R–4494

Sub. Code
511201

M.Sc. DEGREE EXAMINATION, APRIL 2021

Second Semester

Mathematics

LINEAR ALGEBRA

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define linear combination of vectors.

2. If F is a field, verify that the n-tuple space, F^n is a vector space over the field F.

3. In R^3, let $\alpha_1 = (1, 0, 1)$, $\alpha_2 = (0, 1, -2)$, $\alpha_3 = (-1, -1, 0)$. If f is a linear functional on R^3 such that $f(\alpha_1) = 1$, $f(\alpha_2) = -1$, $f(\alpha_3) = 3$ and if $\alpha = (a, b, c)$ find $f(\alpha)$?

4. Define dual space of a vector space.

5. If f and g are non-zero polynomials over F. Show that (a) fg is a non-zero polynomial (b) $\deg(fg) = \deg f + \deg g$.

6. Find the g.c.d. of the following pairs of polynomials $3x^4 + 8x^2 - 3$, $x^3 + 2x^2 + 3x + 6$.

7. Find characteristic values of $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$.

8. Define n-linear function and give an example for two linear function.

9. If V be a real vector space and E an idempotent linear operator on V. Find $(I+E)^{-1}$.

10. Define T-annihilator of a vector.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) If A is an $m \times n$ matrix over F and B, C are $n \times p$ matrices over F, show that $A(dB+C) = d(AB) + AC$ for each scalar d in F.

Or

(b) If W is a proper subspace of a finite-dimensional vector space V. Prove that W is finite-dimensional and $\dim W < \dim V$.

12. (a) If W_1 and W_2 are subspaces of a finite-dimensional vector space, prove that $W_1 = W_2$ if and only if $W_1^0 = W_2^0$.

Or

(b) Suppose that A be an $n \times n$ matrix with real entries, prove that $A = 0$ if and only if trace $(A'A) = 0$.

13. (a) Prove that a polynomial f of degree n over a field F has at most n roots in F.

Or

(b) Suppose $f \equiv g \mod p$ and $f_1 \equiv g_1 \mod p$. Prove that $f + f_1 \equiv g + g_1 \mod p$, $f f_1 \equiv g g_1 \mod p$.

14. (a) Suppose that K be a commutative ring with identity, and let A and B be $n \times n$ matrices over K. Prove that $\det(AB) = (\det A)(\det B)$.

Or

(b) Prove that the determinant of the Vandermonde matrix is $(b-a)(c-a)(c-b)$.

15. (a) If V is a finite-dimensional vector space and let W_1 be any subspace of V, prove that there is a subspace W_2 of V such that $V = W_1 \oplus W_2$.

Or

(b) State and prove primary decomposition theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If W_1 and W_2 are finite-dimensional subspaces of a vector space V, prove that $W_1 + W_2$ is finite-dimensional and $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

17. If V be an n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Prove that the space $L(V, W)$ is finite-dimensional and has dimension mn.

18. If f, d are polynomials over a field F and d is different from 0 then there exist polynomials q, r in $F[X]$ such that (a) $f = dq + r$ (b) either $r = 0$ or $\deg r < \deg d$. Show that the polynomials q, r satisfying (a) and (b) are unique.

19. State and prove Cayley-Hamilton theorem.

20. State and prove Cyclic Decomposition theorem.

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R–4495

Sub. Code
511202

M.Sc. DEGREE EXAMINATION, APRIL 2021

Second Semester

Mathematics

REAL ANALYSIS — II

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define rectifiable curve.

2. If $f(x)=0$ for all irrational x, $f(x)=1$ for all rational x, prove that $f \notin R$ on $[a, b]$ for any $a<b$.

3. Define monotonically increasing function.

4. Give an example for $\{f_n\}$ and $\{g_n\}$ converge uniformly on some set E, but their product $\{f_n g_n\}$ does not converge uniformly on E.

5. Prove that $\{f_n\}$ is uniformly bounded on [0, 1], where

$$f_n(x)=\frac{x^2}{x^2+(1-nx)^2}, \ (0 \le x \le 1, \ n = 1, 2, \ldots)$$

6. Define equicontinuous functions.

7. Define radius of convergence of a power series.

8. Determine the interval of convergence for the power series $\sum_{n=0}^{\infty} n!(2x+1)^n$.

9. Prove that $|\sin nx| \le n|\sin x|$ for $n = 0, 1, 2, ..$ and x real.

10. Define Gamma function.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) If f is continuous on $[a, b]$ prove that the $f \in R(\alpha)$ on $[a, b]$.

Or

(b) State and prove the fundamental theorem of calculus.

12. (a) Suppose $\{f_n\}$ is a sequence of functions defined on E and suppose $|f_n(x)| \le M_n \, (x \in E, n = 1, 2, 3, ...)$ Prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

Or

(b) For $n = 1, 2, 3 ...., x$ real, put $f_n(x) = \frac{x}{1 + nx^2}$. Show that $\{f_n\}$ converges uniformly to a function f, and the equation $f'(x) = \lim_{n \to \infty} f'_n(x)$.

13. (a) Let B be the uniform closure of an algbera A of bounded functions then prove that B is uniformly closed algebra.

Or

(b) If $\{f_n\}$ and $\{g_n\}$ converges uniformly on a set E, prove that $\{f_n + g_n\}$ converges uniformly on E. If, in addition, $\{f_n\}$ and $\{g_n\}$ are sequence of bounded functions, Prove that $\{f_n g_n\}$ converges uniformly on E.

14. (a) Let e^x be defined on real field R by $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ prove that

(i) e^x is continuous and differentiable for all x.

(ii) $\lim_{x \to +\infty} x^n e^{-x} = 0$, for every n.

Or

(b) Show that (i) $\lim_{n \to \infty} (1 + x/n)^n = e^x$, (ii) $\lim_{x \to 0} \frac{\log(1+x)}{x}$ 1.

15. (a) If the sequence of complex functions $\{\phi_n\}$ is orthonormal on $[a, b]$ and if $f(x) \sim \sum_{n=1}^{\infty} c_n \phi_n(x)$,

show that $\sum_{n=1}^{\infty} |c_n|^2 \leq \int_a^b |f(x)|^2 \, dx$.

Or

(b) If f is continuous (with period 2π) and if $\in > 0$, prove that there a trigonometric polynomial P such that $|P(x) - f(x)| \leq \in$ for all real x.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If γ is continuous on $[a, b]$ prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)| dt$.

17. Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E and suppose that

$\lim_{t\to x} f_n(t) = A_n, (n = 1,2, 3,...)$. Prove that $\{A_n\}$ converges and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.

18. Let A be an algebra of real continuous functions on a compact set K. If A separates points on K and if A vanishes at no point of K, prove that the uniform closure B of A consists of all real continuous functions on K.

19. Suppose $a_0, a_1, .., a_n$ are complex numbers, $n \geq 1, a_n \neq 0$

$$P(z) = \sum_{n=0}^{n} a_k z^k$$

Prove that $P(z) = 0$ for some complex numbers z.

20. If $x > 0$ and $y > 0$ then prove that, $\int_0^t t^{x-1} (1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

R–4496

Sub. Code
511203

M.Sc. DEGREE EXAMINATION, APRIL 2021.

Second Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define holomorphic function with suitable example.

2. If $g(w)$ and $f(z)$ are holomorphic functions, prove that $g(f(z))$ is also holomorphic function.

3. Prove that $n(-\gamma, a) = -n(\gamma, a)$ is the index of the point a with respect to a closed curve γ.

4. State Cauchy's theorem on a circular disk.

5. Find the poles of $f(z) = \frac{1}{z^2 + 5z + 6}$.

6. Define removable singularity.

7. Define harmonic functions.

8. How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$.

9. Develop log $(\sin z/z)$ in powers of z upto the term z^4.

10. Prove that the Laurent development is unique.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove Cauchy-Reimann equations.

Or

(b) Expand $\frac{2z+3}{z+1}$ in powers of $z-1$ and find its the radius of convergence.

12. (a) Compute (i) $\int_{|z|=1} \frac{e^z}{z} dz$ and (ii) $\int_{|z|=2} \frac{1}{(z^2+1)} dz$.

Or

(b) if $f(z)$ is analytic in an open disk Δ, prove that $\int_\gamma f(z) dz = 0$ for every closed curvce γ in Δ.

13. (a) State and prove the maximum principle theorem.

Or

(b) Show that the function e^z, $\sin z$ and $\cos z$ have essential singularities at ∞.

14. (a) Let $f(z)$ be analytic except for isolated singularities a_j in a region Ω, prove that $\frac{1}{2\pi i}\int_\gamma f(z) dz = \sum_j n(\gamma, a_j) \operatorname{Re} s_{z=a_j} f(z)$.

Or

(b) Evaluate the following integral by the method of residues $\int_0^{\pi/2} \frac{1}{a+\sin^2 x} dx, \ |a| > 1.$

15. (a) Express the Taylor development of $\tan z$ and the Laurent development of $\cos z$ in terms of the Bernoulli numbers.

Or

(b) If the function $f_n(z)$ are analytic and $\neq 0$ in a region Ω, and if $f_n(z)$ converges to $f(z)$, uniformly on every compact subset of Ω, prove that $f(z)$ is either identically zero or never equal to zero in Ω.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Abel's limit theorem.

17. Suppose that $\varphi(\zeta)$ is continuous on the arc γ, prove that the function $F_n(z) = \int_\gamma \frac{\varphi(\zeta)}{(\zeta - z)^n} d\zeta$ is analytic in each of the region determined by γ, and derivative is $F_n'(z) = nF_{n+1}(z)$.

18. (a) State and prove Cauchy's theorem on a rectagular disk.

(b) Determine $\oint_C \frac{6}{z(z-3)} dz$, where C is the curve $|z-3| = 5$.

19. Evaluate $\int_0^\infty (1+x^2)^{-1} \, log \; x \; dx$.

20. State and prove the Weierstrass's theorem.

R–4497

Sub. Code
511501

M.Sc. DEGREE EXAMINATION, APRIL 2021

Second Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

Each question carries 2 marks.

1. Find the equation of the osculating plane at a general point on the cubic curve given by $r = (u, u^2, u^3)$.

2. Prove that $[r', r'', r'''] = \kappa^2 \tau$.

3. Define osculating circle.

4. Find the coordinates of the cylindrical helix whose intrinsic equations are $\kappa = \tau = \frac{1}{s}$.

5. Write down the parametric or freedom equations of a surface.

6. Define the anchor ring.

7. Write the condition for two family of curves are orthogonal.

8. Define isometric or applicable.

9. Define normal curvature.

10. Prove that geodesic curvature vector of any curve is orthogonal to the curve.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that the length of the common perpendicular d of the tangents at two near points distance s apart is approximately $d = \dfrac{\kappa \tau s^3}{12}$.

Or

(b) Show that $[\dot{r}, \ddot{r}, \dddot{r}] = 0$ is a necessary and sufficient condition that the curve be plane.

12. (a) Explain locus of the center of spherical curvature.

Or

(b) Show that the torsion of an involute of a curve is equal to $\dfrac{\rho(\sigma \rho' - \sigma' \rho)}{(\rho^2 + \sigma^2)\,(c - s)}$.

13. (a) A helicoid is generated by the screw motion of a straight line skew to the axis. Find the curve coplanar with the axis which generates the same helicoid.

Or

(b) Derive the angle between two parametric curves.

14. (a) If θ is the angle at the point (u, v) between the two directions given by $Pdu^2 + 2Qdudv + Rdv^2 = 0$, prove that $\tan\theta = \dfrac{2H(Q^2 - PR)^{\frac{1}{2}}}{ER - 2FQ + GP}$.

Or

(b) Find a surface of revolution which is isometric with a region of the right helicoid.

15. (a) If (λ, μ) is the geodesic curvature vector, prove that $\kappa_g = \dfrac{-H\lambda}{Fu' + Gv'} = \dfrac{-H\mu}{Eu' + Fv'}$.

Or

(b) Prove that every point P of a surface has a neighbourhood which is convex and simple.

Part C (3 × 10 = 30)

Answer any **three** questions.

Each question carries 10 marks.

16. Find the curvature and torsion of the cubic curve given by $r = (u, u^2, u^3)$.

17. Show that the intrinsic equations of the curve given by $x = ae^u \cos u, y = ae^u \sin u, z = be^u$ are $\kappa = \dfrac{\sqrt{2}\,a}{(2a^2 + b^2)^{\frac{1}{2}}} \dfrac{1}{s}, \tau = \dfrac{b}{(2a^2 + b^2)^{\frac{1}{2}}} \dfrac{1}{s}$.

18. Show that the parametric curves on the sphere given by $x = a \sin u \cos v$, $y = a \sin u \sin v$, $z = a \cos u$, $0 < u < \frac{1}{2}\pi, 0 < v < 2\pi$, form an orthogonal system. Determine the two families of curves which meet the curves v = constant at angles of $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$.

19. Prove that, on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$, when $v = c$, for all values of u.

20. Prove that every point P of the surface has a neighbourhood N with the property that every point of N can be joined to P by a unique geodesic arc which lies wholly in N.

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R5369

Sub. Code
511401

M.Sc. DEGREE EXAMINATION, APRIL – 2021

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define Banach space.

2. Define bounded linear operator.

3. Define Hilbert space with an example.

4. State Parallelogram equality.

5. Define Hilbert adjoint operator.

6. Define unitary and normal operators.

7. Define adjoint operator.

8. Show that a norm on a vector space X is a sublinear functional on X.

9. State open mapping theorem.

10. Prove that every subsequence of (x_n) converges weakly to x.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the Riesz's lemma.

Or

(b) In a finite dimensional normed space $X, M \in X$ is compact then prove that if and only if M is closed and bounded.

12. (a) State and prove Schwarz inequality.

Or

(b) Let M be a compact subspace y and $x \in X$ fixed then prove that $z = x - y$ is orthogonal to y.

13. (a) State and prove the existence of Hilbert adjoint operator T^* of T with the norm $\|T^*\| = \|T\|$.

Or

(b) Let the operators $U : H \to H$ and $V : H \to H$ be unitary (Here, H is a Hilbert space) prove that

(i) $\|U\| = 1$, provided $H \neq \{0\}$

(ii) U is normal.

14. (a) State and prove Zorn's lemma.

Or

(b) Prove that every Hilbert space is reflexive.

15. (a) Prove that strong convergence implies weak convergence with some limit.

Or

(b) Prove that A sequence (T_n) of operator $T_n \in B(X,Y)$ where X,Y are Banach space is strongly operator convergent if and only if

(i) The sequence $(\|T_n\|)$ is bounded.

(ii) The sequence $(T_n x)$ is Cauchy in y for every x is a total subset M of X.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove inverse operator theorem.

17. State and prove Bessel inequality.

18. State and prove the properties of Hilbert adjoint operators.

19. State and prove Hann-Banach theorem.

20. State and prove closed graph theorem.

R5370

Sub. Code
511402

M.Sc. DEGREE EXAMINATION, APRIL –2021

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2019 Onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all the** questions.

1. Define conditional probability.

2. Define moment generating function.

3. Define random vector.

4. Define expected value of a random vector.

5. Define Bernoulli distribution

6. Write the pdf of $\Gamma(\alpha, \beta)$ distribution.

7. Define t-distribution.

8. Define order statistic.

9. Define convergence in probability.

10. State weak law of large numbers

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11\. (a) What is the probability of getting six different faces when throwing six perfect dice?

Or

(b) Cast a die a number of independent times until a six appears on the upside of the die. Let Y be the random variable defined by the number of casts needed to obtain the first six, Let $Z = e^{-Y}$. Calculate $E[Z]$.

12\. (a) Let (X_1, X_2) be a random vector, Let $Y_1 = g_1(X_1, X_2)$ and $Y_2 = g_2(X_1, X_2)$ be random variables whose expectations exist. Prove that for all real numbers k_1 and k_2, $E(k_1 Y_1 + k_2 Y_2) = k_1 E[Y_1] + k_2 E[Y_2]$.

Or

(b) Let the continuous type random variables X and Y have the joint pdf $f(x,y) = \begin{cases} e^{-y} \text{ if } 0 < x < y < \infty \\ 0 \quad \text{elsewhere} \end{cases}$.

Determine the marginal density functions of X and Y.

13\. (a) The mgf of a random variable X is $e^{4(e^t - 1)}$. Show that $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.931$.

Or

(b) Let X have a $\chi^2(r)$ distribution. Prove that if $k > -r/2$, then $E(X^k)$ exists and is given by $E(X^k) = \dfrac{2^k \Gamma(r/2 + k)}{\Gamma(r/2)}$.

14. (a) Let Y_1, Y_2, Y_3 be the order statistics of a random sample of size 3 from a distribution having pdf $f(x) = \begin{cases} 1 \text{ } if \text{ } 0 < x < 1 \\ 0 \text{ } elsewhere \end{cases}$ .Determine the pdf of the sample range $Z = Y_3 - Y_1$

Or

(b) Let $f(x) = \frac{1}{6}, x = 1,2,3,4,5,6$ and zero elsewhere, be the pmf of a distribution of the discrete type. Show that the pmf of the smallest observation of a random sample of size 5 from this distribution is $g(y) = \left(\frac{7-y}{6}\right)^5 - \left(\frac{6-y}{6}\right)^5$, $y = 1,2,3,4,5,6$ and zero elsewhere

15. (a) Let T_n have a t-distribution with n degrees of freedom. $n = 1,2,3.......$ Prove that the limiting distribution of T_n is standard normal distribution.

Or

(b) Compute an approximate probability that the mean of a random sample of size 15 from a distribution having pdf $f(x) = 3x^2, 0 < x < 1$, zero elsewhere, is between $\frac{3}{5}$ and $\frac{4}{5}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Chebyshev's inequality.

17. If the random variables X_1 and X_2 have the joint pdf $f(x_1, x_2) = 2e^{-x_1 - x_2}, 0 < x_1 < x_2, 0 < x_2 < \infty$, zero elsewhere, show that X_1 and X_2 are dependent.

18. Prove that if the random variable X is $N(\mu, \sigma^2)$ then the random variable $V = (X - \mu)^2 / \sigma^2$ *is* $x^2(1)$.

19. State and prove Student's Theorem.

20. State and prove Central Limit Theorem.

R5371

Sub. Code
511403

M.Sc. DEGREE EXAMINATION, APRIL – 2021.

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS – 2019 onwards)

Time : Three Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define Isomorphism on Graphs.

2. Define eccentricity of a vertex in a graph.

3. Define Eulerian Tour.

4. Define block and k-chromatic graph.

5. Define Perfect graph.

6. Write down the hall's Theorem (Marriage theorem)

7. Define edge chromatic number.

8. What is a Hajo's graph? Explain it with diagram.

9. Define faces of a graph G.

10. Write down the Euler's formula with clear explanation.

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) Show that a graph is connected iff it contains a spanning tree.

Or

(b) For an edge e of a connected graph G, then show that $G-e$ is connected iff e is a cycle edge of G.

12. (a) Prove that two different blocks of a graph can have atmost one vertex in common.

Or

(b) Prove that if a connected graph G has $2k$ vertices of odd degree, then there is a set of k-edge disjoint trails that use all the edges of G.

13. (a) Show that for every 3-regular graph without cut edge has perfect matching.

Or

(b) Show that $r_n \le n(r_{n-1} - 1) + 2$.

14. (a) Show that the join of the graphs G and H has chromatic number $\chi(G+H) = \chi(G) + \chi(H)$.

Or

(b) Show that for a chromatically $k-$critical graph G, then no vertex of G has degree less than $k-1$.

15. (a) (i) Show that the dual of any planar graph is connected

(ii) If G is a plane graph, then show that $\sum_{f \in F} d(f) = 2m$.

Or

(b) For a simple planar graph G on atleast three vertices, then show that $m \leq 3n - 6$. Further more, $m = 3n - 6$ if and only if every planar embedding of G is a triangulation.

Part C (3 × 10 = 30)

Answer any **Three** questions.

16. For a graph T with n vertices, Prove that the following are equivalent:

(a) T is a tree.

(b) T contains no cycles and has $n-1$ edges.

(c) T is connected and has $n-1$ edges.

(d) T is connected and every edge is cut edge.

(e) Any two vertices of T are connected by exactly one path.

17. For a simple n vertex graph G, where $n \geq 3$, such that $\deg(x) + \deg(y) \geq n$ for each pair of non-adjacent vertices x and y, then show that G is Hamiltonian.

18. State and Prove Turan's theorem.

19. State and Prove the Brook's Theorem.

20. Prove that every loopless Planar graph is 5-colourable.