D–1437

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

First Semester

ALGEBRA – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define the greatest common divisor.

2. Define a subgroup of G.

3. Find the product of the permutation, $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}$.

4. Define a normalizer of a group.

5. State the Pigeonhole principle.

6. Define integral domain.

7. What is meant by the associates of ring?

8. Define principal ideal ring.

9. Define unique factorization domain.

10. Define the integer monic.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If G is a finite group, then prove that $o(a)/o(G)$.

Or

(b) If 'a' is relatively prime to b, but a/bc, then prove that a/c.

12. (a) Prove that $n(k) = 1 + p + \ldots + p^{k-1}$.

Or

(b) Prove that every permutation is the product of its cycle.

13. (a) Prove that the number of non isomorphic abelian groups of order p^n, p is a prime, equals the number of partitions of n.

Or

(b) Show that the commutative ring D is an integral domain if and only if for $a,b,c \in D$ with $a \neq 0$, the relation $ab = ac$ implies that $b = c$.

14. (a) If R is a commutative ring with unit element and M is an ideal of R, then M is a maximal is an ideal of R if and only if R/M is a field.

Or

(b) State and prove Fermat's theorem.

15. (a) If R is integral domain, then prove that $R[x]$ is integral domain.

Or

(b) State and prove Gaus's lemma.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If H and K are finite subgroups of G of order $O(H)$ and $O(K)$, respectively, then prove that $O(HK) = \frac{O(H) \cdot O(K)}{O(H \cap K)}$.

17. Let G be a group and suppose that G is the internal direct product of $N_1, N_2, \ldots, N_n$. Let $T = N_1 \times N_2 \times \ldots \times N_n$, then G and T are isomorphic.

18. Prove that a finite integral domain is a field.

19. (a) If $f(x), g(x)$ are two non zero elements of $F[x]$, then prove that $\deg(f(x) \cdot g(x)) = \deg f(x) + \deg g(x)$.

    (b) State and prove the division algorithm.

20. Prove that $J[i]$ is a Euclidean ring.

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31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define ordered set.

2. What do you mean by absolute value?

3. Write a short notes on metric space.

4. Write the statement of Weierstrass theorem.

5. Define subsequence.

6. What do you mean by complete metric space?

7. Define derived set.

8. What do you mean by Rearrangement?

9. What is meant by discontinuity of first kind?

10. Define uniformly continuous function.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that every neighbourhood is an openset.

Or

(b) If K_n is a sequence of compact sets in X such that $K_n \supset K_{n+1}$, $n = 1,2...$ and if $\lim_{n\to\infty}$ diam $K_n = 0$, then prove that $\bigcap_{n=1}^{\infty} K_n$ consists of exactly one point.

12. (a) State and prove Root test.

Or

(b) Suppose :

(i) $\sum_{n=0}^{\infty} a_n$ converges absolutely

(ii) $\sum_{n=0}^{\infty} a_n = A$

(iii) $\sum_{n=0}^{\infty} b_n = B$

(iv) $C_n = \sum_{k=0}^{n} a_k b_{n-k}$, $n = 0,1,2...$

Prove that $\sum_{n=0}^{\infty} C_n = AB$.

13. (a) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Prove that $f(X)$ is compact.

Or

(b) Let f be monotonic on (a,b). Prove that the set of points of (a,b) at which f is discontinuous is at most countable.

14. (a) Prove that a set S in R^n is closed if and only if it contains all its adherent points.

Or

(b) Let f be defined on an open interval (a,b) and assume that for some C in (a,b) we have $f'(c) > 0$ (or) $f'(c) = +\infty$. Prove that there is a 1–ball $B(c) \subseteq (a,b)$ in which $f(x) > f(c)$ if $x > c$, and $f(x) < f(c)$ if $x < c$.

15. (a) State and prove Rolle's theorem.

Or

(b) Assume f' exists and is monotonic on an open interval (a,b). Then prove that f' is continuous on (a,b).

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Bolzano-Weierstrass theorem.

17. Let S be a subset of R^n. Prove that the following statement are equivalent :

(a) S is compact

(b) S is closed and bounded

(c) Every infinite subset of S has an accumulation point in S.

18. State and prove chain rule for derivatives.

19. State and prove Intermediate value theorem.

20. State and prove Inverse function theorem.

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31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Find all solutions of $y'' + y' - 2y = 0$.

2. State the existence theorem of linear equation with constant coefficients.

3. Determine whether the functions $\phi_1(x) = \cos x$ and $\phi_2(x) = \sin x$, $x \in (-\infty, \infty)$ are linearly dependent or independent.

4. Find all real-valued solutions of the equation $y^{(4)} + y = 0$.

5. Verify that the function $\phi_1(x) = x^3$, $x > 0$ satisfies the equation $x^2 y'' - 7xy' + 15y = 0$.

6. Prove that $P_n(-1) = (-1)^n$.

7. Define regular singular and irregular singular points.

8. How would you demonstrate the Bessel function of order α of the first kind?

9. Find all real valued solutions of the equation $y' = 3y^{2/3}$.

10. State Lipschitz condition.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find all solutions of the equation $y'' - 7y' + 6y = \sin x$.

Or

(b) Consider the equation $y^{(5)} - y^{(4)} - y' + y = 0$.

(i) Compute fine linearly independent solutions.

(ii) Compute the Wronskian of the solutions found in (i).

12. (a) One solution of $x^2 y'' - 2y = 0$ on an interval $0 < x < \infty$ is $\phi_1(x) = x^2$. Find all solutions of $x^2 y'' - 2y = 2x - 1$ on the interval $0 < x < \infty$.

Or

(b) Show that $\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0$, $(n \neq m)$.

13. (a) Find all solutions of the equation $x^2 y'' - (2 + i) xy' + 3iy = 0$ for $|x| > 0$.

Or

(b) Find the singular points of the equation $(1 - x^2) y'' - 2xy' + 2y = 0$ and determine whether regular singular points or not.

14. (a) Obtain two linearly independent solutions of the following equation which are valid near $x = 0$, $x^2 y'' + 2x^2 y' - 2y = 0$.

Or

(b) With the usual notations, prove that $x^{1/2} J_{-1/2}(x) = \frac{\sqrt{2}}{\sqrt{\left(\frac{1}{2}\right)}} \cos x$.

15. (a) Find the solution of $y' = 2y^{1/2}$ passing through the point (x_0, y_0), where $y_0 > 0$. Also find all solutions of this equation passing through $(x_0, 0)$.

Or

(b) Consider the problem $y' = 1 - 2xy,\ y(0) = 0$.

(i) Since the differential equation is linear, an expression can be found for the solution. Find it.

(ii) Consider the above problem on $R : |x| \le \frac{1}{2}$, $|y| \le 1$, if $f(x, y) = 1 - 2xy$, show that $|f(x, y)| \le 2$, $((x, y)$ in $R)$, and their graphs remain in R.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

17. Let $L(y) = y^{(n)} + a_1 y^{(n-1)} + ... + a_n y = 0$ on an interval I containing a point x_0. Prove that for all x in I.
$\|\phi(x_0)\| e^{-K|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{K|x-x_0|}$, where
$K = 1 + |a_1| + |a_2| + ... + |a_n|$.

18. With the usual notations, prove that $J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{x}{2}\right)^{2m}$.

19. (a) State the theorem of "Necessary and sufficient condition for an equation to be exact".

(b) Find an integrating factor of $(e^y + xe^y)dx + xe^y dy = 0$ and hence solve it.

20. Show that a function ϕ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ in an interval I if and only if it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)\, dt$.

D–1440

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31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2025.

First Semester

TOPOLOGY – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Prove that a countable union of countable sets in countable.

2. Define basis for a topology.

3. Prove that every finite point set in a Hausdorff space X is closed.

4. Define standard bounded metric.

5. Let A be a connected subspace of X. Prove that if $A \subset B \subset \overline{A}$, then B is also connected.

6. Define locally path connected at x.

7. Prove that the image of a compact space under a continuous map is compact.

8. Define isolated point of x.

9. Define normal space.

10. Define completely regular.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies $\mathcal{T}$ and $\mathcal{T}'$, respectively, on X. Then prove that the following are equivalent.

(i) Z' is finer than Z.

(ii) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.

Or

(b) Let X be a topological space. Then prove the following conditions hold :

(i) ϕ and X are closed.

(ii) Arbitrary intersections of closed sets are closed.

(iii) Finite unions of closed sets are closed.

12. (a) State and prove the pasting lemma.

Or

(b) Let $\{X_\alpha\}$ be an indexed family of spaces; let $A_\alpha \subset X_\alpha$ for each α. If ΠX_α is given either the product or the box topology, then prove that $\Pi \overline{A}_\alpha = \overline{\Pi A_\alpha}$.

13. (a) Prove that a finite Cartesian product of connected spaces is connected.

Or

(b) State and prove Intermediate value theorem.

14. (a) State and prove Extreme value theorem.

Or

(b) Let X be locally compact Hausdorff; let A be a subspace of X. If A is closed in X or open in X_1 then prove that A is locally compact.

15. (a) Prove that a subspace of a regular space is regular; a product of regular spaces is regular.

Or

(b) Prove that every compact Hausdorff space is normal.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Rules for constructing continuous functions.

17. Prove that if L is a linear continum in the order topology, than L is connected, and so are intervals and rays in L.

18. State and prove the tube lemma.

19. Let X be a metrizable space. Then prove the following are equivalent.

(a) X is compact.

(b) X is a limit point compact.

(c) X is sequentially compact.

20. State and prove the Urysohn lemma.

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D–1441

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31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Second Semester

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a subspace of a vector space. Give an example.

2. When will you say that a vector space V is said to be finite dimensional?

3. Define dual space.

4. Define an algebraic extension of F.

5. What is meant by normal extension of F?

6. What is a singular element?

7. Define the companion matrix of $f(x)$.

8. Define the following terms :

   (a) Symmetric matrix;

   (b) Skew-symmetric matrix.

9. If N is a normal linear transformation and if $vN = 0$ for $v \in V$, then prove that $vN^* = 0$.

10. Define a finite filed.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) With the usual notations, prove that $L(S)$ is a subspace of V.

Or

(b) State and prove the Schwarz inequality.

12. (a) If $p(x) \in F[x]$ and if K is an extension of F, then prove that for any element $b \in K$, $p(x) = (x-b)q(x) + p(b)$ where $q(x) \in K[x]$ and where $\deg q(x) = \deg p(x) - 1$.

Or

(b) Show that it is impossible to trisect 60° by ruler and compass.

13. (a) If V is finite-dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

Or

(b) If $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{pmatrix} \in F_3$, then prove that $A^3 - 6A^2 + 11A - 6 = 0$.

14. (a) If T in $A_F(V)$ has as minimal polynomial $p(x) = q(x)^e$, where $q(x)$ is a monic, irreducible polynomial in $F[x]$, then prove that a basis of V over F can be found in which the matrix of T is of the form

$$\begin{pmatrix} C(q(x)^{e_1}) & & & \\ & C(q(x)^{e_2}) & & \\ & & \ddots & \\ & & & C(q(x)^{e_r}) \end{pmatrix}$$

where $e = e_1 \geq e_2 \geq \ldots . \geq e_r$.

Or

(b) If F is of characteristic O and if S and T, in $A_F(V)$, are such that $ST - TS$ commutes with S, then prove that $ST - TS$ is nilpotent.

15. (a) Show that interchanging two rows of A changes the sign of its determinant.

Or

(b) Let F be a finite field with q elements and suppose that $F \subset K$ where K is also a finite field. Prove that K has q^n elements where $n = [K : F]$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If A and B are finite-dimensional subspaces of a vector space V, then prove that $A + B$ is finite-dimensional and $d(A + B) = \dim(A) + \dim(B) - \dim(A \cap B)$.

17. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Also prove that $[L : F] = [L : K][K : F]$.

18. State and establish the fundamental theorem of Galois theory.

19. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

20. (a) Show that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

(b) If N is normal and $AN = NA$, then prove that $AN^* = N^*A$.

D–1442

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31122

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Second Semester

ANALYSIS – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours — Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define the Riemann integral of *f* over $[a,b]$.

2. Define equicontinuous family of functions.

3. Let e^x be defined on R'. Prove that $\lim_{x\to+\infty} x^n e^{-x} = 0$ for every n.

4. State the Dirichlet Kernel.

5. If $0 < t < 2\pi$, then prove that $E(it) \neq 1$.

6. Define a measurable set.

7. Define the characteristic function χ_A of the set *A*.

8. Define a simple function.

9. State the Lebesgue convergence theorem.

10. When will you say that a sequence $\langle f_n \rangle$ of measurable functions is said to be converge to f in measure?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $f_1, f_2 \in \mathcal{R}(\alpha)$ on $[a,b]$ then prove that

(i) $f_1 + f_2 \in \mathcal{R}(\alpha)$;

(ii) $cf \in \mathcal{R}(\alpha)$;

(iii) $\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$;

(iv) $\int_a^b cf d\alpha = c \int_a^b f d\alpha$.

Or

(b) State and prove the fundamental theorem of calculus.

12. (a) Show that compactness is really needed in Dini's theorem. Justify your answer by means of example.

Or

(b) State and prove Arzele-Ascoli theorem.

13. (a) Suppose Σc_n converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n$ $(-1 < x < 1)$. Prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

Or

(b) State and prove the Parseval's theorem.

14. (a) Let $\langle E_n \rangle$ be an infinite decreasing sequence of measurable sets, that is a sequence with $E_{n+1} \subset E_n$ for each n. Let mE_1 be finite. Prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} mE_n$.

Or

(b) State and prove the Lusin's theorem.

15. (a) Let ϕ and ψ be simple functions which vanish outside a set of finite measure. Prove that $\int (a\phi + b\psi) = a\int \phi + b\int \psi$. Also prove $\int \phi \geq \int \psi$ if $\phi \geq \psi$ almost everywhere.

Or

(b) State and establish the bounded convergence theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If γ' is continuous on $[a,b]$, then prove that γ is rectifiable and length of $\gamma = \wedge(\gamma) = \int_a^b |\gamma'(t)| dt$.

17. State and prove the Stone-Weierstrass theorem.

18. For the gamma function, prove that the functional equation $\Gamma(x+1) = x\Gamma(x)$ holds if $0 < x < \infty$. Also show that $\log \Gamma$ is convex on $(0, \infty)$. Further if f is a positive function on $(0, \infty)$ such that (a) $f(x+1) = xf(x)$; (b) $f(1) = 1$; (c) $\log f$ is convex, then prove that $f(x) = \Gamma(x)$.

19. Prove that the outer measure of an interval is its length.

20. State and prove the Fatou's lemma. Also deduce that monotone convergence theorem.

D–1443

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31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Second Semester

TOPOLOGY — II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State Tychonoff Theorem.

2. When the two compactification of X are equivalent?

3. Define locally finite with an example.

4. Prove that every metrizable space is para compact.

5. Define completion.

6. Define totally bounded with an example.

7. Define point-open topology.

8. State Ascoli's theorem.

9. Prove that any open subspace Y of a Baire space X is itself a Baire space.

10. Let X be a space having finite dimension. If Y is a closed subspace of X, then prove that Y has finite dimension and $dim Y \leq dim X$.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a completely regular space. Then prove that there exists a compactification Y of X having the property that every bounded continuous map $f : X \to \mathbb{R}$ extends uniquely to a continuous map of Y into $\mathbb{R}$.

Or

(b) Prove that every paracompact Hausdorff space X is normal.

12. (a) Let X be the product space $X = \prod X_\alpha$; Let x_n be a sequence of points of X. Then prove that $X_n \to x$ if and only if $\prod_\alpha (x_n) \to \prod_\alpha (x)$ for each α.

Or

(b) Prove that a metric space (X, d) is compact if and only if it is complete and totally bounded.

13. (a) Let X be locally compact Hausdorff. Let $\mathcal{C}(X, Y)$ have the compact open topology. Then prove that the map $\mathcal{C}$: $X_X \mathcal{C}(X, Y) \to Y$ defined by the equation $\mathcal{C}(x, f) = f(x)$ is continuous.

Or

(b) Show that the general version of Ascoli's theorem implies the classical version when X is Hausdorff.

14. (a) Let $C_1 \supset C_2 \supset \ldots$ be a nested sequence of nonempty closed sets in the complete metric space X. If *diam* $C_n \to 0$, then prove that $\bigcap C_n \neq \phi$.

Or

(b) Show that every locally compact Hausdorff space is a Baire Space.

15. (a) Show that $\bigcap U_n$ consists of nowhere-differentiable function.

Or

(b) Prove that any compact subspace X of $\mathbb{R}^2$ has topological dimensions atmost 2.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Tietze extension theorem.

17. State and prove Nagata-Smirnov metrization theorem.

18. Let X be regular. Then prove the following conditions on X are equivalent :

Every open covering of X has a refinement that is :

(a) An open covering of X and countably locally finite.

(b) A covering of X and locally finite.

(c) A closed covering of X and locally finite.

(d) A closed covering of X and locally finite.

19. State and prove Baire Category Theorem.

20. Let $X = Y \cup Z$, where Y and Z are closed subspaces of X having finite topological dimension. Then prove that $dim X = max\{dim Y, dim Z\}$.

D–1444

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31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Pfaffian differential equation.

2. What is meant by orthogonal trajectories?

3. Eliminate the arbitrary function f from the equation $z = xy + f\left(x^2 + y^2\right)$.

4. Define complete solution of partial differential equation.

5. Find a complete integral of the equation.

$$(p+q)(z-xp-yq)=1$$

6. Define integral transform.

7. Write short note on axial symmetry.

8. Define Helmholtz's equation.

9. State the exterior Dirichlet problem.

10. Show that $y = f(x+ct)+g(x-ct)$ is a solution of the one-dimensional wave equation.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the integral curves of the equation.

$$\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$$

Or

(b) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 tan^2 \alpha$ of its intersections with the family of planes parallel to $z = 0$.

12. (a) If u is a function of x, y and z which satisfies the partial differential equation.

$$(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial z} = 0$$

Show that u contains x, y and z only in combinations $x + y + z$ and $x^2 + y^2 + z^2$.

Or

(b) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$.

13. (a) Explain the Jacobi's method.

Or

(b) Find a particular integral of the equation

$(D^2 - D')z = A\,cos(lx + my)$ where A, l, m are constants.

14. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

Or

(b) Solve the equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

15. (a) If $\rho > 0$ and $\psi(r)$ is given by equation $\int_V \frac{\rho(r')dr'}{|r - r'|}$ where the volume V is bounded. Prove that $\lim_{r \to \infty} r\psi(r) = M$ where $M = \int_V \rho(r')dr'$.

Or

(b) Find the potential function $\psi(x, y, z)$ in the region $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ satisfying the conditions :

(i) $\psi = 0$ on $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$

(ii) $\psi = f(x, y)$ on $z = c$, $0 \le x \le a$, $0 \le y \le b$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Verify that the equation $z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$ is integrable and find its primitive.

17. Find the general solution of the differential equation.

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = (x + y)z.$$

18. Find the complete integral of the equation $p^2x + q^2y = z$.

19. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

20. The points of trisection of a string are pulled aside through a distance $\in$ on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest.

D–1445

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Find the arc length of the curve.

2. Define the tension of the curve.

3. Write down the intrinsic equations of the curve.

4. Define the following terms :

   (a) Direction coefficients;

   (b) Direction ratios.

5. Write a short notes on isometric correspondence.

6. State the canonical equations for geodesics.

7. Write down the second fundamental form a surface.

8. Define an umbilic.

9. Define the characteristic point.

10. What is meant by the osculating developable?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) With the usual notations, prove that $[\vec{r}', \vec{r}'', \vec{r}'''] = \kappa^2 \tau$.

Or

(b) If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$.

12. (a) Define the anchor ring. Also find the area of the anchor ring.

Or

(b) On a paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes z = constant.

13. (a) Derive the Christoffel symbols of the first kind.

Or

(b) Show that . $\kappa_g = [\vec{N}, \vec{r}', \vec{r}''] = \dot{s}^{-3}[\vec{N}, \dot{\vec{r}}', \ddot{\vec{r}}'']$

14. (a) Prove that the principle curvatures are given by the roots of the equation

$$\kappa^2(EG - F^2) - \kappa(EN + GL - 2FM) + LN - M^2 = 0.$$

Or

(b) Derive the Rodrigue's formula.

15. (a) Prove that a developable consists of two sheets, which are tangents to the edge of regression along the sharp edge.

Or

(b) State and prove the Monge's theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and establish the Serret-Frenet formula.

17. Show that the intrinsic equations of the curve given by $x = ae^u \cos u$, $y = ae^u \sin u$, $z = be^u$ are $\kappa = \dfrac{\sqrt{2}a}{s\left(2a^2 + b^2\right)^{1/2}}$,

$\tau = \dfrac{b}{s\left(2a^2 + b\right)^{\frac{1}{2}}}$.

18. State and prove the Gauss-Bonnet theorem.

19. (a) Enumerate the Dubin indicatrix.

(b) State and prove the Euler's theorem.

20. Show that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

D–1446

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Draw the network defined by $N = \{1, 2, 3, 4, 5\}$ and $A = \{(1, 2), (1, 3), (2, 3), (2, 5), (3, 4), (3, 5), (4, 2), (4, 5)\}$.

2. What is meant by enumeration of cuts?

3. Define total float.

4. The following table provides the associated activities and their durations. Construct the project nework.

Activity	Predecessor(s)	Duration (days)
A	—	10
B	—	7
C	A	5
D	C	3
E	D	2
F	B, E	1
G	E, F	14

5. Define surplus variable with an example.

6. What is meant by optimality condition?

7. Find the value of the game for the payoff matrix.

	B_1	B_2	B_3	B_4
A_1	8	–2	9	–3
A_2	6	5	6	8
A_3	–2	4	–9	5

8. Define Lagrange Multipliers.

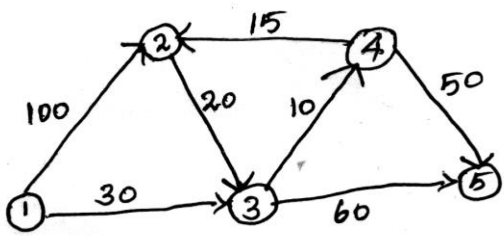
9. Write down the KKT necessary conditions for minimization problem.

10. Define Separable.

PART B — (5 × 5 = 25 marks)

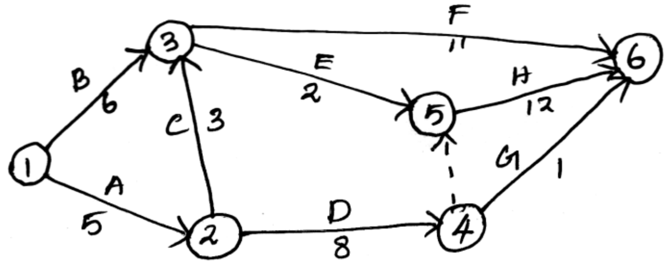
Answer ALL questions, choosing either (a) or (b).

11. (a) The network in figure gives the permissible routes and their lengths in miles between city 1 (node 1) and four other cities (nodes 2 to 5). Determine the shortest routes between city 1 and each of the remaining four cities.



Or

(b) Compute the floats for the noncritical activities of the network in the given figure.



12. (a) Solve the game graphically. The payoff is for player A.

	B1	B1
A1	5	8
A2	6	5
A3	5	7

Or

(b) Explain the Floyd's Algorithm.

13. (a) Determine and classify (as feasible and infeasible) all the basic solutions of the following system of equations.

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}.$$

Or

(b) Explain the Bounded Variables Algorithm.

14. (a) Determine the constrained extreme point for the function minimize $z = x_1^2 + x_2^2 + x_3^2$

Subject to

$g_1(x) = x_1 + x_2 + 3x_3 - 2 = 0$

$g_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0$.

Or

(b) Write down the Newton-Raphson Method.

15. (a) Show that a necessary condition for X_0 to be an extreme point of $f(X)$ is that $\nabla f(X_0) = 0$.

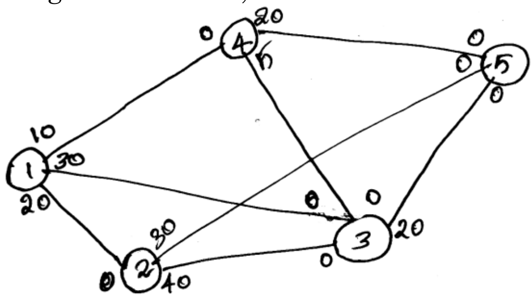
Or

(b) Explain the Golden Section Method.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. From the given network, determine the maximal flow.



17. Solve the game by Linear Programming :

	B_1	B_2	B_3	B_4
A_1	3	–2	1	2
A_2	2	3	–3	0
A_3	–1	2	–2	2
A_4	–1	–2	4	1

18. Solve the LPs by he revised Simplex Method :

Max $z = 5x_1 + 4x_2$

Subject to

$6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$

$x_2 \leq 2$

and $x_1, x_2 \geq 0$.

19. Solve the LPs by Separable Programming.

Max $z = x_1 + x_2^4$

Subject to

$3x_2 + 2x_2^2 \leq 9$

$x_1, x_2 \geq 0$.

20. Solve the Quadratic Programming :

Max $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to

$x_1 + 2x_2 \leq 2$

$x_1, x_2 \geq 0$.

D–1447

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. State the Euclid's lemma.

2. Define the Mobius function $\mu(n)$.

3. If f is multiplicative then prove that $f(1)=1$.

4. State the Liouville's function $\lambda(n)$.

5. Write down the Bell series of f modulo p.

6. Write a short notes on the big on notation.

7. Find out whether are congruence $2x \equiv 3 \pmod 4$ has an integral solution or not. Justify your answer.

8. State the Lagrange theorem.

9. Write down the principle of cross-classification theorem.

10. Determine the value of $\left(\frac{7}{11}\right)$ and $\left(\frac{22}{11}\right)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\sum_{d/n} \phi(d) = n$ if $n \geq 1$.

Or

(b) With the usual notations, prove that $\log n = \sum_{d/n} \wedge(d)$ if $n \geq 1$.

12. (a) State and establish the generalized Mobius inversion formula.

Or

(b) If f and g are arithmetical function, then prove the following :

(i) $(f + g)' = f' + g'$;

(ii) $(f * g)' = f' * g + f * g'$;

(iii) $\left(f^{-1}\right)' = -f' * (f * f)^{-1}$, provided that $f(1) \neq 0$.

13. (a) If $x \geq 0$, prove that $\sum_{n \leq x} \frac{1}{n} = \log x + c + o\left(\frac{1}{x}\right)$.

Or

(b) Prove that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$.

14. (a) If $a \equiv b(\mathrm{mod}\, m)$ and $\alpha \equiv \beta(\mathrm{mod}\, m)$, then prove the following :

(i) $ax + \alpha y = bx + \beta y(\mathrm{mod}\, m)$ for all integers x and y.

(ii) $a\alpha \equiv b\beta(\mathrm{mod}\, m)$.

(iii) $a^n = b^n(\mathrm{mod}\, m)$ for every positive integer n.

Or

(b) Solve the congruence $25x \equiv 15(\mathrm{mod}\, 120)$.

15. (a) Prove that the product formula for Euler's totient can be derived from the cross-classification principle.

Or

(b) Determine whether 219 is a quadratic residue or nonresidue mod 383.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove the fundamental theorem of arithmetic.

17. (a) For $n \geq 1$, prove that $\phi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$.

(b) For $n \geq 1$, prove that $\sigma_x^{-1}(n) = \sum_{d/n} d^{\alpha} \mu(d) \mu\left(\frac{n}{d}\right)$.

18. (a) For all $x \geq 1$, prove that $\left|\sum_{n \leq x} \frac{\mu(n)}{n}\right| \leq 1$, with equality holding only if $x < 2$.

(b) State and establish the Legendre's identity.

19. (a) State and prove the Wolstenholme's theorem.

(b) State and establish the Chinese Remainder Theorem.

20. State and prove the Gauss' lemma.

D–1448

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define a Stochastic graph.

2. State Basic limit theorem of renewal theory.

3. What is meant by time-dependent Poisson processes?

4. Define a Wiener Process.

5. Define Bienayame-Galton-Watson Process.

6. What is meant by Probability of Extinction?

7. Define traffic intensity.

8. Draw the state-transition-rate diagram for M/M/1 queue.

9. Write down the Erlang's second formula.

10. What do you mean by expected busy period for [M/M(1, b)/1]?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain Chapman-Kolmogorov equation.

Or

(b) State and prove additive properties of Poisson process.

12. (a) Prove that state j is persistent if and only if $\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty$.

Or

(b) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that $P\{N(S) = K \mid N(t) = n\} = \binom{n}{k}(S \mid t)^k [1 - (S \mid t)]^{n-k}$.

13. (a) Let $\{X(t), 0 \le t \le T\}$ be a Wiener process with $X(0) = 0$ and $\mu = 0$. Let $M(T)$ be the maximum of $X(t)$ in $0 \le t \le T$ (i.e.) $M(T) = \underset{0 \le t \le T}{Max}\, X(t)$. Then prove that for any $a > 0$ $P\{M(T) \ge a\} = 2P\{X(T) \ge a\}$.

Or

(b) Let $\{X(t), t \ge 0\}$ be a Wiener process with $\mu = 0$ and $X(0) = 0$. Find the distribution of T_{a+b} for $0 < a < a + b$.

14. (a) If $M = 1$, $\sigma^2 < \infty$, then prove that (i) $\lim_{n \to \infty} nP\{X_n > 0\} = \frac{2}{\sigma^2}$ (ii) $\lim_{n \to \infty} E\left\{\frac{X_n}{n} \Big/ X_n > 0\right\} = \frac{\sigma^2}{2}$.

Or

(b) Show that the generating function $F(t, s) = \sum_{K=0}^{\infty} p\{X(T) = K\} S^K$ of an age-dependent branching process $\{X(t), t \geq 0\}$, $X_0 = 1$ satisfies the integral equation

$$F(t, s) = [1 - G(t)]S + \int_0^t P[F(t-u, s)]dG(u).$$

15\. (a) Derive M | M | S | S queueing model.

Or

(b) Illustrate $M^{(X)} | M | 1$ queueing model.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16\. Let $\{X_n, n \geq 0\}$ be a Markov chain having state space $S = \{1, 2, 3, 4\}$ and transition matrix $P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Show that state 1 and 2 are ergodic.

17\. Elaborate Yule-Furry Process.

18\. Show that $P_n(S) = P_{n-1}(P(S))$ and $P_n(S) = P(P_{n-1}(S))$ in branching processes.

19\. Describe M | M | 1 | K queuing model.

20\. Discuss the Birth and Death processes in queueing theory.

D–1449

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define an adjacency matrix of a graph. Give an example.

2. Define a block of a graph with an example.

3. Find the number of different perfect matching in K_{2n}.

4. Define the Ramsey numbers. Give an example.

5. Draw a 3-chromatic graph.

6. Write a short notes on Hajo's conjecture.

7. Prove that $K_{3,3}$ is nonplanar.

8. What is meant by bridge of a graph?

9. Define the following terms :
   (a) Directed graph
   (b) Tournament of a graph.

10. Write down the conservation condition of networks.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define an isomorphism between two graphs with an example. Also prove that $\delta \leq \frac{2\varepsilon}{\nu} \leq \Delta$.

Or

(b) If e is a link of G, then prove that $\tau(G) = \tau(G-e) + \tau(G \cdot e)$.

12. (a) If G is a simple graph with $\gamma \geq 3$ and $\delta \geq \frac{\nu}{2}$, then prove that G is Hamiltonian.

Or

(b) State and prove the Berge theorem.

13. (a) If $\delta > 0$, then prove that $\alpha' + \beta' = \gamma$.

Or

(b) Define the edge chromatic number of a graph. Also prove that $\chi' = \Delta$ if G is bipartite graph.

14. (a) If G is simple graph, then prove that

$\pi_k(G) = \pi_k(G-e) - \pi_k(G \cdot e)$ for any edge e of G.

Or

(b) State and prove the Euler's formula for a connected plane graph.

15. (a) Prove that a digraph D contains a directed path of length $\psi - 1$.

Or

(b) Define flow and cut in a network with an example for each. Also for any flow f and any cut $(S, \overline{S})$ in N, prove that $Val\,f = f^{+}(S) - f^{-}(S)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that a graph is bipartitle if and only if it contains no odd cycle.

17. Show that a nonempty connected graph is Eulerian if an only it it has no vertices of odd degree.

18. With the usual notations, prove that $\gamma(k, k) \geq 2^{k/2}$.

19. State and prove the Brook's theorem.

20. State and establish the five-colour theorem.

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D–1450

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, DECEMBER 2025.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define Banach space.

2. What do you mean by equivalent norm?

3. Define linear operator.

4. Define algebraic reflexive.

5. What do you mean by dual space?

6. Write the statement of polarization identity.

7. Define Annihilator.

8. Define Hilbert space.

9. What do you mean by Reflexivity of Hilbert Space?

10. Define Self adjoint operator.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) State and prove Translation invariance property.

Or

(b) State and prove Riesz's lemma.

12. (a) Let $T: X \to Y$ and $S: Y \to Z$ be bijective linear operators, where X, Y, Z are vector spaces. Prove that the inverse $(ST)^{-1}: Z \to X$ of the product ST exists and $(ST)^{-1} = T^{-1}S^{-1}$.

Or

(b) Prove that the dual space X^1 of a normed space X is a Banach space.

13. (a) Let $T : \mathfrak{D}(T) \to Y$ be a linear operator, where $\mathfrak{D}(T) \subset X$ and X, Y are normed spaces. Prove that T is continuous if T is a continuous at a single point.

Or

(b) State and prove Parallelogram law.

14. (a) Prove that for any subset $M \neq \phi$ of a Hilbert Space H, the span of M is dense in H if and only if $M^{\perp} = \{0\}$.

Or

(b) Let H_1, H_2 be Hilbert spaces, $S: H_1 \to H_2$ and $T = H_1 \to H_2$ bounded linear operator's and α be any scalar. Prove that

(i) $(S+T)^* = S^* + T^*$

(ii) $(\alpha T)^* = \overline{\alpha} T^*$.

15. (a) State and prove self-adjointness of Product theorem.

Or

(b) Prove that in a normed space X, $x_n \xrightarrow{w} x$ if and only if

(i) the sequence $(\|x_n\|)$ is bounded.

(ii) for every element f of a total subset $M \subset X^1$ we have $f(x_n) \to f(x)$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that every finite dimensional subspace Y of a normed space X is complete. In particular, every finite dimensional normed space is complete.

17. If Y is a Banach space, then prove that $B(X, Y)$ is a Banach Space.

18. Prove that the Hilbert adjoint operator T^* of T is exists, is unique and is a bounded linear operator with norm $\|T^*\| = \|T\|$.

19. State and prove Uniform boundedness theorem.

20. State and prove closed graph theorem.

D–1451

Sub. Code
31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Write down the secant method.

2. Define the efficiency index of an iterative-method.

3. Define the spectral norm.

4. Write down the condition for convergence of an iterative method.

5. Define a cubic spline.

6. State the Weierstrass approximation theorem.

7. Define error of approximation.

8. Write down the formula for the Newton-Cotes methods.

9. When will you say that a method is said to be convergent?

10. Define periodic stability.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the number of real and complex roots of the polynomial equation $P_3(x) = x^3 - 5x + 1 = 0$ using Sturm sequences.

Or

(b) Use synthetic division and perform two iterations of the Birge-Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$. Use the initial approximation $p_0 = 0.5$.

12. (a) Determine the condition number of the matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$ using the maximum absolute row sum norm.

Or

(b) Find the largest eigen value in modulus and the corresponding eigen vector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using the power method.

13. (a) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data :

x :	1	2	4	8
$f(x)$:	3	7	21	73

Hence, estimate the values of $f(3)$ and $f(7)$.

Or

(b) Obtain the least squares polynomial approximation of degree one and two for $f(x) = x^{1/2}$ on [0, 1].

14. (a) The following data for the function $f(x) = x^4$ is given.

x :	0.4	0.6	0.8
$f(x)$:	0.0256	0.1296	0.4096

Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation. Compare with the exact solution. Obtain the bound on the truncation errors.

Or

(b) Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$, using (i) Trapezoidal rule, (ii) Simpson's rule. Obtain a bound for the errors.

15. (a) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ using the mid-point method, with $h = 0.2$, over the interval $[0, 1]$. Use the Taylor series method of second order to compute $u(0.2)$. Determine the percentage relative error at $t = 1$.

Or

(b) Find the three term Taylor series solution for the third order initial value problem $W''' + WW'' = 0$, $W(0) = 0$, $W'(0) = 0$, $W''(0) = 1$. Find the bound on the error for $t \in [0, 0.2]$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find all the roots of the polynomial $x^3 - 4x^2 + 5x - 2 = 0$ using the Graeffe's root squaring method.

17. Using the Jacobi method find all the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}.$$

18. The following data for a function $f(x, y)$ is given

$y \mid x$	0	1	3
0	1	2	10
1	2	4	14
3	10	14	28

Construct the bivariate interpolating polynomial and hence find $f(0.5, 0.5)$.

19. Find the quadratic formula

$$\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$$

which is exact for polynomials of highest possible degree. Then use the formula on $\int_0^1 \frac{dx}{\sqrt{x - x^3}}$ and compare with the exact value.

20. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$. Use the second order implicit Runge-Kutta method.

D–1452

Sub. Code
31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2025.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Prove that the probability of the null set is zero.

2. Define a continuous random variable.

3. Let X have the p.d.f. $f(x) = 4x^3,\ 0 < x < 1,$ $= 0,$ elsewhere find $E(X)$.

4. Write down the formula for the correlation coefficient of X and Y.

5. Define the binomial distribution. Also find the mean of the binomial distribution.

6. State any two properties of the normal curve.

7. What do you mean by the change-of-variable technique?

8. Define the t-distribution.

9. Define the concept of "converges in distribution".

10. What do you mean by degenerate distribution?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If C_1 and C_2 are subsets of the sample space X, then prove that

$$P(C_1 \cap C_2) \le P(C_1) \le P(C_1 \cup C_2) \le P(C_1) + P(C_2).$$

Or

(b) Let X be the p.d.f. $f(x) = \frac{x+2}{18}$, $-2 < x < 4$, zero elsewhere. find $E(X)$ and $E\left[(X+2)^3\right]$.

12. (a) Define Gamma p.d.f. Obtain its moment generating function. Deduce its mean and variance.

Or

(b) If the random variables X and Y have the joint p.d..f. $f(x, y) = \begin{cases} x + y, & 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$ Find the correlation coefficient between X and Y.

13. (a) Derive the moment generating function of the Poisson distribution. Deduce its mean, variance and μ_3.

Or

(b) Establish that for $N(\mu, \sigma^2)$ random variable X, $E|X - \mu| = 0.8$ S.D. nearly.

14. (a) Let X have the p.d.f. $f(x) = \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3, .., $ zero elsewhere. Find the p.d.f. of $Y = X^3$.

Or

(b) Let X be $N(0, 1)$. Use the moment generating function technique to show that $Y = X^2$ is $\chi^2(1)$.

15. (a) Find the mean and variance of $S^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2 / n$, where $X_1, X_2, ..., X_n$ is a random sample from $N(\mu, \sigma^2)$.

Or

(b) Let $F_n(u)$ denote the distribution function of a random variable U_n whose distribution depends upon the positive integer n. Further, let U_n converge in probability to the positive constant c and let $P_r(U_n < 0) = 0$ for every n. Prove that the random variable $\sqrt{U_n}$ converges in probability to $\sqrt{c}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Markoff's inequality and deduce Chebyshev's inequality. Show that Chebyshev's inequality cannot be improved.

17. (a) If the correlation coefficient ρ of x and y exist, then prove that $-1 \leq \rho \leq 1$.

(b) Show that the random variables X_1 and X_2 with joint probability density function

$$f(x_1, x_2) = \begin{cases} 12x_1x_2(1-x_2), & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

are stochastically independent.

18. (a) Find the mean and variance of a normal distribution.

(b) Derive the moment generating function of the Chi-square distribution. Deduce its mean and variance.

19. Derive the p.d.f. of F-distribution.

20. (a) State and prove the Central Limit Theorem.

(b) Let X be $\chi^2(50)$ random variable. Determine approximate value of $P_r(40 < x < 60)$.