R–2952		Sub. Code
		511201

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

RINGS AND FIELDS

(CBCS – 2018 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define commutative ring.

2. Define characteristics O.

3. Define imbedded.

4. Define principal ideal ring.

5. State Gauss lemma.

6. Define unique factorization domain.

7. Define splitting field.

8. Define simple extension.

9. Define fixed field.

10. Define solvable group.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If P is a prime number, prove J_P the ring of integers mod P, is a field.

Or

(b) Prove that any field is an integral domain.

12. (a) Let R be a Euclidean ring and let A be an ideal of R. Prove there exists an elements $a_0 \in A$ such that A consists exactly of all $a_0 x$ as x ranges over R.

Or

(b) Prove a Euclidean ring possesses a unit element.

13. (a) If $a \in K$ is algebraic of degree in over F, prove $[F(a):F] = n$.

Or

(b) If L is an algebraic extension of K and if K is an algebraic extension of F, prove L is an algebraic extension of F.

14. (a) If $p(x) \in F[x]$ and if K is an extension of F, prove for any element $b \in K$, $p(x) = (x-b)\,q(x) + p(b)$, where $q(x) \in K(x)$ and where $\deg q(x) = \deg p(x) - 1$.

Or

(b) Prove that if α, β are constructible, prove so are $\alpha \pm \beta, \alpha\,\beta$ and α / β (where $\beta \neq 0$).

15. (a) Prove $G(K, F)$ is a subgroup of the group of all automorphisms of K.

Or

(b) Prove K is a normal extension of F if and only if K is the splitting field A some polynomial over F.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If R is a ring, prove for all $a,b \in R$

(a) $a_0 = 0\,a = 0$

(b) $a(-b) = (-a)\,b = -(ab)$

(c) $(-a)(-b) = ab$. If in addition , R has a unit element 1, then

(d) $(-1)\,a = -a$

(e) $(-1)(-1) = 1$.

17. (a) Let R be a Euclidean ring. Suppose that for $a,b,c \in R$, a/bc but $(a,b) = 1$. Prove a/c.

(b) Find all the units in $J[i]$.

18. (a) State and prove Gauss's lemma.

(b) If R is an integral domain, prove so is $R[x]$.

19. (a) If $p(x)$ is a polynomial in $F(x)$ of degree $n \geq 1$ and is irreducible over F, prove that is an extension E of F, such that $[E, F]$, in which P has a not.

(b) If $f(x) \in F[x]$, prove that is a finite extension E of F in which $f(x)$ has a root.

20. (a) Using the Eisenstein criterion, prove that $x^4+x^3+x^2+x+1$ is irreducible over the field of rational numbers.

(b) If G is a solvable group and if $\overline{G}$ is a homomorphic image of G, prove $\overline{G}$ is solvable.

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R–2953

Sub. Code
511202

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

REAL ANALYSIS — II

(CBCS – 2018 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   (10 × 2 = 20)

Answer **all** questions.

1. Define Riemann-Stieltjes Integral.

2. If $f \in \mathfrak{R}(a)$ and $g \in \mathfrak{R}(a)$ on $[a, b]$, then prove that $fg \in \mathfrak{R}(\alpha)$.

3. Define uniform convergence with an one example.

4. Give an example of convergent series of continuous functions may have a discontinuous sum and explain it.

5. Define pointwise bounded on *E*.

6. Define equicontinuous on *E*.

7. If $0 < x < \frac{\pi}{2}$, prove that $\frac{2}{\pi} < \frac{\sin x}{x} < 1$.

8. Prove that $e^{x+y} = e^x \cdot e^y$, where $x, y \in \mathbb{R}$.

9. Show that the formula equation $\overline{|}(x+1) = x\overline{|}(x)$ holds if $\theta < x < \infty$.

10. Define orthonormal system of functions on $[a, b]$.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Prove that $\int_{-a}^{b} f\, d\alpha \leq \int_{a}^{b}$.

Or

(b) State and prove Integration by parts.

12. (a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a finite point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n (n = 1, 2, 3, ...)$. Then prove that $\{A_n\}$ converges, and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$.

Or

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

13. (a) Prove that every uniformly convergent sequence of bounded function is uniformly bounded.

Or

(b) Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$ converges uniformly in every bounded interval, but does not converge absolutely for any value of x.

14. (a) Given a double sequence $\{a_{ij}\}$, $i = 1, 2, 3, ...,$ $j = 1, 2, 3,...$ suppose that $\sum_{j=1}^{\infty} \left| a_{ij} \right| = b_i \, (i = 1, 2, 3, ...)$ and Σb_i converges. Then prove that $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}$.

Or

(b) Suppose $a_0, a_1, ..., a_n$ are complex numbers, $n \geq 1$, $a_n \neq 0$. $P(z) = \sum_{k=0}^{n} a_k z^k$. Then prove that $p(z) = 0$ for some complex number z .

15. (a) If, for some x, there are constants $\delta > 0$ and $M < \infty$ such that $\left| f(x+t) - f(x) \right| \leq M \left| t \right|$ for all $t \in (-\delta, \delta)$, the prove that $\lim_{N \to \infty} \delta_N (f; x) = f(x)$.

Or

(b) If f is continuous (with period 2π) and if $\in > 0$, then prove that there is a trignometric polynomial p such that $\left| p(x) - f(x) \right| < \in$ for all real x.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Show that $f \in \Re \; (\alpha)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition p such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

17. Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then prove that $\{f_n\}$ converges uniformly on $[a, b]$, to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x) \quad (a \le x \le b)$.

18. State and prove the Stone-Wierstrass theorem.

19. State and prove Parseval's theorem.

20. If f is a positive function on $(0, \infty)$ such that

    (a) $f(x+1) = xf(x)$

    (b) $f(1) = 1$

    (c) $\log f$ is convex.

    then prove that $f(x) = \Gamma(x)$.

R–2954

Sub. Code
511203

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2018 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define regular function.

2. Obtain the equations of the circular helix $r = (a\cos u,\ a\sin u,\ bu)$, $-\infty < u < \infty$ where $a > 0$ and show that the length of one complete turn of the helix is $2\pi c$, where $c = \sqrt{a^2 + b^2}$.

3. Define osculating circle.

4. If the radius of spherical curvature is constant. Prove that the curve either lies on a sphere or has a constant curvature.

5. Define anchor ring.

6. Write the canonical equations for geodesics.

7. Write characteristic properties of geodesics.

8. Define Christoffel symbols of the second kind.

9. Prove that every helix on a cylinder is a geodesic.

10. State Gauss Bonnet theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) State and derive Serret-Frenet formula.

Or

(b) Calculate the curvature and torsion of the cubic curve given by $r = (u, u^2, u^3)$.

12. (a) Show that the osculating plane at P has, in general, three point contact with the curve at P.

Or

(b) Show that if C is a helix lying on a paraboloid of revolution then C_1 is the involute of a circle, where C_1 is the plane curve obtained by projecting C on a plane orthogonal to its axis.

13. (a) A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

Or

(b) Show that on a right helicoid, the family of curves orthogonal to the curves $u \cos v =$ constant is the family $(u^2 + a^2)\sin^2 v$ = constant.

14. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z = $ constant.

Or

(b) If $g(t)$ is continuous for $0 < t < 1$ and if $\int_0^1 v(t)\, g(t)\, dt = 0$ for all admissible functions $v(t)$ as defined above, then prove that $g(t) = 0$.

15. (a) Find the Gaussian Curvature of anchor ring at a point (u,v) and verify that the total curvature of the whole surface is zero.

Or

(b) Derive Liouville's formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find the equation of the osculating plane at a general point on the cubic curve given by $r = (u, u^2, u^3)$ and show that the osculating planes at any three points of the curve meet at point lying in the plane determined by three points.

17. State and prove the fundamental existence theorem for space curves.

18. The metric of a surface is $v^2du^2 + u^2dv^2$. Find the equation of the family of curves orthogonal to the curves uv = constant and find the metric referred to new parameters so that these two families are parametric.

19. Find the angle at the point (u,v), between the two directions given by $Pdu^2 + 2Qdudv + Rdv^2 = 0$.

20. State and prove Gauss-Bonnet theorem.

R–2955

Sub. Code
511204

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2018 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Define conjugate harmonic function.

2. Define order of the rational function.

3. State the Cauchy's theorem for a rectangle.

4. Write the Cauchy's estimate.

5. Define removable singularities.

6. State the maximum principle.

7. Write the Cauchy's integral formula.

8. Find the poles of cotz.

9. Write the Laurent series.

10. Define integral function.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If $g(w)$ and $f(z)$ are analytic, show that $g(f(z))$ is also analytic.

Or

(b) Prove that of a sequence is convergent if and only if it is a cauchy sequence.

12. (a) If $f(z)$ is analytic in an open disk Δ, prove $\int_{\gamma} f(z)\, dz = 0$ for every closed curve γ in Δ.

Or

(b) Compute $\int_{|z|=1} \frac{e^z}{z}\, dz$.

13. (a) Show that the functions e^z, $\sin z$ and $\cos z$ have essential singularities at ∞.

Or

(b) If $f(z)$ is analytic and non – constant in a design Ω, prove its absolute value $|f(z)|$ has no maximum in Ω.

14. (a) How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2?

Or

(b) Evaluate $\int_0^{\pi/2} \frac{dx}{a + \sin^2 x}$, $|a| > 1$ by the method of residues.

15. (a) State and prove Hurwitz theorem.

Or

(b) Prove that the Laurent development is unique.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Verify Cauchy's Riemann's equation for the function z^2 and z^3.

(b) State and prove Abel's limit theorem.

17. (a) State and prove Cauchy's integral formula.

(b) Compute $\int_{|z|=1} e^z z^{-n} dz$.

18. (a) Show that a function which is analytic in the whole plane and has a non essential singularity at ∞ reduces to a polynomial.

(b) If $f(z)$ is analytic in Ω, prove $\int_{\gamma} f(z)dz = 0$ for a every cycle γ which is homologous to zero in Ω.

19. State and prove the argument principle.

20. State and prove Weierstrass theorem.

R–2956

Sub. Code
511401

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define normed space.

2. Define bounded linear operator.

3. Define Hilbert space.

4. What is orthonormal sequence?

5. Define self-adjoint operator and give an example.

6. What is sesquilinear form?

7. Define reflexive spaces.

8. What is adjoint operator? Give an example.

9. Define weak convergence.

10. State open mapping theorem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Write the properties of normed spaces.

Or

(b) Prove that every finite dimensional subspace Y of a normed space X is closed in X.

12. (a) Prove that the space $C\,[a,\,b]$ is not an inner product space, hence not a Hilbert space.

Or

(b) Prove that an orthonormal set is linearly independent.

13. (a) Write the properties of Hilbert-adjoint operators.

Or

(b) Let $U : H \to H$ and $V : H \to H$ be unitary operator on Hilbert space H. Then prove that UV is unitary and U is normal.

14. (a) If a normed space X is reflexive, then prove that it is complete.

Or

(b) State and prove category theorem.

15. (a) Define strong and weak* convergence of a sequence of functionals. Give an example.

Or

(b) Show that the null space of a closed linear operator $T : X \to Y$ is a closed subspace of X.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If Y is a Banach space, then prove that $B(X, Y)$ is a Banach space.

17. (a) State and prove Schwartz inequality.

    (b) If Y is a closed subspace of a Hilbert space H, then prove that $Y = Y^{\perp\perp}$.

18. State and prove Riesz representation theorem.

19. State and prove Hahn-Banach theorem.

20. State and prove closed graph theorem.

R–2957

Sub. Code
511402

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. If $P(c)$ is the probability set function defined as a field β of a simple space C, prove that $P(c) = 1 - P(c^*)$.

2. Define hyper geometric distribution.

3. Evaluate $\int_0^1 \int_0^{1-x_1} (x_1 + x_2) dx_2 \, dx_1$.

4. When do you say that two random variables x_1 and x_2 are independent?

5. Find the expected value of Bernoulli distribution.

6. Define a chi-square distribution.

7. Define F-distribution.

8. Let the independent variables X_1 and X_2 have the same p.d.f., $f(x) = \frac{x}{6}$, $x = 1, 2, 3 = 0$ elsewhere

   Find the joint p.d.f. of x_1 and x_2.

9. Let $\bar{x}$ be the mean of a random variable range of size 25 from distribution is $N(75{,}100)$. Find $\overline{X}$.

10. Let * be $\psi^2(50)$. Approximate $\Pr(40 < x < 60)$.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let c_1 and c_2 be independent events with $P(c_1) = 0.6$ and $P(c_2) = 0.3$. Find $P(c_1 \cup c_2)$ and $p(c_1 \cup c_2{*})$.

Or

(b) Let X have the p.d.f.,

$$f(x) = 2(1-x), \quad 0 < x < 1; \qquad = 0, \quad elsewhere$$ find $E(6x + 3x^2)$.

12. (a) Let $f(x_1, x_2) = 4x_1 x_2, -0 < x_1 < 1$, zero elsewhere, be the p.d.f. of x_1 and x_2 find $\Pr\left(0 < x_1 < \frac{1}{2}, \frac{1}{4} < x_2 < 1\right)$ and $\Pr(x_1 = x_2)$.

Or

(b) Let the random variable X and Y have the joint p.d.f. $f(x, y) = \begin{cases} x + y, & o < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

compute the correlation coefficient of x and y.

13. (a) If X is $b(n, p)$. Find

(i) $E\left(\frac{x}{n}\right)$ and (ii) $E\left[\left(\frac{X}{n} - P\right)^2\right]$.

Or

(b) If X has the p.d.f., $f(x)$ $\begin{cases} \frac{1}{4} x e^{-\frac{x}{2}}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$ Find μ, σ^2 and $M(t)$ (with usual notifies).

14. (a) Determine a constant C so that $f(x) = cx(3-x)^4, 0<x<3$, zero elsewhere, is a p.d.f.

Or

(b) Let x_1 and x_2 be independent with normal distribution $N(6, 1)$ and $N(7, 1)$ respectively find $\Pr(X, > X_2)$.

15. (a) Let S_n^2 denote the variance of a random sample of size n from a distribution $N(\mu, \sigma^2)$. Prove that $nS_n^2/(n-1)$ converges in probability to r^2.

Or

(b) Let the random variable y_n have a distribution $b(n, p)$. Prove that $1-\frac{y_n}{n}$ congerges in probability to $1-p$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let $f(x) = \left(\frac{1}{2}\right)^x, x=1,2,3,..,$ zero elsewhere, be the p.d.f. of the random variable *X*. Find the m.g.f., the mean, and the variance of *X*.

17. Discuss the various of a conditional distribution under the assumption that the condition mean is linear.

18. If the random variable *X* is $N(\mu, \sigma^2), \sigma^2>0$ prove that the random variable $V=(x-\mu)^2/\sigma^2$ is $\psi^2(1)$.

19. Let $x_1, x_2, ..., x_n$ denote a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Prove that the random variable $y = \sum_1^n \frac{(x_i - \mu)^2}{\sigma}$ has a chi-square distribution with n degrees of freedom.

20. State and prove central limit theorem.

R–2958

Sub. Code
511502

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define subgraph and spanning subgraph of a graph G.

2. Define cut edge and give an example.

3. If G is a block with $\gamma \geq 3$, prove that any two edges of G lie on a common cycle.

4. Define Hamilton path and Hamilton cycle of a graph G.

5. Show that a 3 – regular graph with cut edges need not have a perfect matching.

6. Define Ramsey number.

7. Define edge chromatic number.

8. Write down any two basic properties of critical graphs.

9. Give an embedding of $K_{3,3}$ on the mobius band.

10. Define digraph with example.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Let G be a graph with v vertices and $\in$ edges prove that $\sum_{v \in V} d(v) = 2t$. Deduce that the number of vertices of odd degree is even.

Or

(b) Show that a connected graph is a tree if and only if every edge is a cut edge.

12. (a) Show that a graph G with atleast 3 vertices is 2 – connected if and only if any two vertices of G are connected by at least two internally – disjoint paths.

Or

(b) Prove that the closure $C(G)$ of a graph G is well defined.

13. (a) Show that every 3 – regular graph without cut edges has a perfect matching.

Or

(b) State and prove Schur's theorem.

14. (a) Prove that, if G is bipartitle, then $\chi' = \Delta$.

Or

(b) If G is simple graph, then show that $\pi_k(G) = \pi_k(G - e) - \pi_k(G . e)$, when $\pi_k(G)$ is the chromatic polynomial of G.

15. (a) State and prove Euler's formula on connected plane graphics.

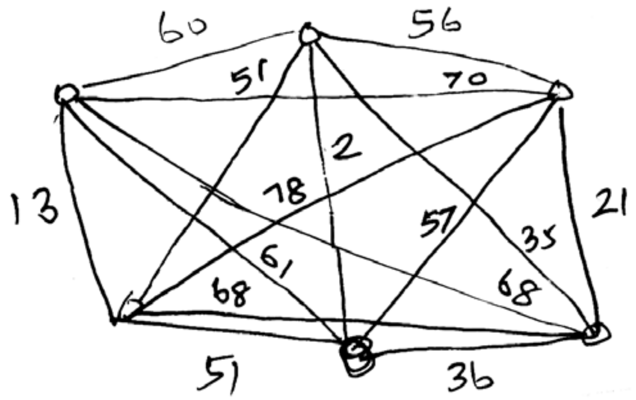
Or

(b) Show that each vertex of a disconnected tournament D with $\gamma \geq 3$ is contained in directed k – cycle, $3 \leq k \leq \gamma$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State Krushal's algorithm and apply this to the weighted graph given below :



17. State and prove Chvatal's sufficient conditions for a simple graph to be hamiltonian.

18. State and prove Taurn's theorem.

19. State and prove Hajo's theorem.

20. State and prove the five – colour theorem.

R–2959

Sub. Code
511504

M.Sc. DEGREE EXAMINATION, APRIL 2019

Fourth Semester

Mathematics

OPTIMIZATION TECHNIQUES

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. What are the steps involved for minimal spanning tree algorithm?

2. What is Floyd's algorithm?

3. Show that the set $c = \{x_1, x_2 / x_1 + x_2 \leq 1, x_1 \geq 0, x_2 \geq 0\}$ is convex.

4. Write bounded dual simplex algorithm.

5. How do you find graphical solution to a game?

6. Let a_{ij} be the (i, j)th element of a play off matrix with m strategies for player A and n strategies for player B. The play off is for player A.

   Prove that $\max_i \min_j a_{ij} \quad \min_j \max_i a_{ij}$.

7. Determine the extreme points of $f(x) = x^4 + x^2$.

8. What is sensitivity analysis in the Jacobian method?

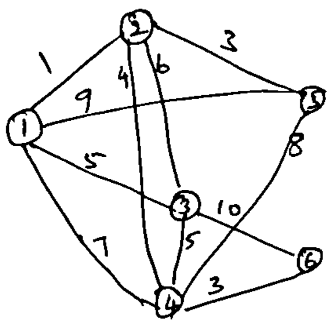
9. Show that in separable convex programming, it is never optimal to have $x_{ki} > 0$ when x_{k-1}, i is not at its upper bound.

10. What is difference between separable programming and separable convex programming?

Part B (5 × 5 = 25)

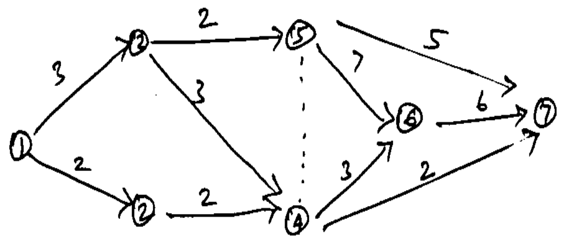
Answer **all** questions, choosing either (a) or (b).

11. (a) Midwest TV cable compand is in the process of providing cable service to five new housing development areas. The figure depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network.



Or

(b) Determine the critical path for the project network in the following figure.



12. (a) Demand and classify all the basic solutions of the following system of equation.

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}.$$

Or

(b) Consider the LP and maximize.

$z = x_1 + 4x_2 + 7x_3 + 5x_4$ Subject to

$2x_1 + x_2 + 2x_3 + 4x_4 = 0$

$3x_1 - x_2 - 2x_3 + 6x_4 = 5,\ x_1,\ x_2,\ x_3,\ x_4 \geq 0.$

13. (a) Solve the following linear programming.

	B1	B2	B3	Row min
A1	3	–1	3	–3
A2	–2	4	–1	–2
A3	–5	–6	2	–6
Column max	3	4	2	

Or

(b) Solve the following games graphically. The play off is for player A

	B1	B2	B3
A1	5	50	50
A2	1	1	1
A3	10	1	10

14. (a) Write the Kuhn-Tncker necessary conditions of the following.

Maximize : $f(x) = x_1^3 - x_2^2 + x_1 x_3^2$ subject to

$x_1 + x_2^2 + x_3 = 5$, $5x_1^2 - x_2^2 - x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$.

Or

(b) Prove that a necessary condition for X_0 to be an extreme of $f(x)$ is that $\nabla f(X_0) = 0$.

15. (a) Find the maximum of the following function by dichotomous search. Assume that $\Delta = 0.05$.

$$f(x) = \frac{1}{\left|(x-3)^3\right|}, \ 2 \leq x \leq 4.$$

Or

(b) Consider the problem maximize

$z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1, \ x_2 - 2x_2^2$

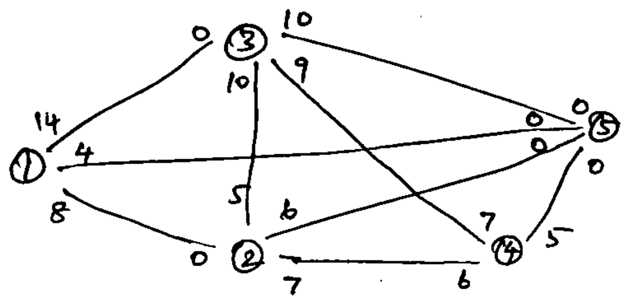
subject to $x_1 + 2x_2 \leq 2, \ x_1, x_2 \geq 0$.

Find an exact optimum solution to this problem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Determine the maximal flow and the optimal flow in each arc for the network bellow.



R–2959

17. Solve the following LP model by the bounded variables algorithm.

Maximize : $z = 3x_1 + 5y + 2x_3$

Subject to : $x_1 + y + 2x_3 \le 14$
$2x_1 + 4y + 3x_3 \le 43$

$0 \le x_1 \le 4$, $7 \le y \le 10$, $0 \le x_3 \le 3$.

18. Consider the following two-person, zero-sum game.

	B1	B2	B3
A1	5	50	50
A2	1	1	1
A3	10	1	10

(a) Verify that the strategies $\left(\frac{1}{6}, 0, \frac{5}{6}\right)$ for A and $\left(\frac{49}{54}, \frac{5}{54}, 0\right)$ for B are optional and determine the value of the game.

(b) Show that the optimal value of the game equals

$$\sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij}\, x_i\, x_j .$$

19. Minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

Subject to : $g_1(x) = x_1 + x_2 + 3x_3 - 2 = 0$

$g_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0$

Determine the extreme points.

20. Consider the problem,

Maximize : $z = x_1 - x_2$

Subject to : $3x^4 + x_2 \leq 243$

$x_1 + x_2^2 \leq 32$

$x_1 \geq 2.1$

$x_2 \geq 3.5$

Find an exact optimum solution to this problem.

R–3239

Sub. Code
511101

M.Sc. DEGREE EXAMINATION, APRIL 2019.

First Semester

Mathematics

GROUPS

(CBCS – 2018 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define Group.

2. Give one example of group.

3. Define right coset.

4. Give an example of right cosets.

5. Define homomorphism.

6. Define automorphism.

7. Find the orbits of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$.

8. Define normalizer.

9. State second part of SYLOW'S theorems.

10. Define internal direct product.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that if every element of the group G is own inverse, prove that G is abolian.

Or

(b) If G is a group, prove that

(i) The identify element of G is unique and

(ii) Every $a \in G$ has a unique inverse in G.

12. (a) Prove that a non-empty subset H of the group G is a subgroup of G if and only if (i) $a, b \in H$ implies that $a, b \in H$ (ii) $a \in H$ implies that $a^{-1} \in H$.

Or

(b) Prove that there is a one-to-one correspondence between any two right cosets of H in G.

13. (a) If ϕ is a homomorphism of G into G, prove

(i) $\phi(e) = \bar{e}$, the unit element of $\overline{G}$

(ii) $\phi(x^{-1}) = \phi(x)^{-1}$ for all $x \in G$.

Or

(b) If H is subgroup of G show that for every $g \in G$, gHg^{-1} is a subgroup of G.

14. (a) Prove every permutation is the product of its cycles.

Or

(b) Determine which of the following are even permutations :

(i) (1, 2, 3) (1, 2)

(ii) (1, 2, 3, 4, 5) (4, 5).

15. (a) If A, B are finite subgroups of G, prove

$$O(AB) = \frac{O(A)O(B)}{O(A \cap xBx^{-1})}.$$

Or

(b) If G is a group of order 231, prove that 11 – Sylow subgroup is in the center of G.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) If the group G has three elements, show it must be abelian.

(b) Do part (a) if G has four elements.

(c) Do part (a) if G has five elements.

17. Prove the relation $a \equiv b \mod H$ is an equivalence relation.

18. State and prove Cauchy's theorem for abelian groups.

19. Prove that conjugacy is an equivalence relation on G.

20. State and prove SYLOW theorem.

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R–3240

Sub. Code
511102

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

REAL ANALYSIS – I

(CBCS – 2018 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. State the least upper bound property and the greatest lower bound property is an ordered set.

2. Define connected space.

3. Define the Uniformly continuous function on a Metric space.

4. Give an example of a function f which has a discontinuity of the second kind at every point x.

5. Prove that f is continuous at x if f is differentiable at a point $x \in [a, b]$.

6. Define a local minimum of a function f at a point p.

7. Prove that $\lim_{n\to\infty}(s_n + t_n) = s + t$ if $\lim_{n\to\infty} s_n = s$ and $\lim_{n\to\infty} t_n = t$.

8. Define Cauchy sequence.

9. State the Root test.

10. Define Rearrangements.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that every Infinite subset of a countable set A is countable.

Or

(b) Show that compact subsets of metric spaces are closed.

12. (a) Prove that $\mathbb{R}^k$ is complete.

Or

(b) If $0 \le x < 1$, then prove that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ and if $x \ge 1$, then series diverges.

13. (a) State and prove Ratio test.

Or

(b) If $\sum a_n = A$ and $\sum b_n = B$, then prove that $\sum (a_n + b_n) = A + B$, and $\sum ca_n = cA$, for any fixed c.

14. (a) Prove that A mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

Or

(b) Let f be monotonic on (a,b). Then prove that the set of points of (a,b) at which f is discontinuous is atmost countable.

15. (a) Prove that composite of two differentiable function is differentiable on $[a,b]$.

Or

(b) Suppose f is a continuous mapping of $[a,b]$ into $\mathbb{R}^k$ and f is differentiable in (a,b). Then prove that there exists $x \in (a,b)$ such that $|f(b)-f(a)| \le (b-a)|f'|$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If a set E in $\mathbb{R}^k$ has one of the following three properties, then prove that it has the other two:

(a) E is closed and bounded

(b) E is compact.

(c) Every infinite subset of E has a limit point in E.

17. Prove that e is irrational.

18. Show that the product of two convergent series is converges if atleast one of the two series converges absolutely.

19. Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X.

20. State and prove L^1 Hospital's rule.

R–3241

Sub. Code
511103

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS – 2018 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define non-homogeneous equation.

2. Define analytic function.

3. Define regular singular point.

4. Write the Euler equation.

5. Eliminate the constants a and b from the equation $\alpha z = (ax + y)^2 + b$.

6. Define complete integral.

7. Write the Laplace's equation.

8. Define complementary function of $F(D, D')z = f(x, y)$.

9. Define boundary value problem for Laplace equation.

10. Discuss an interior chunchill problem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If ϕ is a solution of $\tau(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on I and $\phi_1(x) \neq 0$ on I, a second solution ϕ_2 of (1) on I is given by $\phi_2(x) = \phi_1(x)\int_{x_0}^{x} \frac{1}{(\phi_1(s))^2} \exp\left[-J_{x_0}^{s} a_1(t)dt\right] ds$. Prove that the functions ϕ_1, ϕ_2 form a basis for the solution of (1) on I.

Or

(b) Show that the co-efficient of x^n in $P_n(x)$ is $\frac{(2n)!}{2^n (n!)^2}$.

12. (a) Find all solutions of $x^2y'' + 2xy' - 6y = 0$ for $x > 0$.

Or

(b) Find the singular points of $x^2y'' + (x + x^2)y' - y = 0$ and determine regular singular points.

13. (a) Eliminate the arbitrary function f from $f(x^2 + y^2) + z^2,\ z^2 - \alpha xy = 0$.

Or

(b) Discuss on Cauchy's problem for first order equations.

14. (a) Verify that $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{\alpha z}{x}$ is satisfied by $z = \frac{1}{x}\phi(y-x) + \phi'(y-x)$.

Or

(b) If $\alpha_r D + \beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\varepsilon)$ is an arbitrary function of the single variable ε, prove if $\alpha_r \neq 0$, $u_r = \exp\left(-\frac{\gamma_r x}{\alpha_r}\right)\phi_r(\beta_r x - \alpha_r y)$ is a solution of the equation $F(D, D')z = 0$.

15. (a) If $\rho > 0$ and $\psi(r)$ is given by $\psi(r) = \int_V \frac{\rho(r')dT^1}{|r - r'|}$, where the volume V is bounded, prove that $\lim_{r=\infty} r\psi(r) = M$ where $M = \int_V \rho(r')dT'$.

Or

(b) Derive the d'Alembert's solution of the one-dimension wave equation.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Existence theorem for analytic co-efficients.

17. Explain on Bessel equation.

18. If $u_i(x_1, x_2 \ldots x_n, z) = c_i(i = 1,2,\ldots n)$ are independent solutions of the equations $\frac{dx_1}{p_1} = \frac{dx_2}{p_2} = \cdots = \frac{dx_n}{p_n} = \frac{dz}{R}$, prove that the relative $\Phi(u_1, u_2 \ldots u_n) = 0$, in which the function Φ is arbitrary, is a general solution of the linear partial differential equation $P_1 \frac{\partial z}{\partial x_1} + P_2 \frac{\partial z}{\partial x_2} + \ldots + P_n \frac{\partial z}{\partial x_n} = R$.

19. Find a particular integral of the following equations:

    (a) $(D^2 - D^1) = 2y - x^2$

    (b) $(D^2 - D^1)z = e^{x+y}$.

20. Explain an elementary solutions of the one-dimensional wave equation.

R–3242

Sub. Code
511104

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS – 2018 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Show that $(a, a+2) = 1$ or 2 for any integer a.

2. If $2^n - 1$ is prime, then show that n is prime.

3. State the Euler's summation formula.

4. When do we say two lattice points are visible? What is the density of the set of lattice points visible from the origin?

5. Define the function $H(x)$.

6. Define Chebychev's ψ -function.

7. State the principle of cross-classification.

8. Find all n for which $\phi(n) \equiv 2 (\mathrm{mod}\, 4)$.

9. Define the Legendre's symbol $(n|p)$.

10. State the Reciprocity law for Jacobi symbols.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove Euclid theorem.

Or

(b) State and prove the Division Algorithm.

12. (a) For all $x \geq 1$, prove that

$$\sum_{n \leq x} \sigma_1(n) = \frac{1}{2}\varsigma(2)x^2 + O(x \log x).$$

Or

(b) State and prove the Legendre's Identity.

13. (a) For the n^{th} prime p_n, prove that if $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$, then $\lim_{x \to \infty} \frac{\pi(x) \log \pi(x)}{x} = 1$.

Or

(b) Prove that $\lim_{x \to \infty} \left(\frac{\psi(x)}{x} - \frac{\vartheta(x)}{x} \right) = 0$ for $x > 0$.

14. (a) State and prove Chinese remainder theorem.

Or

(b) Solve the congruence $25x \equiv 15 (\text{mod} 120)$.

15. (a) If P is an odd positive integer, then show that

$(-1|P)=(-1)^{\frac{P-1}{2}}$.

Or

(b) State and prove the Euler's criterion.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove any four properties of Euler's totient function.

17. For all $x \geq 1$, prove that $\sum_{n \leq x} d(n) = x \log x + (2C - 1)x + O(\sqrt{x})$, where C is Euler's constant.

18. Prove that the prime number theorem implies $\lim_{x \to \infty} \frac{M(x)}{x} = 0$.

19. State and prove the Wolstenholme's theorem. Prove all the results which you used in your proof.

20. State and prove the Quadratic Reciprocity law.

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R–3243

Sub. Code
511513

M.Sc. DEGREE EXAMINATION, APRIL 2019

First Semester

Maths

OBJECT ORIENTED PROGRAMMING AND C++

(CBCS – 2018 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Give any two special features of C++ over C.

2. What is the use of cast operator?

3. What do you mean by data encapsulation?

4. Give an example for nesting of classes.

5. What is a pointer?

6. What is copy constructor?

7. Why it is necessary to overload an operator?

8. What do you mean by static binding?

9. What is virtual function?

10. Mention the types of inheritance.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Describe the logical operators available in C++.

Or

(b) Explain c_{in} and c_{out} streams.

12. (a) How can you declare a member function?

Or

(b) Can objects appear as function arguments? Illustrate with an example.

13. (a) With suitable examples, explain new and delete operators.

Or

(b) Write a program in C++ to find and replace a particular character in a string using pointers.

14. (a) Write a note on over loading unary operators.

Or

(b) Write a program to overload the + operators to provide string addition.

15. (a) Explain the multi-level inheritance with an example.

Or

(b) Illustrate the use of abstract base class.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Describe the basic concepts of object oriented programming.

17. Explain the different types of constructors used in C++.

18. Write a program in C++ with menu facility to per form the following string operations:

    (a) String length

    (b) String reverse

    (c) String concatenation.

19. Write a program to perform complex number arithmetic operations using operator overloading.

20. Explain the following with suitable examples:

    (a) Early binding

    (b) Dynamic binding.

R–3244

Sub. Code
511301

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

MECHANICS

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define non holonomic constraints.

2. State the principle of work and kinetic energy.

3. Write the standard form of Lagranges Equation for a holonomic system.

4. Define ignorable coordinates.

5. Write the form of the Hamiltonian function.

6. Write Jacobi's form of the principle of least action.

7. Write Hamilton-Jacobi equation.

8. State stackel's theorem.

9. Define canonical transformation.

10. Write Lagrange brackets.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Write a short note on generalized coordinates.

Or

(b) Explain angular Momentum.

12. (a) Explain integrals of the motion.

Or

(b) Derive the Lagranges equation using Routhian function.

13. (a) Find the stationary values of the function $f = z$ subject to the constraints

$$\phi_1 = x^2 + y^2 + z^2 - 4 = 0$$
$$\phi_2 = xy - 1 = 0$$

Or

(b) Explain Modified Hamilton's principle.

14. (a) Explain pfaffian differential form.

Or

(b) Illustrate the Hamilton-Jacobi method for simple mass-spring system.

15. (a) Write a short note on homogeneous canonical transformation.

Or

(b) Explain point transformation.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove konig's theorem.

17. A double pendulum consists of two particles suspended by massless rods. Assuming that all motion takes place in a vertical plane, find the differential equations of Motion. Linearize these equations assuming small motions.

18. State and prove Hamilton principle.

19. Derive modified Hamilton-Jacobi equation.

20. Explain poission brackets.

R–3245

Sub. Code
511302

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2015 onwards)

Time : 3 Hours — Maximum : 75 Marks

Part A — (10 × 2 = 20)

Answer **all** questions.

1. Define: Basis for a topology.

2. Let A and B denote a subset of a space *X*. Prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

3. When a metric space is said to be metrizable?

4. Let A be connected space of X. If $A \subset B \subset \overline{A}$, then prove B is also connected.

5. State: Maximum and minimum value theorem.

6. Prove that the real in *R* is not compact.

7. Define: Second countable space. Give an example.

8. Give an example showing that a Hausdorff space with a countable basis need not be metrizable.

9. Show that $[0,1]^\infty$ is not locally compact in the uniform topology.

10. Define: Stone-Cech compactification.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $\mathcal{B}$ be a basis for a topology T on a set X. Then prove that T equals the collection of all unions of element of $\mathcal{B}$.

Or

(b) If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y -. Then prove that the collection = $\mathcal{D}$ = $\{B \times C \mid B \in \mathcal{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

12. (a) State and prove sequence lemma.

Or

(b) State and prove uniform limit theorem.

13. (a) Prove that every compact subset of a Hausdorff space is closed.

Or

(b) Prove that the image of a compact space under a continuous map is compact.

14. (a) Prove that every compact Hausdorff space is normal.

Or

(b) Show that the Tietze extension theorem implies the Urysohn lemma.

15. (a) Let X be a Hausdroff space. Then prove that X is locally compact at x if and only if for every neighborhood U of x, there is a neighborhood V of x, such that $\overline{V}$ is compact and $\overline{V} \subset U$.

Or

(b) State and prove the Tychonoff theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) If A is a subspace of X and B is a subspace of Y then prove that the product topology on A × B is the same as the topology A × B inherits as a subspace of X × Y. (5)

(b) Let Y be a subspace of X. If U is open in Y and Y os open in X, then prove that U is open in X. (5)

17. Prove that the Cartesian product of connected spaces is connected.

18. Let X be a metrizable space. Prove that the following are equivalent :

(a) X is compact.

(b) K is limit point compact.

(c) K is sequentially compact.

19. State and prove Urysohn's lemma.

20. Let X be a completely regular space. Prove that a subspace of completely regular space is completely regular and a product of completely regular space is completely regular.

R–3246

Sub. Code
511303

M.Sc. DEGREE EXAMINATION, APRIL 2019

Third Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2015 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   (10 × 2 = 20)

Answer **all** questions.

1. Find the length of the curve given as the intersection of the surface $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $x = a\cos h(z/a)$, from the point $(a,0,0)$ to the point (x,y,z).

2. Define curvature and what is the curvature for a straight line.

3. Define osculating sphere.

4. Define an involute and show that the involutes of a circular helix are plane curves.

5. Define an anchor ring.

6. Define metric.

7. Show that a curve on a plane is a geodesic if and only if, it is a straight line.

8. Write a characteristic property of a geodesic.

9. Define geodesic parallels.

10. Define Gaussian curvature.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Obtain a formula for the arc length of a path given in terms of a parameters and also show that the length of one complete turn of the helix $\vec{r} = (a \cos u, a \sin u, bu)$, $a > 0$, $-\infty < u < \infty$ is $2\pi c$, where $c = \sqrt{a^2 + b^2}$.

Or

(b) Define torsion and show that $[\dot{\vec{r}}, \ddot{\vec{r}}, \dddot{\vec{r}}] = 0$ is a necessary and sufficient condition for the curve to be a plane curve.

12. (a) Show that the osculating plane at 'p' has in general, three point contact with the curve at 'p'.

Or

(b) Show that the torsion of an involute of a curve is equal to $\rho(\sigma\rho' - \sigma'\rho)/(\rho^2 + \sigma^2)(c - s)$.

13. (a) Show that the parametric curve on the sphere given by $x = a\sin u \cos v$, $y = a\sin u \sin v$, $z = a\cos u$, $0 < u < \frac{\pi}{2}$, $0 < v < 2\pi$ form on orthogonal system.

Or

(b) Find the coefficients of the direction which makes an angle $\frac{\pi}{2}$ with the direction whose coefficients are (l, m)

14. (a) Explain : Intrinsic properties of surfaces.

Or

(b) Prove that on the general surface, a necessary condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$. Where $v = c$, for all values of u.

15. (a) Prove that, if the orthogonal trajectories of the curves $u =$constant are geodesic then $\frac{H^2}{E}$ is independent of 'u'.

Or

(b) Calculate the circumstance of geodesic circle of small radius r and to see how it differs from the Euclidean formula $2\pi r$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadrics surfaces by $ax^2 + by^2 + cz^2 = 1$ and $a'x^2 + b'y^2 + c'z^2 = 1$.

17. Prove that the radius of curvature of the locus of the centre of curvature of a curve is given by $\left[\left\{\frac{\rho^2\sigma}{R^3}\frac{d}{ds}\left(\frac{\sigma\rho'}{\rho}\right) - \frac{1}{R}\right\}^2 + \frac{\rho'^2\sigma^4}{\rho^2 R^4}\right]$.

18. A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

19. Prove that if θ is the angle at the point (u, v) between the two directions given by $Pdu^2 + 2Q\,du\,dv + Rdv^2 = 0$ then $\tan\theta = \frac{2H\,(Q^2 - PR) - \frac{1}{2}}{ER - 2FQ + GP}$.

20. Derive the liouville's formula for geodesic curvature.

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R–3247

Sub. Code
511304

M.Sc. DEGREE EXAMINATION, APRIL 2019.

Third Semester

Mathematics

MULTIVARIATE CALCULUS

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define Continuously differentiable functions.

2. Define Partial derivative.

3. Define Jacobian.

4. Define Null space.

5. Define Basic κ — forms in R^n.

6. Define Flip.

7. Show that the simplex Q^k is the smallest convex subset of R^k that contains $0, e_1 \ldots\ldots\ldots e_k$

8. Prove that every closed form is exact in any set which is C^n-equivalent to a convex set.

9. Define closed form and exact form.

10. State Stokes' formula.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If X is a complete metric space, and if φ is a contraction of X into X, then show that there exists one and only $x \in X$ such that $\varphi(x) = x$.

Or

(b) Suppose that f is a real-valued function defined in an open set $E \subset R^n$, and that the partial derivatives $D_1 f \ldots\ldots\ldots D_n f$ are bounded in E. Prove that f is continuous in E.

12. (a) If $[A]$ and $[B]$ are n by n matrices, then det $([B][A]) =$ det$[B]$ det $[A]$.

Or

(b) Suppose f is defined in an open set $E \subset R^2$, suppose that $D_1 f,$, $D_{21} f,$, and $D_2 f,$ exist at every point of E, and $D_{21} f,$ is continuous at some point $(a,b) \in E$. Then show that $D_{21} f(a, b) = D_{12} f(a, b)$

13. (a) For every $F \in C(I^k)$, prove that $L(f) = L'(f)$.

Or

(b) Suppose $\omega = \sum_I b_I(x) dx_I$ is a standard presentation of a k-form w in an open set $E \subset R^n$. If $w = 0$ in E then prove that $b_I(x) = 0$ for every increasing k-index I and for every $x \in E$.

14. (a) State and prove Stoke's formula.

Or

(b) If $f \in C(R^n)$ and the Support of f lies in K, then prove that

$$f = \sum_{i=1}^{s} \psi_i f$$

Each $\psi_i f$ has its Support in Some V_α.

15. (a) State and prove divergence theorem.

Or

(b) Suppose E is an open set in R^3, $u \in C^n(E)$ and F, G are vector fields in E of class C^n.

(i) If $F = \nabla u,$, prove that $\nabla \times F = 0$

(ii) If $F = \nabla \times G,$, prove that $\nabla \cdot F = 0$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Inverse function theorem.

17. State and prove Rank theorem.

18. Suppose F is a C' -mapping of an open set $E \in R^n$ into R^n, $0 \in E$, $F(0) = 0$ and $F'(0)$ is invertible. Then prove that there is a neighborhood of 0 in R^n in which a representation

$F(x) = B_1 .....B_{n-1}\ G_n 0..........0 G_1(x)$

is valid. In above equation, each G_i is primitive C' -mapping in some neighborhood of 0; $G_i(0) = 0, G_i^{'}(0)$ is invertible, and each B_i is either a flip or the identity operator.

19. If $E \subset R^n$ is Convex and open, if $k \geq 1$, , if ω is a k-form of class C' in E, and if $d\omega = 0$, then there is a $(k-l)$ – form of λ in E such that $\omega = d\lambda$.

20. Let

$$\varsigma = \frac{xdy \wedge dz + ydz \wedge dx + zdx \wedge dy}{r^3}$$

where $r = (x^2 + y^2 +)(1/2)$, let D be the rectangle given by $0 \leq u \leq \pi, 0 \leq v \leq 2\pi$ and let Σ be the 2- surface in R^3, with the parameter domain D given by

$x = \sin u \cos v, y = \sin u \sin v, z = \cos u$.

(a) Prove that $d\varsigma = 0$ in $R^3 - 0$.

(b) Is ς exact in the complement of every line through the origin.

(c) Define $\lambda = -(z / r)\eta$,where

$$\eta = \frac{xdy - ydx}{x^2 + y^2}$$

Then λ is a 1-form open set in $V \subset R^3$ in which $x^2 + y^2 > 0$. Show that ς is exact in V by showing that $\varsigma = d\lambda$.

R–3248

Sub. Code
511515

M.Sc. DEGREE EXAMINATION, APRIL 2019

Maths

Third Semester

IMAGE PROCESSING AND PATTERN RECOGNITION

(CBCS – 2015 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   ($10 \times 2 = 20$)

Answer **all** questions.

1. What is sampling?

2. List out the important properties of two dimensional geometric transformation.

3. On what mathematical operation are the two basic approaches for edge detection based on?

4. What are the operators used to detect the crack edge?

5. If the original image size is 256×256 pixels, 8 bits/pixel, it would occupy 65536 bytes. After compression it occupies 6554 bytes. Calculate the compression ratio.

6. What is entropy?

7. What is pattern recognition?

8. What do you mean by decision function?

9. Distinguish between supervised and unsupervised learning methods.

10. What is syntatic pattern recognition?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Describe the anatomy of the human visual system.

Or

(b) Give the procedure involved in enhancing the image using histogram specification.

12. (a) Describe the prewitt operator.

Or

(b) Describe the basic morphological operations.

13. (a) Write about contour coding.

Or

(b) Explain the salient features of fractal compression.

14. (a) With an example, explain the pattern recognition system.

Or

(b) Illustrate the implementation of decision functions.

15. (a) Describe the maximum distance algorithm.

Or

(b) Explain the concepts for formal language theory.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Explain any three basic gray level transformations.

17. Explain the pyramid edge detection approach with an example.

18. What are the types of redundancies normally available in an image? Explain them.

19. Describe the implementation procedure of decision functions.

20. How patterns are classified by distance functions? Explain with suitable examples.

R–3249

Sub. Code
511201

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

ALGEBRA – II

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a subspace.

2. Define a linear dependent set.

3. What is meant by simple extension of F?

4. Define an algebraic number.

5. Define a solvable group.

6. In S_5, show that (1 2) and (1 2 3 4 5) generates S_5.

7. Define left invertiblity.

8. Define cyclic group with an example.

9. If A is invertible then prove that $tr(ACA^{-1}) = trC$.

10. Define transpose of a matrix.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If V is finite-dimensional over F then prove that any two bases of V have the same number of elements.

Or

(b) If V and W are of dimensions m and n respectively over F show that $Hom(V, W)$ is of dimension mn over F.

12. (a) If L is an algebraic extension of K and if K is an algebraic extension of F, prove that L is an algebraic extension of F.

Or

(b) Show that a polynomial of degree n over a field can have at most n roots in any extension field.

13. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) Prove that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

14. (a) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

Or

(b) Prove that if V is finite-dimensional over F, then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

15. (a) If $T \in A(V)$ then prove that $tr\ T$ is the sum of the characteristic roots of T.

Or

(b) If F is of characteristic 0 and if S and T in $A_F(V)$ are such that $ST — TS$ commutes with S, prove that $ST - TS$ is nilpotent.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If V is finite dimensional and $v \neq 0 \in V$, prove that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

17. If $p(x)$ is a polynomial in $F[x]$ of degree $n \geq 1$ and is irreducible over F, prove that there is an extension E of F, such that $[E : F] = n$, in which $p(x)$ has a root.

18. If K is a finite extension of F, prove that $G(K, F)$ is a finite group and its order, $o(G(K, F))$ satisfies $o(G(K, F)) \leq [K, F]$.

19. Show that two nilpotent linear transformations are similar if and only if they have the same invariants.

20. If $T \in A(V)$ is hermitian, prove that all its characteristic roots are real.

R–3250

Sub. Code
511202

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

MEASURE AND INTEGRATION

(CBCS – 2015 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part A　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Define Lebesgue measurable.
2. Define hereditary.
3. Define essential supremum.
4. Define essentially bounded.
5. Define Lebesgue integral.
6. State Lebesgue's Monotone Convergence theorem.
7. Define negative set and null set.
8. Define total variation.
9. Define monotone class.
10. State Fubini's theorem.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) For any sequence of sets $\{E_i\}$, prove that $m^*\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m^*(E_i)$.

Or

(b) Prove that every interval is measurable.

12. (a) Let E be a measurable set. Prove that for each y the set $E + y = [x + y : x \in E]$ is measurable and the measures are the same.

Or

(b) Find the cardinality of the class of measurable sets.

13. (a) Let f and g be non-negative measurable functions. Prove that $\int f\,dx + \int g\,dx = \int (f+g)\,dx$.

Or

(b) Show that every measurable function f there corresponds a Borel measurable function g such that $f = g$ a.e.

14. (a) If γ is a signed measure and $E_1 \subseteq E_2 \subset \ldots.$, prove that $\gamma\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim \gamma(E_i)$.

Or

(b) State and prove Jordan decomposition theorem.

15. (a) Prove that $\mathcal{E}$ is an algebra.

Or

(b) Let f be an $\mathcal{S} \times \mathcal{T}$ measurable function and let $\phi^*(z) = \int\limits_Y |f|_x d\gamma$, $\phi(y) = \int\limits_X |f|^y d\mu$ for each $x \in X$; $y \in Y$; prove that $\phi^* \in L'(\mu)$, $\psi^* \in L'(\gamma)$, $f \in L'(\mu \times \gamma)$ are equivalent.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Prove that the class $\mathcal{M}$ is a σ - algebra.

(b) Describe the ring generated by the finite open intervals.

17. (a) Prove that there exists a non-measurable set.

(b) Show that a nowhere dense perfect set can contain a non-measurable set.

18. Let f and g be a non-negative measurable functions.

(a) If $f \le g$, prove $\int f\,dx \le \int g\,dx$,

(b) If A is a measurable set and $f \le g$ on A, prove $\int\limits_A f\,dx \le \int\limits_A g\,dx$,

(c) If $a \le 0$, prove $\int af\,dx = a\int fdx$,

(d) If A and B are measurable sets and $A \supseteq B$, prove $\int\limits_A fdx \ge \int\limits_B fdx$.

19. State and prove Jordan decomposition theorem.

20. Let $[X, \mathcal{S}, \mu]$ and $[Y, \mathcal{T}, \nu]$ be a $\sigma-$ finite measure spaces. For $V \in \mathcal{S} \times \mathcal{T}$ write $\phi(x) = \gamma(V_x)$, $\psi(y) = \mu(V^y)$, for each $x \in X$, $y \in Y$. Prove that ϕ is $\mathcal{S}$–measurable, ψ is $\mathcal{T}$–measurable and $\int_X \phi \, d\mu = \int_Y \psi d\gamma$.

R–3251

Sub. Code
511203

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Define Pfaffian differential form.

2. Define Homogeneous equations.

3. Define general integral.

4. Define compatible.

5. Define Separable equations.

6. Write a fundamental idea of Charpit's method.

7. Define particular integral.

8. Write a Hankel transform.

9. Define interior Neumann problem.

10. Write down the Cauchy-Riemann equations.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the integral curves of the equations

$$\frac{dx}{y(x+y)+az}=\frac{dy}{x(x+y)-az}=\frac{dz}{z(x+y)}.$$

Or

(b) Find the orthogonal trajectries on the curve $x^2+y^2+2fyz+d=0$ of its curves of intersection with plane parallel to the plane $x \circ y$.

12. (a) Find the general solution of the differential equation

$$x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z.$$

Or

(b) Find the surface which is orthogonal to the one-parameter system $z=cxy(x^2+y^2)$ and which passes through the hyperbola $x^2-y^2=a^2, z=0$.

13. (a) Find the complete integral of the equation $p^2x+q^2y=z$.

Or

(b) Prove that equation of the Clairant form

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=f\left(\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial u}{\partial z}\right)$$ is always soluble by Jacobi's method.

14. (a) If u is the complementary function and z, a particular integral of a linear partial differential equation, prove that $u + z_1$ is a general solution of the equation.

Or

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.

15. (a) If $\rho > 0$ and $\psi(r) = \int_V \frac{\rho(r')\,dr'}{|r - r'|}$, where the volume V is bounded, prove that $\lim_{r=\infty} r\psi(r) = M$, where $\mu = \int_V \rho(r')\,dr'$.

Or

(b) Prove that the solutions of certain Neumann problem can be differ from one another by a constant only.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Solve the equations

$$\frac{dx}{y + \alpha z} = \frac{dy}{z + \beta x} = \frac{dz}{x + \gamma y}.$$

17. Prove that the general solution of the linear partial differential equation $Pp = Qq = R$ is $F(u, v) = 0$ where F is an arbitrary function and $u(x, y, z) = C_1$, $v(x, y, z) = C_2$ from a solution of the equations

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

18. (a) Prove that :

$[f, g] = 0$ where

$$[f, g] = \frac{\partial(f, g)}{\partial(x, p)} + p\frac{\partial(f, g)}{\partial(z, p)} + \frac{\partial(f, g)}{\partial(y, q)} + q\frac{\partial(f, g)}{\partial(z, q)}$$

(b) Show that the equations $xp - yq = x, x^2 p + q = xz$ are compatible and find their solution.

19. (a) Show that the equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ possess solutions of the form $\sum_{n=0}^{\infty} c_n \cos(nx + \varepsilon_n) e^{-kn^2 t}$.

(b) Find the particular integral of the equation

$(D^2 - D^1)z = 2y - x^2$.

20. The faces $x = 0$, $x = a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation $\theta = f\, x)(0 \le x \le a)$. Determine the temperature at a subsequent time t.

R–3252

Sub. Code
511204

M.Sc. DEGREE EXAMINATION, APRIL 2019

Second Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2015 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define conjugate harmonic functions.

2. State Abel's limit theorem.

3. Define line integral.

4. State Cauchy's theorem in a Disk.

5. Define Zeros and Poles.

6. Define simple connectivity.

7. State the Residue theorem.

8. State the Mean–value property.

9. State Weierstrass's theorem.

10. Define the Gamma function.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove Luca's theorem.

Or

(b) If Z_1, Z_2, Z_3, Z_4 are distinct points in the extended plane and T any linear transformation, prove that $(TZ_1, TZ_2, TZ_3, TZ_4) = (Z_1, Z_2, Z_3, Z_4)$.

12. (a) Compute $\int_\gamma x dz$, where γ is the directed line segment from 0 to $1+i$.

Or

(b) If $f(z)$ is analytic in an open disk Δ, prove that $\int_\gamma f(z) dz = 0$ for every closed curve γ in Δ.

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

(b) If $f(z)$ is analytic and non-constant in a region Ω, prove that its absolute value $|f(z)|$ has no maximum in Ω.

14. (a) How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2?

Or

(b) If u_1 and u_2 are harmonic in a region Ω, prove that $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$ for every cycle γ which is homologous to zero in Ω.

15. (a) Develop $\log\left(\frac{\sin z}{z}\right)$ in powers of z up to the term z^6.

Or

(b) Prove that every function which is mesomorphic in the whole plane is quotient of two entire functions.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If $u(x,y)$ and $v(x,y)$ have analytic first order partial derivatives which satisfy the Cauchy – Riemann differential equations, prove that $f(z)=u(z)+iv(z)$ is analytic with continuous derivative $f'(z)$ and conversely.

17. If the function $f(z)$ is analytic on R, prove that $\int_{\partial R} f(z)dz = 0$.

18. Apply the representation $f(z)=w_0+\varepsilon(z)^u$ to $\cos z$ with $z_0=0$. Determine $\varepsilon(z)$ explicitly.

19. Find the poles and residues of the following functions :

(a) $\frac{1}{z^2+5z+6}$

(b) $\cot z$.

20. If $f(z)$ is analytic in the region Ω containing z_0, prove that the representation.

$f(z)=f(z_0)+\frac{f'(z_0)}{1!}(z-z_0)+....+\frac{f^n(z_0)}{n!}(z-z_0{}^n)+....$ is valid in the largest open disk of center z_0 contained in Ω.