D–2444

Sub. Code
31111

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

First Semester

ALGEBRA – I

(CBCS 2018 – 19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define an equivalence relation on a set.

2. Define a subgroup of G. Give an example.

3. What is meant by homomorphism of groups?

4. Define an even permutation. Give an example.

5. Write down the class equation of G.

6. Define a zero-divisor.

7. Define a prime element of R.

8. Define a Euclidean ring.

9. What is meant by Primitive Polynomial?

10. State the Eisenstein criterion theorem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) For any three sets A, B and C. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Or

(b) State and prove the Euler's theorem.

12. (a) If p is a prime number and $p|o(G)$, then prove that G has an element of order p.

Or

(b) State and prove third part of Sylow's theorem.

13. (a) Show that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

Or

(b) Define an integral domain. Also prove that a finite integral domain is a field.

14. (a) If U is an ideal of R and $I \in U$, prove that $U = R$.

Or

(b) Let R be a Euclidean ring. Suppose that for $a, b, c \in R$, $a|bc$ but $(a, b) = 1$. Prove that $a|c$.

15. (a) If $f(x), g(x)$ are two non zero element of $F[x]$, then prove that $\deg(f(x)\ g(x)) = \deg f(x) + \deg g(x)$.

Or

(b) If R is a unique factorization domain, then prove that $R[x]$ is also an unique factorization domain.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If H and K are finite subgroups of G of orders $O(H)$ and $O(K)$, respectively, then prove that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.

17. State and prove the Cayley's theorem.

18. Prove that every integral domain can be imbedded in a field.

19. (a) If p is a prime number of the form $4n+1$, then solve the congruence $x^2 \equiv -1 \mod p$.

(b) State and establish the Fermat's theorem.

20. State and prove the Gauss' lemma.

D–2445

Sub. Code
31112

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

First Semester

ANALYSIS – I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Is the set of all rational numbers countable? Justify your answer.

2. Define the cantor set. Give an example.

3. Define a Cauchy sequence.

4. Find the radius of convergence of the power series $\sum n^3 z^n$.

5. Let E be a non compact set in R^1. Prove that there exists a continuous function on E which is not bounded.

6. What is the difference between continuity and uniform continuity?

7. Define an adherent point. Give an example.

8. Examine for differentiability the function f given by

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

9. What is meant by local maximum and local minimum of a function?

10. State the partial derivative of the function.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $x \in R, y \in R$ and $x > 0$, then prove that there is a positive integer n such that $nx > y$.

Or

(b) Define the neighborhood of a point in a metric space. Also prove that every neighborhood is an open set.

12. (a) Prove that closed subsets of compact sets are compact.

Or

(b) Let P be a non empty perfect set in R^k. Prove that P is uncountable.

13. (a) State and prove Cauchy's condensation test.

Or

(b) State and establish the Ratio test for the convergence of a series.

14. (a) Define monotonically increasing function. Also prove that monotonic functions have no discontinuities of the second kind.

Or

(b) State and prove the representation theorem for open sets in the real line.

15. (a) State and prove the generalized mean-value theorem.

Or

(b) State and prove the implicit function theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. With the usual notations, prove that every k-cell is compact.

17. Define e and prove that it is not rational. Also prove that

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e .$$

18. If f is a continuous mapping of a compact metric space X into a metric space Y, then prove that f is uniformly continuous on X.

19. State and prove the Cantor intersection theorem.

20. State and establish the inverse function theorem.

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D–2446

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. What is an initial value problem for a second-order differential equation?

2. How to test linear independence?

3. Find the Wronskian of $\phi_1(x) = e^x$ and $\phi_2(x) = e^{2x}$.

4. State the existence theorem for n^{th} order ODE.

5. State the uniqueness theorem for n^{th} order ODE.

6. Define n^{th} Legendre polynomial $P_n(x)$.

7. Give an example of an equation with a regular singular point.

8. What is the standard form of the Bessel equation?

9. What is an exact differential equation.

10. Define Lipschitz condition.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the general solution of $y'' + w^2 y = 0$.

Or

(b) State and prove uniqueness theorem for 2nd order ODE.

12. (a) Compute the solution of the initial value problem $y'' + 10y = 0,\ y(0) = \pi,\ y'(0) = \pi^2$.

Or

(b) If $\phi_1, \phi_2, \ldots, \phi_n$ are any n solutions of the nth order linear homogeneous equation $L(y) = y^{(n)} + a, y(n-1) + \ldots. + a_n y = 0$, the prove that there are linearly independent iff $W(\phi_1, \phi_2, .. \phi_n)(x) \neq 0\ \forall x \in I$.

13. (a) Solve the differential equation $y''' - 3r_1 y'' + 3r_1^{\,2} y' - r_1^{\,2} y = 0$. Compute also the Wronskian of the three independent solutions.

Or

(b) Using power series method to solve the Legendre equation. $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$.

14. (a) Using power series method, find two linearly independent solution of $L(y) = y'' - xy = 0$.

Or

(b) Show that the coefficient of x^n in $P_n(x)$ is $\dfrac{(2n)!}{2^n (n!)^2}$.

15. (a) Consider the initial value problem $y' = 1 + xy, y(0) = 0$. Compute the first four approximations $\phi_0, \phi_{1,} \phi_2$ and ϕ_3.

Or

(b) Show that the function $f(x, y) = xy^2$ satisfies the Lipschitz condition on the rectangle $|x| \leq 1, |y| \leq 1$ but does not satisfy a lipschitz condition on a strip $|x| \leq 1$, $|y| < \infty$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let ϕ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0. Then prove that for all x in I $\|\phi(x_0)\| e^{-k|x-x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\| e^{-k|x-x_0|}$ where $\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2\right]^{1/2}, k = 1 + |a_1| + |a_2|$.

17. Find the solution $\phi(x)$ of the equation $\psi''' + \psi'' + y' + y = 1$ which satisfies $\psi(0) = 0$, $\psi'(0) = 1$, and $\psi''(0) = 0$ by the method of variation of parameters.

18. Find all solutions of the following equations:

(a) $x^2 y'' + xy' - 4y = x$ (b) $x^2 y'' + 2xy' - 6y = 0$.

19. Find the singular point of the equation $(1 - x^2)y'' - 2xy' + 2y = 0$ and determine whether they are regular singular point.

20. Let M and N be two real-valued functions which have continuous partial derivatives on $R: |x - x_0| \leq a$, $|y - y_0| \leq b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

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31114

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

First Semester

TOPOLOGY — I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Prove that a finite product of countable sets is countable.

2. Define the topology generated by the sub basis.

3. Prove that every finite point set in a Hausdorff space X is closed.

4. Define box topology.

5. The rationals Q, is it connected? Explain.

6. State Extreme value theorem.

7. Define one-point compactification.

8. Define regular space.

9. Prove that the space $\mathbb{R}_l$ is normal.

10. State Urysohn lemma.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if C is an infinite subset of $\mathbb{Z}_+$, then prove that C is countably infinite.

Or

(b) Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ', respectively, on X. Then prove that the following are equivalent

(i) Z' is finer than Z.

(ii) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}$ such that $x \in B' \subset B$.

12. (a) Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

Or

(b) Let $\{X_\alpha\}$ be an indexed finite of spaces ; let $A_\alpha \subset X_\alpha$ for each α. Prove that if $\prod X_\alpha$ is given either the product or the box topology, then $\prod \overline{A}_\alpha = \overline{\prod A_\alpha}$.

13. (a) Prove that a finite Cartesian product of connected spaces is connected.

Or

(b) Define the following terms with an example for each :

(i) Linear continuum ;

(ii) Path connected.

14. (a) Prove that every compact subspace of a Hausdorff space X is closed.

Or

(b) State and prove the Lebesgue number lemma.

15. (a) Let X be a topological space. Let one-point sets in X be closed. Then prove that X is regular if and only if given a point x of X and neighbourhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$.

Or

(b) Prove that every metrizable space is normal.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let B be a non empty set. Then prove that the following are equivalent :

(a) B is countable.

(b) There is a surjective function $f : \mathbb{Z}_{+} \to B$.

(c) There is an injectuve function $g : B \to \mathbb{Z}_{+}$.

17. State and prove Rules for constructing continuous functions.

18. Prove that the topologies on $\mathbb{R}^n$ induced by the euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

19. State and prove the tube lemma.

20. State and prove Urysohn metrization theorem.

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31121

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Second Semester

ALGEBRA – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. If V is a vector space over F, then prove that

   (a) $\alpha 0 = 0$ for $\alpha \in F$

   (b) $(-\alpha)v = -(\alpha v)$ for $\alpha \in F$, $v \in V$.

2. If $\dim_f v = m$, then find $\dim_f Hom(V,V)$.

3. Define splitting field over F.

4. For any $f(x), g(x) \in F[x]$ and any $\alpha \in F$, prove that $(f(x)+g(x))' = f'(x)+g'(x)$.

5. Define the fixed field of G.

6. Define the following terms :

   (a) Range of T

   (b) Rank of T.

7. Define a characteristic vector of T.

8. State the rational canonical form of T.

9. For all $A \in F_n$, prove that $(A')' = A$.

10. When will you say that the linear transformation is said to be unitary?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $v_1, v_2, ..., v_n$ is a basis v over F and if $w_1, w_2, ..., w_m$ in V are linearly independent over F, then prove that $m \leq n$.

Or

(b) With the usual notations, prove that $A(A(W)) = W$.

12. (a) Let V be a finite-dimensional inner product space. Prove that V has an orthonormal set as a basis.

Or

(b) If α, β are constructible numbers, show that $\alpha \pm \beta, \alpha\, \beta$ and α/β are constructible.

13. (a) If V is finite - dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

Or

(b) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

14. (a) Let V and W be two vector spaces over F and suppose that ψ is a vector isomorphism of V onto W. Suppose that $S \in A_F(V)$ and $T \in A_F(W)$ are such that for any $v \in V$, $(vS)\psi = (v\psi)T$, Prove that S and T have the same elementary divisions.

Or

(b) Show that the determinant of a triangular matrix is the product of its entries on the main diagonal.

15. (a) Define normal linear transformation. If N is normal and if $vN^k = 0$, then prove that $vN = 0$.

Or

(b) For every prime number p and every positive integer m, prove that there exists a field having p^m elements.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. (a) State and establish the Bessel's inequality.

(b) In V, prove that $\|u + v\|^2 + \|u - v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right)$.

17. Show that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

18. (a) Prove that the fixed field of G is a subfield of K.

(b) Show that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

19. Let $V = F^{(3)}$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of $T \in A(V)$ in the basis $v_1 = (1,0,0)$, $v_2 = (0,1,0)$, $v_3 = (0,0,1)$. Find the matrix of T in the basis $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ and $u_3 = (0,0,1)$.

20. If $T \in A(V)$ then prove that $T^* \in A(V)$. Also prove the following :

(a) $(T^*)^* = T$

(b) $(S + T)^* = S^* + T^*$;

(c) $(\lambda S)^* = \bar{\lambda} S^*$

(d) $(ST)^* = T^* S^*$; for all $S, T \in A(V)$ and all $\lambda \in F$.

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31122

DISTANCE EDUCATION

M.Sc. DEGREE EXAMINATION, MAY 2026.

Second Semester

Mathematics

ANALYSIS — II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define a refinement of P.

2. For each position $P = \{x_0, x_1, \cdots, x_n\}$ on $[a, b]$ and to each curve γ on $[a, b]$. Let the number $\wedge(P, r) = \sum_{i=1}^{n} \left| r(x_i) - r(x_{i-1}) \right|$. What does this $\wedge(P, r)$ represent.

3. Prove that limits cannot be interchanged using the double sequence $S_{m,\,n} = \frac{m}{m+n}$, $(m = 1, 2, \cdots; n = 1, 2, \cdots)$.

4. Define an orthogonal system and orthonormal on I.

5. State the Parseval's theorem.

6. Write down the countable subadditivity property.

7. Show that m^* is translation invariant.

8. Define a Borel set. Give an example.

9. Define Lebesgue measurable function.

10. With the usual notations prove that $|f| = f^+ + f^-$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

Or

(b) If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)| dt$.

12. (a) Let $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ (x real, n = 1, 2, ...). Prove that $\{f_n'\}$ does not converge to f' where $f(x) = \lim_{n \to \infty} f_n(x)$.

Or

(b) Show that there exists a real continuous function on the real line which is nowhere differentiable.

13. (a) Given a double sequence $\{a_{ij}\}$, $i = 1, 2, \ldots.,$ $j = 1, 2, \ldots.$ Suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i \ (i = 1, 2, \cdots)$ and Σb_i converges. Prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$

Or

(b) Derive the Stirling's formula.

14. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Prove that $m^*(\cup A_n) \leq \Sigma m^* A_n$.

Or

(b) State and prove the Egoroff's theorem.

15. (a) If f and g are bounded measurable functions defined on a set E of finite measure, then prove that $\int_E (af + bg) = a\int_E f + b\int f$. Also prove that $\int_{A \cup B} f = \int_A f + \int_B f$ if A and B are disjoint measurable sets of finite measure.

Or

(b) Let $\langle f_n \rangle$ be a sequence of measurable functions that converges in measure to f. Prove that there is a subsequence $\langle f_{n_k} \rangle$ that converges to f almost everywhere.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) State and prove the fundamental theorem of calculus.

(b) If $f \in \mathscr{R}(\alpha)$ on $[a, b]$, then prove that $|f| \in \mathscr{R}(\alpha)$ and $\left|\int_a^b f\,d\alpha\right| \leq \int_a^b |f|\,d\alpha$.

17. If f is a continuous complex function on $[a, b]$, then prove that there exists a sequence of polynomials P_n such that $\lim_{n \to \infty} P_n(x) = f(x)$ uniformly on $[a, b]$.

18. Define gamma function. Prove the following :

(a) The functional equation $\Gamma(x+1) = x\Gamma(x)$ holds if $0 < x < \infty$.

(b) $\Gamma(n+1) = n!$ for $n = 1, 2, \cdots$

(c) $\log \Gamma$ is convex on $(0, \infty)$.

(d) $\Gamma(i) = 1$.

19. Let f be an extended real-valued function whose domain is measurable. Prove that the following statements are equivalent :

(a) For each real number α the set $\{x : f(x) > \alpha\}$ is measurable.

(b) For each real number α the set $\{x : f(x) \geq \alpha\}$ is measurable.

(c) For each real number α the set $\{x : f(x) < \alpha\}$ is measurable.

(d) For each real number α the set $\{x : f(x) \leq \alpha\}$ is measurable.

20. (a) State and prove bounded convergence theorem.

(b) State and establish Lebesgue convergence theorem.

D–2450

Sub. Code
31123

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Second Semester

TOPOLOGY — II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. List some limitations of the Tietze extension theorem.

2. State Zorn's lemma.

3. Define countably locally finite.

4. Prove that every closed subspace of a para compact space is para compact.

5. Define evoluation map.

6. Define Pointwise bounded.

7. Define compact open topology.

8. Define Baire space with an example.

9. State Baire category theorem.

10. Define geometrically independent.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let $A \subset X$; let $f : A \to Z$ be a continuous map of A into the Hausdorff space Z. Then prove that there is at most one extension of f to a continuous function $g : \overline{A} \to Z$.

Or

(b) Prove that every regular Lindelof space is paracompact.

12. (a) Prove that Euclidean space $\mathbb{R}^k$ is complete in either of its usual metrics, the euclidean metric d or the square metric ρ.

Or

(b) Let X be a space, let (Y, d) be a metric space. If the subset F of $C(X, Y)$ is totally bounded under the uniform metric corresponding to d then prove that F is equi continuous under d.

13. (a) Prove that if X is locally compact, or if X satisfies the first countability axiom, then X is compactly generated.

Or

(b) Show that the general version of Ascolis theorem implies the classical version when X is Hausdorff.

14. (a) Prove that any open subspace V of a Baire space X is itself a Baire space.

Or

(b) Show that if every point x of X has a neighbourhood that is a Baire space than X is a Baire space.

15. (a) What is meant by the topological dimension of space X ? Is the interval $X = [0, 1]$ has topological dimension 1? Justify your answer.

Or

(b) Prove that given a finite set $\{x_1, x_2, \cdots x_n\}$ of points of $\mathbb{R}^N$ and given $\delta > 0$, there exists a set $\{y_1, \cdots y_n\}$ of points of $\mathbb{R}^N$ in general position in $\mathbb{R}^N$, such that $\left| x_i - y_i \right| < \delta$ for all i.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Tychonoff theorem.

17. Let X be a regular space with a basis $\mathcal{B}$ that is countably locally finite. Then prove that X is normal, and every closed set in X is G_f set in X.

18. State and prove Smirnov metrization theorem.

19. State and prove Ascoli's theorem.

20. Let $h:[0, 1] \to \mathbb{R}$ be a continuous function. Then prove that given $t > 0$, there is a function $g:[0, 1] \to \mathbb{R}$ with $\left| h(x) - g(x) \right| < \varepsilon$ for all x, such that g is continuous and nowhere differentiable.

D–2451

Sub. Code
31124

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018–19 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. What is meant by orthogonal trajectories?

2. Verify that the differential equation $\left(y^2 + yz\right)dx + \left(xz + z^2\right)dy + \left(y^2 - xy\right)dz = 0$ is integrate.

3. Eliminate the arbitrary function f from the equation $z = x + y + f(xy)$.

4. Define general solution of partial differential equation.

5. Find a complete integral of the equation $pq = 1$.

6. Write down the Kernal for the Fourier transform.

7. Find a particular integral of the equation $\left(D^2 - D^1\right)z = e^{2x+y}$.

8. Write short note on axial symmetry.

9. Write all possible solutions of the one-dimensional wave equation. Justify the feasible solution.

10. State the Interior Neumann problem.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the integral curves of the equation $\frac{dx}{y(x+y)+az} = \frac{dz}{x(x+y)-az} - \frac{dz}{z(x+y)}$.

Or

(b) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = 0$.

12. (a) Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$. Which contains the straight line $x + y = 0$, $z = 1$.

Or

(b) Show that the equations $xy = yq$, $z(xp + yq) = 2xy$ are compatible and solve them.

13. (a) Explain the Jacobi's method.

Or

(b) Find a particular integral of the equation $(D^2 - D^1)z = 2y - x^2$.

14. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

Or

(b) Solve the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

15. (a) Derive the general solution for one-dimensional wave equation.

Or

(b) Find the potential function $\psi(\rho, z)$ in the region $0 \le \rho \le 1$, $z \ge 0$ satisfying the conditions

(i) $\psi \to 0$ as $z \to \infty$

(ii) $\psi = 0$ on $\rho = 1$

(iii) $\psi = f(p)$ on $z = 0$ for $0 \le \rho \le 1$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Solve the equation $\frac{dx}{y + \alpha z} = \frac{dy}{z + \beta x} = \frac{dz}{x + \gamma y}$.

17. Find the solution of the equation $z = \frac{1}{2}\left(p^2 + q^2\right) + (p - x)(q - y)$ which passes through the x-axis.

18. Show that only integral surface of the equation $2q(z - px - qy) = 1 + q^2$. Which is circumscribed about the paraboloid $2x = x^2 + z^2$ is the enveloping cylinder which toches it along its section by the plane $y + 1 = 0$.

19. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

20. Determine the temperature $\theta(\rho, t)$ in the infinite cylinder $0 \leq \rho \leq a$ when the initial temperature is $\theta(\rho, 0) = f(p)$ and the surface $\rho = a$ is maintained at zero temperature.

D–2452

Sub. Code
31131

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define an osculating plane.

2. Write down the formula for the radius of spherical curvature.

3. Define a cylindrical helix and right circular helix.

4. Calculate the fundamental coefficients E, F, G for the paraboloid $\vec{r} = (u, v, u^2 - v^2)$.

5. Define an isometric surfaces.

6. Define geodesics.

7. State the whitehead theorem.

8. Write an expression for geodesic curvature K_g.

9. Define the Gaussian curvature.

10. Write down the characteristic line corresponding to the plane u.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $x = a \cosh\left(\frac{z}{a}\right)$, from the point $(a, 0, 0)$ to the point (x, y, z).

Or

(b) If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma \rho') + \frac{\rho}{\sigma} = 0$.

12. (a) Prove that the ratio of the curvature to the torsion is constant at all points.

Or

(b) Find the area of the Anchor ring.

13. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z = \text{constant}$.

Or

(b) Find a surface of revolution which is isometric with a region of the right helicoid.

14. (a) Prove that every helix on a cylinder is a geodesic.

Or

(b) State and prove the Gauss-Bonnet theorem.

15. (a) Derive the second fundamental form of a surface.

Or

(b) State and prove the Monge's theorem.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadratic surfaces $ax^2 + by^2 + cz^2 = 1$, $a'x^2 + b'y^2 + c'z^2 = 1$.

17. Show that the intrinsic equations of the curve given by $x = ae^u \cos u$, $y = ae^u \sin u$, $z = be^u$ are $K = \frac{\sqrt{2}a}{(2a^2+b^2)^{1/2}} \cdot \frac{1}{s}$, $\tau = \frac{b}{(2a^2+b^2)^{\frac{1}{2}}} \cdot \frac{1}{s}$.

18. Derive the geodesic differential equations.

19. Establish the Rodrigue's formula.

20. Show that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

D–2453

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31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Draw the network defined by $N = \{1, 2, 3, 4, 5, 6\}$, $A = \{(1, 2), (1, 5), (2, 3), (2, 4), (3, 4), (3, 5), (4, 3), (4, 6), (5, 2), (5, 6)\}$.

2. Define free float.

3. Define convex set.

4. Determine the saddle-point solution for the game :

	B_1	B_2	B_3	B_4
A_1	8	6	2	8
A_2	8	9	4	5
A_3	7	5	3	5

5. Define Slack Variable.

6. Write down the L.P.P. of the game theory.

7. What is meant by Lagrange multipliers?

8. Write down the KKT necessary conditions for maximization problem.

9. Define separable.

10. Write down the Red-flagging rule.

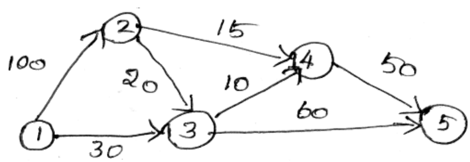
PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain the minimal spanning tree algorithm.

Or

(b) Determine the shortest routes between City 1 and each of the remaining four cities for the following figure :



12. (a) Determine the classify (as feasible and infeasible) all the basic solutions of the following system of equations.

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}.$$

Or

(b) Show that a necessary condition for X_0 to be an extreme point of $f(x)$ is that $\nabla f(x_0) = 0$.

13. (a) Determine the extreme points of the function $f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2 x_3 - x_1^2 - x_2^2 - x_3^2$.

Or

(b) Explain the Lagrangian method.

14. (a) Consider the problem

Minimize $f(X) = X_1^2 + X_2^2 + X_3^2$

Subject to $\begin{array}{l} g_1(X) = x_1 + x_2 + 3x_3 - 2 = 0 \\ g_2(X) = 5x_1 + 2x_2 + x_3 - 5 = 0 \end{array}$.

Determine the constrained extreme point.

Or

(b) Write the KKT necessary condition for the following problem.

Maximize $f(x) = x_1^3 - x_2^3 + x_1 x_3^2$

$$x_1 + x_2^2 + x_3 = 5$$

Subject to $5x_1^2 - x_2^2 - x_3 \geq 2$.

$$x_1,\ x_2,\ x_3 \geq 0$$

15. (a) Find the maximum of the following function :

$f(x) = \begin{cases} 3x, & 0 \leq x \leq 2 \\ \frac{1}{3}(-x + \infty), & 0 \leq x \leq 3 \end{cases}$ using dichotomous method.

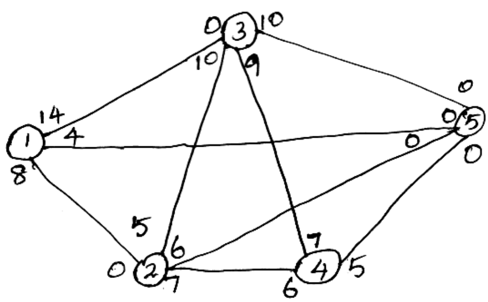
Or

(b) Explain the bounded variable algorithm.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Find the maximum flow and the optimum flow in each acr for the network in figure.



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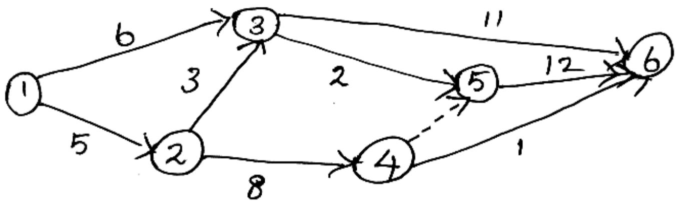
17. Determine critical path for the project network in the following figure. All the durations are in days.



18. Solve the following 2 × 4 games :

	B_1	B_2	B_3	B_4
A_1	2	2	3	–1
A_2	4	3	2	6

19. Solve the problem $g(x) = (3x-2)^2 (2x-3)^2$ by the Newton-Raphson method.

20. Solve the quadratic programming :

Maximize $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to $x_1 + 2x_2 \leq 2$ and

$x_1, x_2 \geq 0.$

D–2454

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours — Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define relatively prime.

2. Define Mobius function.

3. Define Convolution.

4. Define totally multiplicative.

5. Define reduced residue systems.

6. If $a \equiv b \pmod{m}$, then prove that $ac \equiv bc \pmod{mc}$.

7. Define Euler Totient function.

8. Define quadratic residue modulo.

9. Prove that $\left(\frac{7}{11}\right) = (-1)$.

10. Define Jacobi symbol.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find values of x and y to satisfy $43x + 64y = 1$.

Or

(b) Let $f(n)$ be a multiplicative function and $F(n) = \sum_{d/n} f(d)$. Prove that $F(n)$ is multiplicative.

12. (a) If f and g are arithmetic functions and β is a complex valued function then prove that $g(f\beta) = (g * f)(\beta)$.

Or

(b) For each positive integer n, prove that

$$d(n) = \prod_{i=1}^{K} (e_i + 1).$$

13. (a) Prove that any two reduced residue system modulo m have same number of elements.

Or

(b) Let a, b, c, d, x, y are integers

(i) If $a \equiv b \pmod{m}, c \equiv d \pmod{m}$, then prove that

(1) $ac \equiv bd \pmod{m}$

(2) $ax + cy \equiv bx + dy \pmod{m}$

(ii) $a \equiv b \pmod{m}$ and $d \mid m$, $d > 0$ then $a \equiv b \pmod{d}$

14. (a) If P and Q are two distinct odd integers and $P>1$, $Q>1$ such that $(Q, P)=1$, then prove that $\left(\frac{P}{Q}\right)\left(\frac{Q}{P}\right)=(-1)^{\frac{Q-1}{2}\cdot\frac{P-1}{2}}$.

Or

(b) If p is an odd prime, then prove that $\left(\frac{a}{p}\right)\equiv a^{\frac{p-1}{2}}$ (mod p).

15. (a) If m and n are relatively prime, then prove that $\varphi(m, n) = \varphi(m)\cdot\varphi(n)$.

Or

(b) If Q is an odd integer and $Q>1$, then prove that $\left(\frac{-1}{Q}\right)=(-1)^{\frac{Q-1}{2}}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Euclidean Algorithm.

17. All multiplicative functions forms a group with dirichlet product. Prove that they form a subgroup of all arithmetic function and not vanishing at 1.

18. State and prove Fermat's theorem.

19. State and prove Gauss's lemma.

20. Let p be an odd prime and a be any integer $\exists:(a, 2p)=1$, then prove that $\left(\frac{a}{p}\right)=(-1)^t$, where $t=\sum_{j=1}^{\frac{p-1}{2}}\left[\frac{ja}{p}\right]$. Also prove that $\left(\frac{2}{p}\right)=(-1)^{\frac{p^2-1}{8}}$.

D–2455

Sub. Code
31134

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. What is meant by Stochastic graph?

2. Write the statement First Entrance Theorem.

3. Define a Point Process.

4. Define a Wiener-Einstein Process.

5. Write down the formula forward and backward diffusion equation of the Wiener Process.

6. What is meant by Probability of Extinction?

7. Define rate equality principle.

8. Draw the State-transition-rate diagram for M/M/1 queue model.

9. Write down the Erlang's first formula.

10. Explain about the expected busy period.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Suppose that the probability of a dry day (State 0) following a rainy day (State 1) is Y_3 and that the probability of a rainy day following a dry day is Y_2. Given that May 1 is a dry day. Find the probability that (i) May 3 is a dry day (ii) May 5 is a dry day.

Or

(b) State and prove Additive Property of Poisson Process.

12. (a) If State K is a persistent null, then for every j $\lim_{n\to\infty} p_{jk}^{(n)} \to O$ and if state k is aperiodic, persistent non-null then $\lim_{n\to\infty} p_{jk}^{(n)} \to \frac{F_{jk}}{\mu_{kk}}$.

Or

(b) If $\{N(t)\}$ is a Poisson process then prove that the auto-correlation coefficient between $N(t)$ and $N(t+S)$ is $\left[\frac{t}{(t+S)}\right]^{\frac{1}{2}}$.

13. (a) Derive Kolmogorov equations for a Wiener Process.

Or

(b) Suppose that $\{X(t), 0<t\}$ is a Wiener process with $X(0)=0$ and $\mu=0$. Then prove that

$$P\{X(t)\leq x\}=P\{X(t)/\sigma\sqrt{t}\leq x/\sigma\sqrt{t}\}=\phi(x/\sigma\sqrt{t}).$$

14. (a) Show that the p.g.f. $R_n(S)$ of Y_n satisfies the recurrence relation $R_n(S)=SP(R_{n-1}(S))$, $P(S)$ being the p.g.f. of the offspring distribution.

Or

(b) Prove that the generating function $F(t, S) = \sum_{K=0}^{\infty} P\{X(t) = K\} S^K$ of an age-dependent branching process $\{X(t), t \geq 0\}$; $X_0 = 1$ satisfies the integral equation

$$F(t, S) = [1 - G(t)]S + \int_0^t P(F(t-u, S))\, dG(u).$$

15. (a) Explain the M/M/S queueing model.

Or

(b) Illustrate the moments of the distribution of the waiting time T for the system M/M (a, b)/1 model.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let $\{X_n, n \geq 0\}$ be a Markov chain having state space $S = \{1, 2, 3, 4\}$ and transition matrix $P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$.

Show that State 1 and State 2 is ergodic.

17. Explain about the Birth and Death Process.

18. Show that $P_n(S) = P_{n-1}(P(S))$ and $P_n(S) = P(P_{n-1}(S))$.

19. State and prove Yaglom's theorem.

20. Illustrate M/M (a, b)/1 queueing model.

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D–2456

Sub. Code
31141

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define an isomorphic graphs. Give an example.

2. What is meant by cut edge of a graph G?

3. Define a block of a graph with an example.

4. Draw the Herschel graph.

5. Define a perfect matching. Give an example.

6. Define the Ramsey numbers. Also find $r(k, 2)$.

7. Write a short notes on Hajos conjecture.

8. State the Jordan curve theorem in the plane.

9. Draw all the tournaments on four vertices graphs.

10. Write down the conservation condition and capacity constraint in the networks.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define the degree of a vertex in graph G with an example. Also prove that $\delta \leq 2\varepsilon / V \leq \Delta$.

Or

(b) If G is a tree, then prove that $\varepsilon = V - 1$.

12. (a) If e is a link of G, then prove that $\tau(G) = \tau(G - e) + \tau(G \cdot e)$.

Or

(b) With the usual notations, prove that $\mathrm{K} \leq \mathrm{K}' \leq \delta$.

13. (a) State and prove the Berge theorem.

Or

(b) If $\delta > 0$ then prove that $\alpha' + \beta' = \nu$.

14. (a) What is meant by critical graph? Also prove that every critical graph is a block.

Or

(b) Define planar graph with an example. Also prove that K_5 is non planar.

15. (a) If two bridges overlap, then prove that either they are skew or else they are equivalent 3-bridges.

Or

(b) Prove that a digraph D contains a directed path of length $\psi - 1$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that a graph is bipartite if and only if it contains no odd cycle.

17. State and establish the Dirac theorem. Also prove that $C(G)$ is well defined.

18. State and prove the Vizing's theorem.

19. Prove that every planar graph is 5-vertex colourable.

20. State and prove the max-flow min-cut theorem.

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D–2457

Sub. Code
31142

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define the following terms :

   (a) Normed Linear Space.

   (b) Convex set.

2. Define a Schauder basis with an example.

3. What is meant by equivalent norms?

4. Define a sequentially commact.

5. When will you say that the operator T is said to be bounded?

6. Define an orthogonality.

7. Define direct sum.

8. Write down the Pythagorean relation.

9. Define the following terms :

   (a) Hermitian

   (b) Normal.

10. What is meant by weak convergence?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the completion theorem.

Or

   (b) State and prove the compactness theorem.

12. (a) State and prove the Riesz's lemma.

Or

   (b) If a normed space X is finite dimensional, then prove that every linear operator on X is bounded.

13. (a) State and prove Schwarz inequality.

Or

   (b) If Y is a closed subspace of a Hilbert space H, then prove that $Y = Y^{\perp\perp}$.

14. (a) State and prove Hahn-Banach theorem on normed spaces.

Or

   (b) Prove that every Hilbert space H is reflexive.

15. (a) State and prove unitary operator theorem.

Or

(b) In a normed space X, prove that $x_n \xrightarrow{w} x$ if and only if

(i) The sequence $(\| x_n \|)$ is bounded.

(ii) For every element f of a total subset $M \subset X'$ we have $f(x_n) \to f(x)$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let T be a linear operator. Prove the following :

(a) The range $\mathfrak{R}(T)$ is a vector space.

(b) If dim $\mathfrak{D}(T) = n < \infty$, then dim $\mathfrak{R}(T) \leq n$

(c) The null space $\mathcal{N}(T)$ is a vector space.

17. Let $T : \mathfrak{D}(T) \to Y$ be a linear operator, where : $\mathfrak{D}(T) \subset X$ and X, Y are normal spaces. Prove that

(a) T is continuous if and only if T is bounded.

(b) If T is continuous at a single point, it is continuous.

18. If Y is a Banach space, then prove that $B(X, Y)$ is a Banach space.

19. State and prove the Baire's Category theorem.

20. State and establish the open mapping theorem.

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31143

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. What is meant by iteration function?

2. State the Sturm's theorem.

3. Define the term Pivot element.

4. State the properties of Quadratic Spline Interpolation.

5. Define the best approximation.

6. What is meant by Richardson's extrapolation?

7. Write down the formula for the Newton-Cotes methods.

8. Define the local truncation error.

9. Write down the equation of mid-point method.

10. When will you say that a method is said to be consistent?

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) How should the constant α be chosen at ensure the fastest possible convergence with the iteration formula $x_{n+1} = \dfrac{\alpha x_n + x_n^{-2} + 1}{\alpha + 1}$.

Or

(b) Use synthetic division and perform two iterations of the Birgevieta method to find the smallest positive root of the polynomial $P_3(x) = 2x^3 - 5x + 1 = 0$. Use the initial approximation $p_o = 0.5$. Also obtain the deflated polynomial.

12. (a) Determine the condition number of the matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$. Using the maximum absolute row sum norm.

Or

(b) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ by the data :

x :	1	2	4	8
$f(x)$:	3	7	21	73

Hence, estimate the values of $f(3)$ and $f(7)$.

13. (a) Find the least squares polynomials approximation of degree one and two for $f(x) = x^{\frac{1}{2}}$ on [0, 1].

Or

(b) The following table of values is given :

x :	–1	1	2	3	4	5	7
$f(x)$:	1	1	16	81	256	625	2401

Using the formula $f'(x_1)=\frac{(f(x_2)-f(x_0))}{(2h)}$ and the Richardson extrapolation, find $f'(3)$.

14. (a) Find the Jacobian matrix for the system of equations

$f_1(x, y)=x^2+y^2-x=0$;

$f_2(x, y)=x^2-y^2-y=0$ at the point (1, 1).

Or

(b) Solve the initial value problem $u'=-2tu^2$, $u(0)=1$ with $h=0.2$, on the interval [0, 0.4], using the backward Euler method.

15. (a) Find the three term Taylor series solution for the third order initial value problem :

$\begin{matrix} W'''+WW''=0, & W(0)=0, \\ W'(0)=0, & W''(0)=1 \end{matrix}$. Find the bound on the error for $t\in[0, 0.2]$.

Or

(b) Explain Stability analysis.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find all the roots of the polynomials $x^3-4x^2+5x-2=0$. Using the Graeffe's root squaring method.

17. Using the Jacobi method find all the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}.$$

18. Given the following values of $f(x) = \ln x$, find the approximate value of $f'(2.0)$ and $f''(2.0)$, using the methods based on linear and quadratic interpolation. Also obtain an upper bound on the error.

i :	0	1	2
x_i :	2.0	2.2	2.6
f_i :	0.69315	0.78846	0.95551

19. Find the quadratic formula

$$\int_0^1 f(x) = \frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$$

Which is exact for polynomials of highest possible degree. Then use the formula on $\int_0^1 \frac{dx}{\sqrt{x - x^3}}$ and compare with the exact value.

20. Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$, with $h = 0.2$ on the interval [0, 0.4]. Use the second order implicit Runge-Kutta method.

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31144

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2026.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. What is meant by the probability set function?

2. Let X have the p.d.f $f(x)=\begin{cases} \frac{x}{6} & x=1,2,3 \\ 0 & \text{elsewhere} \end{cases}$. Find $E(X^3)$.

3. Define correlation coefficient of X and Y.

4. If the moment generating function of a random variable X is $M(t)=\left(\frac{2}{3}+\frac{1}{3}e^t\right)^5$, then find mean and variance.

5. Let X be $n(2, 25)$. Find $P(0<X<10)$.

6. Define a random sample.

7. What is meant by F-distribution?

8. Write down the properties of $\overline{X}$ and S^2 distribution.

9. Define a limiting distribution.

10. Let Y have a binomial distribution with $n = 50$ and $p = -\frac{1}{25}$. Find $P(Y \leq 1)$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If C_1 and C_2 are subsets of ζ, then prove that $P(C_1 \cup C_2) = P(C_1) + P(C_2) - P(C_1 \cap C_2)$.

Or

(b) Let X have the p.d.f. $f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$.

Then find (i) $E(X)$ (ii) $E(X^2)$ (iii) $E(6X + 3X^2)$.

12. (a) A hand of 5 cards is to be dealt at random and without replacement from an ordinary deck of 52 playing cards. Find the conditional probability of an all-space hand (C_2), relative to the hypothesis that there are atleast 4 spaces in the hand (C_1).

Or

(b) The binomial distribution with p.d.f.

$$f(x) = \begin{cases} \binom{7}{x}\left(\frac{1}{2}\right)^x \left(1 - \frac{1}{2}\right)^{7-x} & x = 0, 1, \cdots 7 \\ 0 & \text{elsewhere} \end{cases}.$$

Compute m.g.f., mean, variance and $P(0 \leq X \leq 1)$.

13. (a) Let X have a gamma distribution with $\alpha = \frac{r}{2}$, where r is a positive integer, and $\beta > 0$. Define the random variable $Y = \frac{2X}{\beta}$. Find the p.d.f. of Y.

Or

(b) If the random variable X is $n(\mu, \sigma^2), \sigma^2 > 0$, then prove that the random variable $V = \frac{(X-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

14. (a) Define the t-distribution.

Or

(b) Let Y_1, Y_2, Y_3 be the order Statistics of a random sample of size 3 from a distribution having p.d.f. $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$, show that the p.d.f. of the sample range $Z_1 = Y_3 - Y_1$.

15. (a) Let $\overline{X}_n$ have the distribution function $F_n(\bar{x}) = \int_{-\infty}^{\bar{x}} \frac{1}{\sqrt{Y_n}\sqrt{2\pi}} e^{-nw^2/2} dw$. Find the limiting distribution of $\overline{X}_n$

Or

(b) Let Z_n be $\chi^2(n)$, then prove that the moment generating function of Z_n is $(1-2t)^{-\frac{n}{2}}$, $t < \frac{1}{2}$. The mean and the variance of Z_n are respectively, n and $2n$. The limiting distribution of the random variable $Y_n = \frac{(Z_n - n)}{\sqrt{2n}}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Chebychev's inequality.

17. Let the random variables X and Y have the joint p.d.f. $f(x, y)=\begin{cases} x+y & 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$. Compute the correlation coefficient of X and Y.

18. Let X and Y have a bivariate normal distribution with means μ_1 and μ_2, positive variances σ_1^2 and σ_2^2 and correlation coefficient P. Then prove that X and Y are stochastically independent if and only if $P=0$.

19. Let $X_1, X_2, \cdots X_n$ denote random variables that have means $\mu_1, \mu_2, \cdots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \cdots \sigma_n^2$. Let P_{ij}, $i \neq j$ denote the correlation coefficient of X_i and X_j and let $K_1, K_2, \cdots K_n$ denote real constants. Show taht the mean and the variance of the linear function $Y=\sum_{i=1}^{n} K_i X_i$ are respectively $\mu_Y=\sum_{i=1}^{n} K_i \mu_i$ and $\sigma_Y^2=\sum_{i=1}^{n} K_i^2\, \sigma_1^2+2 \sum\sum_{i<j} K_i K_j P_{ij} \sigma_i \sigma_j$.

20. Let $X_1, X_2, \cdots X_n$ denote the items of a random sample from a distribution that has mean μ and positive variance σ^2. Then prove that the random variable $Y_n=\dfrac{\left(\sum_{i=1}^{n} X_i-n\mu\right)}{\sqrt{n}\,\sigma}=\sqrt{n}\,\dfrac{\left(\overline{X}_n-\mu\right)}{\sigma}$ has a limiting distribution that is normal with mean zero and variance 1.
