D–5511

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

First Semester

ALGEBRA – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the Euclidean algorithm.

2. Define automorphism of a group.

3. Show that the group of order 21 is not simple.

4. If $o(G) = p^2$, where p is a prime, show that G is abelian.

5. Define an integral domain.

6. If U is an ideal of R and $1 \in U$, then prove that $U = R$.

7. Define a left ideal of R.

8. When will you say a polynomial is integermonic?

9. Prove that any field is an integral domain.

10. Find the g.c.d of the polynomial $x^2 + 1$ and $x^6 + x^3 + x + 1$ in $Q[x]$.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ for any three sets A, B and C.

Or

(b) Let $\theta : G \to H$ be a onto group homomorphism with Kernal K. Prove that G / K is isomorphic to H.

12. (a) State and prove the Cauchy's theorem for abelian group.

Or

(b) If P is a prime number and $P / o(G)$, then prove that G has an element of order P.

13. (a) Show that any two p-Sylow subgroups of a group G are conjugate.

Or

(b) If D is an integral domain and D is of finite characteristic, prove that the characteristic of D is a prime number.

14. (a) Show that the set of multiples of a fixed prime number p form a maximal ideal of the ring of integers.

Or

(b) If $[a,b] = [a',b']$ and $[c,d] = [c',d']$, then prove that $[a,b][c,d] = [a',b'][c',d']$.

15. (a) Prove that $J[i]$ is a Euclidean ring.

Or

(b) State and prove the Einstein criterian theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If G is a group, N a normal subgroup of G , then prove that G/N is also a group.

17. Prove that every group is isomorphic to a subgroup of $A(S)$ for some appropriate S .

18. If Q is a homomorphism of R into R' with Kernal $I(Q)$ then prove the following

(a) $I(Q)$ is a subgroup of R under addition

(b) If $a \in I(Q)$ and $\gamma \in R$, then both $a\gamma$ and γa are in $I(Q)$.

19. If U is an ideal of a ring R, then prove that R/U is a ring and is a homomorphic image of R .

20. If R is an integral domain, then prove that $R[x_1, x_2, \ldots, x_n]$ is an integral domain.

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D–5512

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define countable and uncountable set.

2. For $x \in R'$ and $y \in R'$, define $d_1(x,y) = (x-y)^2$, determine, whether it is a metric or not.

3. Define compact set. Give an example.

4. When do you say that a series converge? Give an example of a divergent series.

5. State the root test.

6. Define bounded and unbounded function.

7. Give an example of a function which has second kind discontinuity at every point.

8. Find the radius of convergence of the power series $\sum \frac{2^n}{n^2} z^n$.

9. State the generalized mean value theorem.

10. Let f be defined on $[a,b]$. If f is differentiable at a point $x \in [a,b]$, then prove that f is continuous at x.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $|z+w| \le |z| + |w|$, where z and w are complex numbers.

Or

(b) Prove that a set E is open if and only if its complement is closed.

12. (a) Define a Cauchy sequence. If X is a compact metric space and if $\{p_n\}$ is a Cauchy sequence in X, then prove that $\{p_n\}$ converges to some point of X.

Or

(b) State and prove the Weierstrass theorem.

13. (a) Derive the partial summation formula.

Or

(b) Suppose (i) the partial sums A_n of $\sum a_n$ form a bounded sequence (ii) $b_0 \ge b_1 \ge \ldots \ge b_n \ge \ldots$, (iii) $\lim_{n\to\infty} b_n = 0$. Prove that $\sum a_n \ b_n$ converges.

14. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.

Or

(b) Suppose f is a continuous mapping of metric space X into a metric space Y. If E is a connected subset of X, then prove that $f(E)$ is connected.

15. (a) State and prove chain rule for differentiation.

Or

(b) State and prove Cauchy mean-value theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that for every real $x > 0$ and every integer $n > 0$, there is one and only one real y such that $y^n = x$.

17. Prove that every k-cell is compact.

18. Investigate the behaviour (convergence or divergence) of $\sum a_n$ if

(a) $a_n = \dfrac{\sqrt{n+1} - \sqrt{n}}{n}$

(b) $a_n = \left(\sqrt[n]{n} - 1\right)^n$.

19. Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

20. State and prove the inverse function theorem.

D–5513

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State the existence theorem for linear equations with constant coefficients.

2. Solve $y'' - 4y = 0$.

3. Compute the Wronskian of $\varphi_1(x) = x^2$, $\varphi_2(x) = 5x^2$.

4. Define indicial polynomial.

5. Show that $P_n(-x) = (-1)^n P_n(x)$.

6. Define singular point.

7. Find all real valued solution of the equation $y' = x^2 y$.

8. Show that $f(x, y) = y^{1/2}$ does not satisfy the Lipschitz condition on $R : |x| \le 1,\ 0 \le y \le 1$.

9. Consider the initial value problem $y' = y_2^{(2)} + 1$, $y_2' = y_1^{(2)}$; $y_1(0) = 0$, $y_2(0) = 0$, Compute the first three successive approximations $\varphi_0, \varphi_1, \varphi_2$.

10. State the non-local existence theorem for $y' = f(x, y)$, $y(x_0) = y_0$, $(|y_0| < \infty)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve : $y'' + 10y = 0$, $y(0) = \pi$, $y'(0) = \pi^2$.

Or

(b) Find all solutions of $y'' - 4y' + 5y = 3e^{-x} + 2x^2$.

12. (a) Show that $\int_{-1}^{1} P_n^2(x)\, dx = \frac{2}{2n+1}$.

Or

(b) Let φ_1, φ_2 be two solutions of $L(y) = 0$ on an interval I and let x_0 be any point in I. Prove that φ_1, φ_2 are linearly independent on I if and only if $W(\varphi_1, \varphi_2) \neq 0$.

13. (a) Find all solutions of the equation $x^2 y'' + xy' - 4\pi y = x$ for $x > 0$.

Or

(b) Show that $\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0$ when $n \neq m$.

14. (a) Show that –1 and +1 are regular singular points for the Legendre equation $(1-x^2)y''-2xy'+\alpha(\alpha+1)y=0$.

Or

(b) Compute the first four successive approximation $\varphi_0, \varphi_1, \varphi_2, \varphi_3$ for the equation $y'=1+xy,\ y(0)=1$.

15. (a) Let $f(x, y)=\frac{\cos y}{1-x^2}$, $(|x|<1)$. Show that f satisfies a Lipschitz condition on every strip $S_\alpha : |x| \leq \alpha$, where $0<\alpha<1$.

Or

(b) Prove that a function φ is a solution of the initial value problem $y'=f(x,y),\ \ y(x_0)=y_0$ on an interval I if and only if it is a solution of the integral equation $y=y_0+\int_{x_0}^{x} f(t, y)\,dt$ on z.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let φ be a solution of $L(y)=y^{(n)}+a_1 y^{(n-1)}+....+a_n y=0$ on an interval I containing a point x_0. For all x in I, prove that $\|\varphi(x_0)\| e^{-k|x-x_0|} \leq \|\varphi(x)\| \leq \|\varphi(x_0)\| e^{k|x-x_0|}$, where $k=1+|a_1|+|a_2|+....+|a_n|$.

17. Find the two linearly independent power series solutions of the equation $y''-xy'+y=0$.

18. Derive Bessel's function of first kind of order $\alpha_i\ J_\alpha(x)$.

19. Find all solutions of the equation

$$x^2 y'' - (2+i)xy' + 3iy = 0 \text{ for } x > 0$$

20. Let f be a real-valued continuous function on the strip $S: |x - x_0| \leq a$, $|y| < \infty$, $a > 0$ and f satisfies on S a Lipschitz condition with constant $k > 0$. Prove that the successive approximations $\{\varphi_k\}$ for the problem $y' = f(x, y)$, $y(x_0) = y_0$ exist on the entire interval $|x - x_0| \leq a$ and converge there to a solution φ of $y' = f(x, y)$, $y(x_0) = y_0$.

D–5514

Sub. Code
31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define one-one function, Give an example.

2. Define equivalence class.

3. Define well-ordered set.

4. What is meant by quotient map?

5. Is the rational Q connected? Justify your answer.

6. Define compact space. Give an example.

7. What is meant by the path component of the space X?

8. Define limit point compact.

9. Define regular space.

10. Whether the space $\mathbb{R}_k$ is Hausdorff or not? Justify your answer.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that a countable union of countable set is countable.

Or

(b) State and prove strong induction principle.

12. (a) Let A be a subset of a topological space X and let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

Or

(b) State and prove the uniform limit theorem.

13. (a) Let X be locally path connected. Prove that every connected open set X is path connected.

Or

(b) State and prove the sequence lemma.

14. (a) Let Y be a subspace of X. Prove that Y is compact if and only if every covering of Y by sets open in X contains a finite subcollection covering Y.

Or

(b) Prove that compactness implies limit point compactness, but not conversely.

15. (a) Prove that every locally compact Hausdorff space is completely regular.

Or

(b) Define regular space. Prove that a subspace of a regular space is regular.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Define the Cartesian product of an indexed family of sets $\{A_\alpha\}_{\alpha \in J}$. Prove that a finite product of countable set is countable.

17. Prove that every finite point set in a Hausdorff space is closed.

18. If L is a linear continuum in the order topology, then prove that L is connected, and so are intervals and rays in L.

19. State and prove tube lemma.

20. State and prove Urysohn metrization theorem.

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D–5515

Sub. Code
31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Second Semester

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define dual space.

2. Define the orthogonal complement.

3. Find the degree of the splitting field of $x^4 - 2$ over F.

4. Express the polynomial $x_1^3 + x_2^3 + x_3^3$ in the elementary symmetric functions in x_1, x_2, x_3.

5. When will you say that an element in $A(V)$ is singular?

6. Define Hermition and skew hermition.

7. What is meant by index of nilpotence?

8. Define the elementary divisors of T in $A(V)$.

9. Define symmetric and skew symmetric matrix.

10. Define unitary transformation.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $L(S)$ is a subspace of a vector space V.

Or

(b) With usual notations, prove that $F^{(n)}$ is isomorphic with $F^{(m)}$ if and only if $n = m$.

12. (a) State and prove the remainder theorem.

Or

(b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

13. (a) For any $f(x), g(x) \in F[x]$, prove that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.

Or

(b) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

14. (a) Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $V \neq 0$ in V, $vT \equiv \lambda v$.

Or

(b) Let V be finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of minimal polynomial for T is not 0.

15. (a) Prove that every $A \in F_n$ satisfies its characteristic equation.

Or

(b) Prove that $T \in A(V)$ is unitary if and only if $TT^* = 1$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If V and W are of dimensions m and n respectively, over F, then prove that $Hom(V,W)$ is of dimension mn over F.

17. Let V be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.

18. Prove that the number e is transcendental.

19. If $p(x) \in F[x]$ is solvable by radicals over F, then prove that the Golois group over F of $p(x)$ is a solvable group.

20. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

D–5516

Sub. Code
31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Second Semester

ANALYSIS – II

(CBCS 2018-19 Academic Year onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Prove that any constant function is Riemann integrable.

2. Define an equicontinuous function on a set.

3. Show that $\left(e^{x}\right)^{1} = e^{x}$.

4. Define pointwise bounded function.

5. On what intervals does the series $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2 x}$ converge uniformly?

6. Define an orthogonal system of functions on $[a,b]$.

7. Define the gamma function.

8. Define Borel set.

9. Define measurable function. Give an example.

10. Define an integrable function over the measurable set.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if f is continuous on $[a,b]$ then $f \in R(\alpha)$. on $[a,b]$.

Or

(b) State and prove the fundamental theorem of calculus.

12. (a) State and prove the Cauchy criterion for uniform convergence.

Or

(b) Let β be the uniform closure of an algebra A of bounded functions. Prove that B is a uniformly closed algebra.

13. (a) If f is a positive function on $(0, \infty)$ such that

(i) $f(x+1) = x(f(x))$

(ii) $f(1) = 1$

(iii) $\log f$ is convex, then show that $f(x) = \sqrt{x}$.

Or

(b) If f is a continuous and if $\in > 0$, then prove that there is a trigonometric polynomial P such that $|p(x) - f(x)| < \in$ for all real x.

14. (a) State and prove Egoroff's theorem.

Or

(b) Let $< E_n >$ be an infinite decreasing sequence of measurable sets. Let ${}_m E_1$ be finite, then prove that $m\left(\overset{\infty}{\underset{i=1}{\wedge}} \in i\right) = \lim\limits_{n \to \infty} {}_m E_n$.

15. (a) Let φ and ψ be simple functions which vanish outside a set of finite measure. Prove that $\int(a\varphi + b\psi) = a\int\varphi + b\int\psi$ and if $\varphi \geq \psi$ almost everywhere then prove that $\int\varphi \geq \int\psi$.

Or

(b) State and prove Fatou's lemma.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that there exists a real continuous function on the real line which is no where differentiable..

17. State and prove Parseval's theorem.

18. Prove that the outer measure of an interval is its length.

19. Let f and g be integrable over E. Prove the following:

(a) The function *cf* is integrable over E and $\int_E cf = c\int_E f$.

(b) The function $f+g$ is integrable over E and $\int_E f+g = \int_E f + \int_E g$.

(c) If $f \leq g$ almost everywhere, then $\int_E f \leq \int_E g$.

20. State and prove Lebesque monotone convergence theorem.

D–5517

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Second Semester

TOPOLOGY – II

(CBCS 2018-19 Academic Year onwards)

Time : Three hours — Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a completely regular space.
2. When will you say two compactification is equivalent?
3. Define locally finite in a topological space. Give an example.
4. Define paracompact space.
5. What is meant by point open topology?
6. Define an equicontinuous function.
7. Define compactly generated space.
8. What is meant by compact open topology?
9. Show that the set Q of rationals is not a G_8-set in the reals.
10. Define a finite dimensional space.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the subspace of a completely regular space is completely regular.

Or

(b) If X is completely regular then prove that x can be imbedded in $[0, 1]^J$ for some J.

12. (a) Prove that a metric space x is complete if every Cauchy sequence in x has a convergent subsequence.

Or

(b) Show that the metric space (x, d) is complete if and only if for any nested sequence $A_1 \supset A_2 \supset \ldots$ of non empty closed sets of X such that diameter $A_n \to 0, \underset{n \in z_+}{n} A_n \neq \varphi$.

13. (a) Prove that every metrizable space is paracompact.

Or

(b) Let X be a compactly generated space; let (y, d) be a metric space, prove that $\zeta(x, y)$ is closed in y^x in the topology compact convergence.

14. (a) If X is a compact Hausdorff space, or a complete metric space, then prove that x is a Baire space.

Or

(b) Show that every locally compact Hausdorff space in a Baire space.

15. (a) Show that the sets $B_c(f,t)$ form a basis for a topology on Y^x .

Or

(b) If Y is a closed subset of x, and if x has finite dimension, then prove that Y has finite dimension and $\dim Y \leq \dim x$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove the Tietze extension theorem.

17. State and prove the sufficiency of the Nagata Smirnov Metrization theorem.

18. Let $I = [0, 1]$. Prove that there exists a continuous map $f : I \rightarrow I^2$ whose image fills up the entire square I^2 .

19. Let X be a space and let (y, d) be a metric space. For the space $\varsigma(x, y)$. Prove that the compact open topology and the topology of compact convergence coincide.

20. State and prove Baire category theorem.

D–5518

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Pfaffian differential equation.

2. Define orthogonal trajectories of a system of curves on a surface.

3. Find the complete integral of the equation $p^2z^2 + q^2 = 1$.

4. Eliminate the arbitrary function f from the equation $z = f(x^2 + y^2)$.

5. When we say that a first order partial differential equation is separable?

6. Write down the telegraphy equation.

7. Solve $(D^2 - D^1)z = 0$.

8. Write down the fundamental idea of Jacobi's method.

9. Define interior Neumann Problem.

10. Write down the d'Alembert solution of the one dimensional wave equation.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

Or

(b) Verify whether the equation :

$z(z + y)dx + z(z + x)dy - 2xydz = 0$ is integrable or not.

12. (a) Find the general integral of the linear partial differential equation $z.(xp - yq) = y^2 - x^2$.

Or

(b) Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.

13. (a) Solve the equation $p^2x + q^2y = z$ using Jacobi's method.

Or

(b) Find the complete integral of the partial differential equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0, x^2 = 4y$.

14. (a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

Or

(b) Find a particular integral of $(D^2 - D^1)z = 2y - x^2$.

15. (a) Solve the equation $r + s - 2t = e^{x+y}$.

Or

(b) Prove that the solution of a certain Neumann problem can differ from one another by a constant only.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Verify that the equation $yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$ is integrable and find its solution.

17. Find the integral surface of the equation $px + qy = z$ passing through $x + y = 1$ and $x^2 + y^2 + z^2 = 4$.

18. Find the complete integral of $q = (z + px)^2$.

19. Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$.

20. A uniform string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form of the curve $y = k \sin^3\left(\frac{\pi x}{l}\right)$ and then releasing it from this position at time $t = 0$. Find the displacement of the point of the string at a distance x from one end at any time t.

D–5519

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Radius of Curvature of a Curve.

2. Define an involute.

3. Write down the equation of the osculating plane at a point of inflexion.

4. State the fundamental existence theorem for space curves.

5. State the intrinsic properties.

6. Give an example for a surface.

7. Write the expression for Geodesic Curvature Kg.

8. When will you say that the vector is called the geodesic vector of the curve?

9. What is meant by the characteristic line?

10. What do you mean by osculating developable of the curve?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Calculate the curvature and torsion of the cubic curve given by $\bar{r} = (u, u^2, u^3)$.

Or

(b) Derive the equation of involute.

12. (a) Find the osculating circle at p, a point to the curve.

Or

(b) Find the area of the Anchor Ring.

13. (a) Prove that the metric is invariant under a parameter transformation.

Or

(b) Enumerate Isometric correspondence.

14. (a) Derive the Canonical equations for geodesics.

Or

(b) Show that a curve on surface in Geodesic if and only if its Gaussian Curvature vector is zero.

15. (a) Show that the characteristics point of the plane u is determined by the equations $\bar{r}.\bar{a} = p$, $\bar{r}.\dot{\bar{a}} = \dot{p}$, $\bar{r}.\ddot{\bar{a}} = \ddot{p}$.

Or

(b) Enumerate the polar developable.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Find the intrinsic equations of the curve given by $x = ae^u \cos u, y = ae^u \sin u, z = be^u$.

17. Prove that $\tau = \dfrac{[\bar{r}'\,\bar{r}''\,\bar{r}''']}{|\bar{r}' \times \bar{r}''|^2} = \dfrac{[\dot{\bar{r}}\,\ddot{\bar{r}}\,\dddot{\bar{r}}]}{|\dot{\bar{r}} \times \ddot{\bar{r}}|^2}$

18. Find the surface of revolution which is isometric with the region helicoid.

19. Prove that $K_g = \dfrac{1}{H\dot{s}^3}\left(\dfrac{\partial T}{\partial \dot{u}} V(t) - \dfrac{\partial T}{\partial \dot{v}} U(t)\right)$.

20. Derive the Rodrigues formula.

D–5520

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Explain triple operation.
2. Define a cut and the cut capacity in a network.
3. Define purchasing cost.
4. What is the difference between PERT and CPM?
5. Define the effective lead time.
6. Define two-person zero-sum game.
7. Write down the Floyd's algorithm.
8. Explain Kendall's notation.
9. Define the transition-rate diagram.
10. What is separable programming?

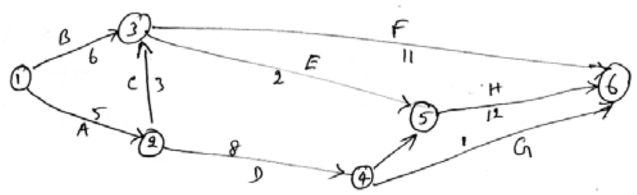
PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain Three-Jug puzzle with an illustration.

Or

(b) Determine the critical path for the project network.



All the durations are in days

12. (a) Solve the game whose payoff matrix is given by

Player B

$$\text{Player A}\begin{bmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix}$$

Or

(b) Solve the following game and determine the value of the game $\overset{\displaystyle B}{A\begin{bmatrix} 2 & 5 \\ 7 & 3 \end{bmatrix}}$.

13. (a) Solve the following L.P.P.:

Maximize: $z = 2x_1 + 3x_2 + 10x_3$ subject to the constraints

$x_1 + 2x_3 = 0;\ x_2 + x_3 = 1;\ x_1, x_2, x_3 \geq 0$

Or

(b) What are the total and free floats of a critical activity?

14. (a) Enumerate the no-setup model.

Or

(b) Show, how the following problem can be made separable

Maximize: $z = x_1, x_2 + x_3 + x_1 x_3$

Subject to $x_1, x_2 + x_3 + x_1 x_3 \leq 0$

$$x_1, x_2, x_3 \geq 0.$$

15. (a) Use the method of Lagrangian multiples to solve the non-linear programming problem:

Minimize

$z = 2x_1^2, x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

Subject to the constraints $x_1 + x_2 + x_3 = 20$

Or

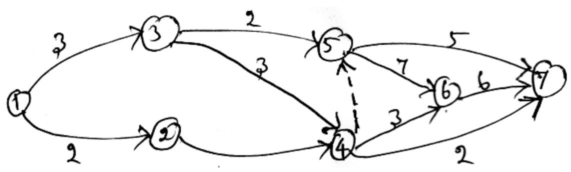
(b) Determine the extreme point of the function

$f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2 x_3 - x_1^2 - x_2^2 - x_3^2.$

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find the critical path for the following project network:



17. Using the bounded variable technique, solve the following L.P.P.:

Maximize: $z = 4x_1 + 10x_2 + 9x_3 + 11x_4$

Subject to the constraints :
$2x_1 + 2x_2 + 2x_3 + 2x_4 \le 5$
$48x_1 + 80x_2 + 160x_3 + 240x_4 \le 257,$
$0 \le x_j \le 1$ for $i = 1,2,3,4.$

18. Use two-phase revised simplex method to solve the L.P.P:

Minimize : $z = 3x_1 + x_2$ Subject to the constraints

$x_1 + x_2 \ge 1$
$2x_1 + 3x_2 \ge 2,\ x_1,\ x_2 \ge 0$

19. Solve the non-linear programming problem:

Minimize: $z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$

Subject to the constraints $x_1 + x_2 + x_3 = 11.$

20. Solve the quadratic programming problem:

Maximize : $z = 2x_1 + x_2 - x_1^2$ Subject to the constraints.

$2x_1 + 3x_2 \le 6,\ 2x_1 + x_2 \le 4;\ x_1,\ x_2 \ge 0.$

D–5521

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is meant by divisibility? Give an example.

2. When will you say a number is composite? Give an example.

3. If a prime p does not divide a, then prove that $(p,a)=1$.

4. State the Euler totient function $\varphi(n)$.

5. Define Dirichlet convolution.

6. If $a = b(\mathrm{mod}\, m)$ and if $0 \leq |b-a| < m$, then prove that $a = b$.

7. State the Legendre's identity.

8. State the Wilson's theorem.

9. What are the quadratic residue and non-residues mod13?

10. Write down the diophantic equations with an example.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any integer x, Prove that $(a,b)=(b,a)=(a,-b)=(a,b+ax)$.

Or

(b) Prove that if 2^n+1 is prime, then n is a power of 2.

12. (a) Let f be multiplicative. Prove that f is completely multiplicative if and only if $f^{-1}(n)=\mu(n)f(n),$, for all $n \geq 1$.

Or

(b) If $n \geq 1$, prove that $\phi(n)=\sum_{d/n}\mu(d)\frac{n}{d}$.

13. (a) Prove that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$.

Or

(b) State and prove that Euler's summation formula.

14. (a) Solve the congruence $25x \equiv 15(\text{mod}\,120)$.

Or

(b) State and prove Wolstenholme's theorem.

15. (a) Show that, the Legendre's symbol (n/p) is a completely multiplicative function of n.

Or

(b) Prove that, for any prime p all the coefficients of the polynomial $f(x)=(x-1)(x-2)....(x-p+1)-x^{p-1}+1$ are divisible by p.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove the Euclidean algorithm.

17. If $2^n - 1$ is prime, then prove that n is a prime.

18. Derive Dirichlet's asymptotic formula.

19. State and prove Lagrange theorem.

20. State and prove the quadratic reciprocity law.

D–5522

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Stochastic processes.

2. Define order of a Markov chain.

3. State the stochastic matrix.

4. Define diffusion equation.

5. Is Wiener process, a Gaussian process? Justify your answer.

6. Define sample paths.

7. Define Markov renewal branching process.

8. State the Bellman–Harris process.

9. What is meant by Poisson queue?

10. Define birth and death process.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If state j is persistent non-null, then prove that, as $n \to \infty$.

(i) $p_{jj}^{(nt)} \to t/\mu_{jj}$ when state j is periodic with period t and

(ii) $p_{jj}^{(n)} \to 1/\mu_{jj}$ when state j is a periodic

Or

(b) Let $\{x_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution $\Pr(X_0 = i) = \frac{1}{3}, i = 0,1,2$.

Find:

(i) $\Pr(X_2 = 2, x_1 = 1 | x_0 = 2)$

(ii) $\Pr(X_2 = 2, x_1 = 1, x_0 = 2)$.

12. (a) A coin is tossed, p being the probability of head in a toss. Let $\{x_n, n \geq 1\}$ have the two states 0 or 1 according as the accumulated number of heads and tails in n tosses are equal or unequal. Show that the states are transient when $p \neq 1/2$, and persistent null when $p = 1/2$.

Or

(b) Prove that the internal between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter τ has a negative exponential distribution with near $1/\tau$.

13. (a) Let $\{X(t), t \geq 0\}$ be a Wiener process with $\mu = 0$ and $x(0) = 0$. Find the distribution of T_{a+b}, for $0 < a < a + b$.

Or

(b) If $X(t)$, with $X(0)$ and $\mu = 0$, is a Wiener process, show that $Y(t) = \sigma \times (t/\sigma^2)$ is also a Wiener process. Find its covariance function.

14. (a) Find the waiting time density and expected waiting time for $M/M(1,b)/1$ model.

Or

(b) Prove that $P_n(s) = P_{n-1}(P(s))$

15. (a) Derive the Fokker–Planck equation.

Or

(b) Find the moments of the distribution of the waiting time T.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Ergodic theorem.

17. Find the differential equation of pure death process. If the process starts with i individuals, then find the mean and variance of the number $N(t)$ present at time t.

18. If $m = 1, \sigma^2 < \infty,$ then prove that

(a) $\lim_{n \to \infty} n \Pr(x_n > 0) = 2/\sigma^2$

(b) $\lim_{n \to \infty} E\left[\frac{X_n}{n} \Big/ x_n > 0\right] = \frac{\sigma^2}{2}$

(c) $\lim_{n \to \infty} \Pr\left(\frac{X_n}{n} > u \Big/ X_n > 0\right) = \exp\left(\frac{-2u}{\sigma^2}\right), u \geq 0.$

19. Prove that the generating function

$F(t, s) = \sum_{k=0}^{\infty} \Pr\{X(t) = k\} s^k$ of an age-dependent branching process $\{X(t), t \geq 0\}$; $x(0) = 1$ satisfies the integral equation $F(t, s) = [1 - G(t)]s + \int_0^t [F(t - u, s] dG(u).$

20. Derive Pollaczek-Khinchine formula.

D–5523

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Fourth Semester

GRAPH THEORY

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define an induced sub graph of a graph with an example.

2. Give an example of a closed walk of even length which does not contain a cycle.

3. Define Ramsey numbers.

4. Define centre of a tree.

5. Define chromatic number of a graph.

6. What is meant by a critical graph?

7. State the Kuratowski graph.

8. Define a critical graph. Give an example.

9. Define directed graph.

10. Define minimum in degree and maximum outdegree.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain adjacency matrix and incidence matrix with examples.

Or

(b) If k-regular bipartite graph with $k > 0$ has bipartition (x, y), then prove that $|x| = |y|$.

12. (a) Prove that $C(G)$ is well defined.

Or

(b) If G is a k-regular bipartite graph with $k > 0$ then prove that G has a perfect matching.

13. (a) State and prove the Berge theorem.

Or

(b) Let G be a connected graph that is not an odd cycle, prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree atleast two.

14. (a) Prove that the complete graph k_5 is non planar.

Or

(b) Prove that every critical graph is a block.

15. (a) If two digraphs are isomorphic then prove that the corresponding vertices have the same degree pair.

Or

(b) If a digraph D is strongly connected then prove that D contains a directed closed walk containing all its vertices.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that $\tau(k_n) = n^{n-2}$.

17. State and prove Brook's theorem.

18. Prove that every planar graph is 5- colourable.

19. State and prove Euler's theorem.

20. Prove that the edges of a connected graph G can be oriented so that the resulting digraph is strongly connected if and only if every edge of G is contained in atleast one cycle.

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D–5524

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours　　　　　　　　Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define convex set. Give an example.

2. Define the normed dual.

3. Define bounded linear map.

4. Define inner product space.

5. Let X, Y be metric spaces. If $f : X \to Y$ is continuous and $g : X \to Y$ is closed, then prove that $f + g : X \to Y$ is closed.

6. Define completely continuous linear transformation.

7. Define adjoint and self-adjoint operators.

8. Show that the vector space of all null sequences c_0 is not reflexive.

9. Define weak convergence in a Hilbert space.

10. State the closed graph theorem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If (X,d) and (Y,d') are metric spaces and $f : X \to Y$, then prove that f is continuous if and only if $f^{-1}(F)$, where F is any closed set in Y, is closed set in X.

Or

(b) If a Cauchy sequence (x_n) in a normed space has a convergent subsequence (x_{n_k}), then prove that (x_n) is convergent.

12. (a) Prove that the linear functional f on the normed linear space X is bounded if and only if it is continuous.

Or

(b) If x and y are two elements in a normed linear space X, then prove that $\left| \|x\| - \|y\| \right| \leq \|x - y\|$.

13. (a) Prove that the Schwarz inequality $\|< x, y >\| \leq < x, x >^{1/2} . < y, y >^{1/2}$ for all x, y in an inner product space X.

Or

(b) Prove that the inner product is jointly continuous. In particular $y_n \to y \Rightarrow < x, y_n > \to < x, y >$.

14. (a) Prove that an orthonormal set in an inner product space is linearly independent.

Or

(b) Let the dual space of the normed linear space X be $\tilde{X}$. Prove that if $\tilde{X}$ is separable then X is separable

15. (a) Let A be a linear transformation on the finite dimensional space X. Prove that A is completely continuous.

Or

(b) Suppose $A : X \to Y$. If A is completely continuous then prove that the range of A, $R(A)$ is separable.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that all compact sets are countably compact.

17. In the normed linear space X, suppose that $\{x_\alpha\}$ is summable to $x \in X$, where α runs through an index set $\wedge$. Prove that all but a countable number of the x_α must be zero.

18. If A is completely continuous then prove that its conjugate map A' is completely continuous.

19. State and prove Hahn-Banach theorem.

20. State and prove open mapping theorem.

D–5525

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define simple root.

2. What is meant by rate of convergence of iterative method.

3. Define spectral radius of a matrix A.

4. What is meant by interpolating polynomial $P(x)$?

5. Write the Hermite interpolating polynomial.

6. State any two properties of cubic spline interpolation.

7. Write the general Euler's formula.

8. Write down the formula for numerical differentiation of y with respect to x once.

9. Define mesh points.

10. What is the advantages of Runge-Kutta method.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find a real root $f(x) = x^3 - 5x + 1 = 0$ in the interval $(0,1)$. Calculate the first six iterations only.

Or

(b) Perform one iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$. Use initial approximation $P_0 = 0.5, q_0 = 0.5$.

12. (a) Solve the following equations by using Gauss elimination method :

$x + y + z = 6$

$3x + 3y + 4z = 20$

$2x + y + 3z = 13$

Or

(b) Using the Jacobi method find all the eigen values and the corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

13. (a) From the following $f(x, y)$ data, find $f(0.25, 0.75)$ using linear interpolation.

y \ x	0	1
0	1	1.414214
1	1.732051	2

Or

(b) Find the least square approximation of second degree for the following data:

x	–2	–1	0	1	2
$f(x)$	15	1	1	3	19

14. (a) A differentiation rule of the form $hf'(x_2) = \alpha_0 f(x_0) + \alpha_1 f(x_1) + \alpha_2 f(x_3) + \alpha_3 f(x_4)$ where $x_j = x_0 + jh,\ j = 0,1,2,3,4$ is given. Determine the values of α_0, α_1, α_2 and α_3 so that the rule is exact for a polynomial of degree 4 and find an expression for the round-off error in calculating $f'(x_2)$.

Or

(b) Using the following data, find $f'(6.0)$, error = $o(h)$ and $f''(6.3)$, error = $o(h^2)$.

x	6.0	6.1	6.2	6.3	6.4
$f(x)$	0.1750	–0.1998	–0.2223	–0.2422	–0.2596

15. (a) Find the appropriate value of $I = \int_0^1 \frac{\sin x}{x} dx$ using mid point rule and two point open type rule.

Or

(b) Evaluate $\int_0^\infty (3x^3 - 5x + 1)e^{-x}\,dx$ using the Gauss-Laguerre two point formula.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Obtain the complex root of the equation $f(z) = z^3 + 1 = 0$ correct to eight decimal places. Use the initial approximation to a root as $(x_0, y_0) = (0.25, 0.25)$.

17. Find all the roots of the polynomial equation $x^3 - 4x^2 + 5x - 2 = 0$ using the Graeffe's root squaring method.

18. Given the following values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
–1	1	–5
0	1	1
1	3	7

estimate the values of $f(-0.5)$ and $f(0.5)$ using the Hermite interpolation.

19. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$ using backward Euler method.

20. Solve the initial value problem $u'' = (1 + t^2)\, u$, $u(0) = 1$, $u'(0) = 0$, $t \in [0, 0.4]$ by using Runge-kutta method with $h = 0.2$.

D–5526

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2022.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018-19 Academic Year onwards)

Time : Three hours | Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Let A and B be independent events with $P(A) = 0.7, P(B) = 0.2$. Find $P(A \cap B)$.

2. Prove that the expected value of the product of two random variables is equal to the product of their expectations plus their covariance.

3. Define a negative binomial distribution.

4. Write down the moment generating function of $Y = \sum_{i=1}^{n} X_i$.

5. If the p.d.f of X is $f(x) = \begin{cases} 2xe^{-x^2}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$. Determine the p.d.f of $y = x^2$.

6. If the random variable x has a Poisson distribution such that $\Pr(x = 1) = \Pr(x = 2)$, , then find $\Pr(x = 4)$.

7. Let $\overline{X}$ be the mean of a random sample of size 5 from a normal distribution with $\mu = 0$ and $\sigma^2 = 125$. Determine c so that $P(\overline{X} < c) = 0.90$.

8. Define covariance in distribution.

9. When we say that a sequence of random variables $X_1, X_2, ...$ converges in probability to a random variable X?

10. Let Zn be $\chi^2(n)$. Find the mean and variance of Zn.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Chebyshev's inequality.

Or

(b) Two cards are drawn successively and without replacement from an ordinary deck of playing cards. Compute the probability of drawing :

(i) two hearts

(ii) A heart on the first draw, a club on the second draw.

12. (a) Let $f(x_1, x_2) = \frac{1}{16}, x_1 = 1, 2, 3, 4$ and $x_2 = 1, 2, 3, 4$ and zero elsewhere be the joint p.d.f. of X_1 and X_2. Show that X_1 and X_2 are independent.

Or

(b) Let $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ be the p.d.f of X. Find the distribution function and the p.d.f. of $y = \sqrt{x}$.

13. (a) Let X have a Poisson distribution with $\mu = 100$. Use Chebyschev's inequality to determine a lower bound for $P_r(75 < x < 125)$.

Or

(b) If the correlation coefficient ρ of X exists then prove that $-1 \le \rho \le 1$.

14. (a) Let $\overline{X}$ denote the mean of a random sample of size 36 from an exponential distribution with mean 3. Find $P(2.5 \le \bar{x} \le 4)$.

Or

(b) Let X and Y be random variables with $\mu_1 = 1$, $\mu_2 = 4$, $\sigma_1^2 = 4$, $\sigma_2^2 = 6$, $S = 1/2$. Find the mean and variance of $z = 3x - 2y$.

15. (a) Let X have the uniform distribution over the interval $\left(-\pi/2, \pi/2\right)$. Show that $y = \tan x$ has a Cauchy distribution.

Or

(b) Let $F_n(y)$ be the distribution function of a random variable y_n whose distribution depends upon the positive integer n. Prove that the sequence $y_n, n = 1, 2, \ldots$ converges in probability to the constant c if and only if the limiting distribution of y_n degenerate at $y = c$.

SECTION C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let X have the p.d.f $f(x) = \begin{cases} \frac{x+2}{18}, & -2 < x < 4 \\ 0, & \text{elsewhere} \end{cases}$. Find $E(X), E\left[(X+2)^3\right]$ and $E\left[6X - 2(x+2)^3\right]$.

17. Find the m.g.f of a normal distribution and hence find the mean and variance of a normal distribution.

18. Let the random variables X and Y have the joint p.d.f $f(x, y) = \begin{cases} x + y, & 0 < x < 1; 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

Compute the correlation coefficient of X and Y.

19. Derive the p.d.f of the beta distribution with parameters α and β.

20. State and prove central limit theorem.

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