (z-c)^2 \tan^2 \alpha$ and

(c) $z = f(x^2 + y^2)$, show that they have same partial differential equation.

18. Find the integral surface of the equation $px + qy = z$ passing through $x + y = 1$ and $x^2 + y^2 + z^2 = 4$.

19. A uniform string of line density ρ stretched to tension ρc^2 and executes a small transverse vibration in a place through the undisturbed line of the string. The ends $x = 0$ and $x = l$ of the string are fixed. The string is at rest, with the point $x = b$ drawn aside through a small distance E and released at time $t = 0$. Find the transverse displacement y at any time t by Fourier series method.

20. The faces $x = 0$ and $x = a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation $\theta = f(x)$, $0 \le x \le a$. Determine the temperature at a subsequent time t.

D–6879

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Third Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define torsion of a curve.
2. Define evolute of a space curve.
3. Define osculating sphere.
4. What is meant by right helicoid?
5. Define the tangential components.
6. State the fundamental existence theorem for space curves.
7. Define the geodesic curvature.

8. When will you say that a vector is called the geodesic vector of the curve?

9. Define the characteristic line.

10. Define osculating developable of the curve.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If the radius of spherical curvature is constant, then prove that the curve either lies on a sphere or has constant curvature.

Or

(b) With the usual notations, prove that $[\vec{v}', \vec{v}'', \vec{v}'''] = k^2 \tau$

12. (a) Show that the involutes of a circular helix are plane curves.

Or

(b) Find the area of the anchor ring.

13. (a) Show that a curve on a surface is geodesic if and only if its Gaussian curvature vector is zero.

Or

(b) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z = constant.

14. (a) Derive the Liouville's formula for Kg.

Or

(b) Prove that every helix on a cylinder is a geodesic.

15. (a) Discuss the Dupin's indicatrix.

Or

(b) Enumerate the polar developable.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Discuss an isometric correspondence in detail.

17. Find the intrinsic equations of the curve given by $x = a\ e^{u} \cos\ u, y = a\ e^{u} \sin\ u, z = b\ e^{u}$

18. If θ is the angle at the point (u,v) between the two directions given by $P\ du^2 + 2Q\ dudv + R\ dv^2 = 0$, then prove that $\tan\theta = \dfrac{2H\sqrt{Q^2 - PR}}{ER - 2FQ + GP}$.

19. Derive the Christoffel symbols of the second kind.

20. Derive the Rodrigue's formula.

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D–6880

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Third Semester

Mathematics

OPTIMIZATION TECHNIQUES

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Write down the types of algorithm in Network models.

2. Define a critical activity.

3. Define the effective lead time.

4. Write the formula for purchasing cost per unit time in EOQ with price breaks.

5. Define a queue discipline.

6. Write down the P-K formula.

7. Draw the transition - rate diagram.

8. Define strategies of a game and value of the game.

9. When we say that a function $f(x_1, x_2, ..., x_n)$ is separable.

10. What is quadratic programming?

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Describe Dijkstra's algorithm.

Or

(b) Construct the network diagram comprising activities B, C, ...Q and N such that the following constraints are satisfied. B<E, F; C<G,L; E,G<H; L,H<I; L<M; H<N; H<J; I,J,<P; P<Q. The notation X<Y means that the activity x must be finished before y can begin.

12. (a) For what value of λ, the game with the following payoff matrix is strictly determinable?

Player B

$$\text{Player A}\quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} \lambda & 6 & 2 \\ -1 & \lambda & -7 \\ -2 & 4 & \lambda \end{pmatrix} \end{array}$$

Or

(b) Solve the following game and determine the value of the game

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \end{array}$$

13. (a) Solve the following L.P.P:

Maximize $z = 3x_1 + 9x_2$

Subject to the constraints:

$x_1 + 4x_2 \le 8,$

$x_1 + 2x_2 \le 4;$

$x_1, x_2 \ge 0$

Or

(b) Explain the bounded variable algorithm.

14. (a) Show, how the following problem can be made separable.

Maximize: $z = x_1, x_2, + x_3 + x_1 x_3$

Subject $x_1, x_2 + x_3 + x_1 x_3 \leq 0$

$x_1, x_2, x_3 \geq 0$.

Or

(b) Explain the solution of mixed strategy.

15. (a) Solve the non-linear programming problem:

Minimize: $f(x_1, x_2) = 3x_1^2 + x_2^2 + 2x_1 x_2 + 6x_1 + 2x_2$

Subject to the constraints: $2x_1 - x_2 = 4$

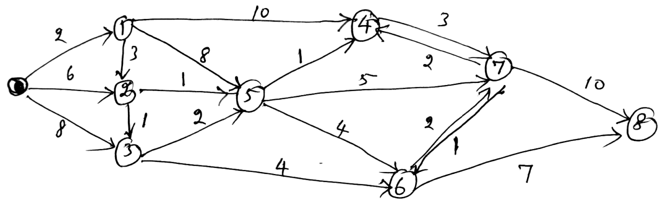
Or

(b) Discuss briefly about "separable convex programming".

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions

16. Find the shortest route from 0 to 8. The numbers on the links represent the distance in kilometres.



17. Using the bounded variable technique, solve the following L.P.P.

Maximize : $z = 3x_1 + x_2 + x_3 + 7x_4$

Subject to the constraints

$2x_1 + 3x_2 - x_3 + 4x_4 \leq 40$

$-2x_1 + 2x_2 + 5x_3 - x_4 \leq 35$

$x_1 + x_2 - 2x_3 + 3x_4 \leq 100;$

$x_, \geq 2,\ x_2 \leq 1, x_3 \geq 3, x_4 \geq 4.$

18. Solve by revised simplex method:

minimize: $z = x_1 + 2x_2$

Subject to the constraints:

$2x_1 + 5x_2 \geq 6,$

$x_1 + x_2 \geq 2;$

$x_1 \geq 1$ and $x_2 \geq 0$

19. Use the method of Lagragian multipliers to solve the non-linear programming problem:

Maximize $z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

Subject to the constraints: $x_1 + x_2 + x_3 = 20$

20. Solve the quadratic programming problem:

Maximize: $z = 2x_1 + 3x_2 = 2x_1^2$

Subject to the constraints:

$x_1 + 4x_2 \leq 4.$

$x_1 + x_2 \leq 2;$

$x_1, x_2 \geq 0$

D–6881

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Third Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS 2018–19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. If a prime p does not divide a, then show that $(p,a) = 1$.

2. Find all integers n such that $\varphi(n) = 12$.

3. State Mobius inversion formula.

4. Define Liouville's function $\lambda(n)$ and the divisor function $\sigma_\alpha(n)$.

5. If f is multiplicative then show that; $f(1) = 1$.

6. Prove that $\alpha \circ (\beta \circ F) = (\alpha * \beta) \circ F$.

7. Define average order of $\mu(n)$ and of $\wedge(n)$.

8. If $a \equiv b(\bmod m)$ and if $0 \leq |b-a| < m$, then show that $a = b$.

9. If p is odd, $p > 1$, prove that $1^2.3^2.5^2...(p-2)^2 \equiv (-1)^{\frac{p+1}{2}} (\bmod p)$

10. State Wolskenholme's theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) State and prove Euclid's lemma.

Or

(b) Prove that if $n \geq 1$, $\sum_{d/n} \mu(d) = \begin{cases} 1 & 1\ if\ n = 1 \\ 0 & if\ n > 1 \end{cases}$.

12. (a) Prove that $n \geq 1$ $\varphi(n) = \sum_{d/n} \mu(d)(n/d)$.

Or

(b) State and prove division algorithm.

13. (a) If f and g are multiplicative, then prove that its Dirichlet product is $f * g$.

Or

(b) State and prove generalized Mabius inversion formula.

14. (a) Assume $(a, m) = d$. Prove that the linear congruence $ax \equiv b(\bmod m)$ has solutions if, and only if d/b.

Or

(b) State and prove Euler's summation formula.

15. (a) Lets p be an off prime. Then prove that for all n, $(n/p) = n^{(p-1)/2} (\mathrm{mod}\, p)$.

Or

(b) State and prove Wilson's theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Fundamental theorem of arithmetic.

17. (a) Prove that if $n \geq 1$, $\varphi(n) = n \underset{p/n}{\pi} (1 - \frac{1}{p})$.

(b) Prove that , if $2^n - 1$ is prime, then n is prime.

18. If both g and $f * g$ are multiplicative, then prove that f is also multiplicative.

19. Prove that for all $x \geq 1$, $\sum_{n \leq x} d(n) = x \log x + (2c - 1)x + 0(x)$ where C is Euler's constant.

20. State and prove Lagrange' theorem.

D–6882

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define order of a Markov chain.

2. Define non-full persistent state.

3. Define transition densities.

4. State the Brownian motion.

5. Write the forward diffusion equation.

6. Write the equation of motion of a Brownian particle.

7. Define traffic intensity.

8. Define probability of extinction.

9. Define idle period.

10. Write the Erlang's second formula.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) The t.p.m of a Markov chain $\{X_n, n = 1,2,\ldots\}$ having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $\prod_0 = (0.7, 0.2, 0.1)$. Find

(i) $\Pr(X_2 = 3)$

(ii) $\Pr\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

Or

(b) If $\{N(t)\}$ is a Poisson process then prove that the autocorrelation coefficient between $N(t)$ and $N(t+s)$ is $\left(\dfrac{t}{t+s}\right)^{1/2}$.

12. (a) If $X(t)$ with $X(0)$ and $\mu = 0$ is a Wiener process and $0 < s < t$, show that for atleast one λ satisfying $s \le \lambda \le t$ $\Pr\{X(\tau) = 0\} = (2/\pi)\cos^{-1}\left(\left(\dfrac{s}{t}\right)^{1/2}\right)$.

Or

(b) Let $\{X(t), t \ge 0\}$ be a Wiener process with $\mu = 0$ and $X(0) = 0$. Find the distribution of T_{a+b} for $0 < a < a + b$.

13. (a) Prove that the p.g.f $R_n(s)$ of Y_n satisfies the recurrence relation $R_n(s) = s\,P(R_{n-1}(s))$; $P(s)$ being the p.g.f of the offspring distribution.

Or

(b) Show that the p.g.f. of the conditional distribution of X_n, given $X_n > 0$, is $\frac{P_n(s) - P_n(0)}{1 - P_n(0)}$.

14. (a) Prove that $E\{X_{n+r} / X_n\} = X_n m^r$ for $r, n = 0,1,2,\ldots$.

Or

(b) Derive the state solution.

15. (a) Derive the necessary and sufficient condition for the existence of a steady state of the infinite series $\sum_{n=1}^{\infty} \prod_{K=1}^{n} \frac{\lambda_{k-1}}{\mu k}$.

Or

(b) Show that the average number of busy channels in the system for $M / M / \infty$ model is λ/μ.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the Chapman – Kolmogorov equation.

17. Let a finite Markov chain with state space $S = \{0,1,2,\ldots,l\}$ be a martingale. Prove that, as $n \to \infty$,

$$P_{ij}^{(n)} = 0, \qquad j = 1,2,\ldots,l-1 \qquad \text{and} \qquad \left.\begin{aligned} P_{il}^{(n)} &= \frac{i}{l} \\ P_{io}^{(n)} &= 1 - \frac{i}{l} \end{aligned}\right\},$$

$i = 1,2,\ldots,l-1$.

18. Show that Ornstein – Uhlenbech process as a transformation of Wiener process.

19. Suppose that $m = 1$ and $\sigma^2 < \infty$, then prove that

(a) $\lim_{n\to\infty} n \cdot \Pr\{X_n > 0\} = \dfrac{2}{\sigma^2}$

(b) $\lim_{n\to\infty} E\left\{\dfrac{X_n}{n} / X_n > 0\right\} = \dfrac{\sigma^2}{2}$ and

(c) $\lim_{n\to\infty} \Pr\left\{\dfrac{X_n}{n} > u / X_n > 0\right\} = \exp\left\{-\dfrac{2u}{\sigma^2}\right\}, \ u \geq 0$.

20. A mechanic looks after 8 automatic machines, a machine breaks down, independently of others, in accordance with a Poisson process, the average length of time for which a machine remains in working order being 12 hours. The duration of time required for repair of a machine has an exponential distribution with mean 1 hour.

Find

(a) The probability that 3 or more machines will remain out of order at the same time.

(b) The average number of machines in working order and

(c) For what fraction of time, on the average, the mechanic will be idle?

D–7328

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define a complete bipartite graph. Give an example.
2. Prove that in a tree any two vertices are connected by a unique path.
3. Define disconnected graph with an example.
4. Find the number of different perfect matchings in K_{2n}.
5. Define edge chromatic number.
6. When will you say a graph is critical graph?
7. Define $\gamma(k,l)$.
8. Prove that K_5 is non-planar.

9. Define indegree and outdegree of a vector v in a digraph.

10. Define a Hamiltonian path in a digraph.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If a k-regular bipartite graph with $k > 0$ has bipartition (X, Y), then prove that $|X| = |Y|$.

Or

(b) If G is a tree, prove that $\varepsilon = \gamma - 1$.

12. (a) Prove that $\gamma(m, n) = \gamma(n, m)$.

Or

(b) Let G be a k-regular bipartite graph with $k > 0$. Prove that G has a perfect matching.

13. (a) Find the edge chromatic number of K_n and $K_{m,n}$.

Or

(b) If G is uniquely n-colourable, then prove that $\delta(G) \geq n - 1$.

14. (a) Show that there is no map of five regions in the plane such that every pair of regions are adjacent.

Or

(b) Explain the four colour conjecture.

15. (a) Prove that the $(i, j)^{th}$ entry A^n is the number of walks length n from v_i and v_j.

Or

(b) Prove that in a simple digraph, every vertex lies in exactly one strong component.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that a vertex V of a tree G is a cut vertex of G if and only if $\deg(v) > 1$.

17. Prove that a connected graph has an Euler trial if and only if it has atmost two vertices of odd degree.

18. State and prove Vizing's theorem.

19. Prove that every planar graph is 5-vertex colourable.

20. State and prove max-flow, min-cut theorem.

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D–7329

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Banach space. Give an example.

2. Define quotient norm on X/Y.

3. Give an example of a discontinuous linear functional.

4. Define Hamel basis.

5. Define inner product space. Give an example.

6. Define annihilator.

7. Define unitary operator.

8. Define orthogonal projection.

9. Write Schwartz inequality.

10. State the open mapping theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions. Choosing either (a) or (b).

11. (a) Show that the real linear space $\mathbb{R}$ and the complex linear space $\mathbb{C}$ are Banach spaces under the norm $\|x\| = |x|$, $x \in \mathbb{R}$ or $\mathbb{C}$

Or

(b) Let x be a normal space, and let the subset $\{x \in X / \|x\| \leq 1\}$ be compact in x. prove that X is finite dimensional.

12. (a) Prove that if x is a finite dimensional linear space then all linear functionals are bounded.

Or

(b) Let X and Y be normed spaces. Prove that if X is finite dimensional then every linear map from X to Y is continuous.

13. (a) Let $< \cdot >$ be an inner product on a linear space X. Prove that $4 < x, y > = < x + y, x + y > - < x - y, x - y > + i < x + iy, x + iy > - i < x - iy, x - iy >$ for all $x, y \in X$.

Or

(b) Prove that every completely continuous linear transformation is a continuous linear transformation.

14. (a) If f is a linear functional on the Hilbert space X, with null space N, then prove that f is continuous if and only if N is a closed subspace.

Or

(b) Prove that $\|A^*A\| = \|A\|^2, A$ is a bounded linear transformation.

15. (a) State and prove the closed graph theorem.

Or

(b) Let A be a symmetric operator. Support that $R(A) = x$ that is; A is an onto mapping. prove that A is self-adjoint.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions

16. Let X be a normed linear space. then show that the following conditions are equivalent

(a) Every closed and bounded subset of X is compact.

(b) The subset $\{x \in X / \|x\| \le 1\}$ of X is compact

(c) X is finite dimensional.

17. Let $\widetilde{X}$ denote the normed linear space of all bounded linear functionals over the normed linear space X. Prove that $\widetilde{X}$ is a Banach space.

18. Let $\{u_1, u_2, ...\}$ be a countable orthonormal set in an inner product space X and $x \in X$. Prove that $\sum_n |\langle x, u_n \rangle|^2 \le \|x\|^2$, where equality holds if and only if $x = \sum_n \langle x, u_n \rangle u_n$.

19. State and prove Hahn-Banach theorem.

20. State and prove open mapping theorem.

D–7330

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Jacobian matrix.
2. Define a linear system with an example.
3. Define Eigen function.
4. Define the matrix norm.
5. State the interpolating conditions.
6. Define the shape function $N_i(x)$.
7. Write the Hermite interpolating polynomial.
8. State Weierstrass approximation theorem.
9. Write down the order of the single step method.
10. State the boundary value problem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the multiple root of the equation $f(x)=27x^5+27x^4+36x^3+28x^2+9x+1=0$ by Newton-Raphson method.

Or

(b) Use synthetic division and perform two iteration by Birge-Vieta method to find the smallest positive root of the equation $x^4-3x^3+3x^2-3x+2=0$.

12. (a) Solve the following systems of equations buy Gauss Elimination method.

$$4x_1+x_2+x_3=4$$
$$x_1+4x_2-2x_3=4$$
$$3x_1+2x_2-4x_3=6$$

Or

(b) Estimate the Eigen values of the matrix $A=\begin{bmatrix}1 & 2 & -1\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$, using the Gerschgorin bounds.

13. (a) Find the inverse of the matrix $A=\begin{bmatrix}2 & 3 & -1\\ 3 & 1 & 2\\ -1 & 2 & -1\end{bmatrix}$ by Cholesky method.

Or

(b) Using piecewise linear interpolation, find the interpolating polynomial for the following data:

x	0	1	2
y=f(x)	1	3	35

14. (a) Determine the best minimax approximation to $e^{|x|}$ with a polynomial of degree 0 and 1 for $|x| \le 1$

Or

(b) Evaluate $\int_0^{\frac{1}{2}} \frac{x}{\sin x} dx$ using Gauss-Legendre three point formula.

15. (a) Use the Euler method to solve numerically the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0,1]$.

Or

(b) Apply the Taylor's series second order method to integrate $y' = 2t + 3y$, $y(0) = 1$, $t \in (0,0.4)$ with $h = 0.1$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find all the roots of the polynomial $x^3 - 4x^2 + 5x - 2 = 0$ using the Graeffe's root squaring method.

17. Find the largest Eigen value in modulus and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$, using the power method.

18. Obtain the cubic Spline approximation for the function given in the tabular form

x	0	1	2	3
f(x)	1	2	33	244

19. Calculate $\int_0^{0.8}(1+\frac{\sin x}{x})dx$, correct to 5 decimal places.

20. Solve the initial value problem $u' = -2tu^2$, $u(0)=1$ with $h=0.2$ on the interval $[0,1]$, by using the second order implicit Runge-Kutta method.

D–7331

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION.

MAY 2021 EXAMINATION

&

MAY 2020 ARREAR EXAMINATION

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS 2018-19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. For each $c \subset \mathcal{C}$, prove that $p(c) - 1 - p(c^*)$

2. Define conditional probability.

3. Let $f(x, y) = \begin{cases} 6x^2 y, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

   Find $\Pr\left(0 < x < \frac{3}{4}, \frac{1}{3} < y < 2\right)$

4. Prove that the expected value of the product of two random variables is equal to the product of their expectations plus their covariance.

5. Write the p.d.f of Gamma distribution.

6. Define covariance of X and Y.

7. If the p.d.f of X is $f(x) = \begin{cases} 2x\, e^{-x^2}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$, then find the p.d.f. of $Y = X^2$.

8. Write down the m.g.f. of $Y = \sum_{i=1}^{n} X_i$.

9. Define $t-$distribution.

10. Define convergence in probability distribution.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the Baye's theorem.

Or

(b) Find the mean and variance of the p.d.f.

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

12. (a) Let the joint p.d.f of X_1 and X_2 be

$$f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Show that the random variables X_1 and X_2 are dependent.

Or

(b) Let $f(x_1, x_2) = \begin{cases} 4x_1\, x_2, & 0 < x_1 < 1;\ 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$

be the p.d.f. of X_1 and X_2. Find $\Pr\left(0 < X_1 < \frac{1}{2},\ \frac{1}{4} < X_2 < 1\right)$.

13. (a) Find the measures of skewness and kurtosis of the Binomial distribution $b(n,\ p)$.

Or

(b) Let X have a gamma distribution with $\alpha = \frac{\gamma}{2}$ where γ is a positive integer, and $\beta > 0$. Define the random variable $Y = 2X/\beta$. Show that the p.d.f. of Y is $\chi^2(\gamma)$.

14. (a) Derive the p.d.f. of Chi-square distribution.

Or

(b) Let X and Y be random variables with $\mu_1 = 1$, $\mu_2 = 4$, $\sigma_1^2 = 4$, $\sigma_2^2 = 6$, $\rho = \frac{1}{2}$ Find the mean and variance of $Z = 3X - 2Y$.

15. (a) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

Or

(b) Let Z_n be $\chi^2(n)$ and let $W_n = \frac{Z_n}{n^2}$, find the limiting distribution of W_n.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Chebyshev's inequality theorem.

17. Let $f(x, y) = \begin{cases} e^{-x-y}, & 0 \le x < \infty, 0 \le y < \infty \\ 0, & \text{elsewhere} \end{cases}$

    be the joint p.d.f of X and Y.

    Find $P[X < 1]$, $P[X > Y]$ and $P[X + Y = 1]$

18. Find the m.g.f. of a normal distribution and hence find the mean and variance of a normal distribution.

19. Derive the p.d.f. of F-distribution.

20. State and prove the central limit theorem.

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