R5794

Sub. Code
511101

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2021

First Semester

Mathematics

GROUPS AND RINGS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Cyclic subgroup.

2. Define Normal subgroup.

3. Write kernel of homomorphism.

4. What is Even permutation?

5. Write the second part of Sylow's theorem.

6. Define Solvable.

7. Define an Isomorphic.

8. Define Maximal ideal.

9. What is mean by Integer monic polynomial?

10. Define an Euclidean ring.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that, every subgroup of an abelian group is normal.

Or

(b) Prove that, the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

12. (a) If G is a finite group then prove that, $c_a = o(G)/o(N(a))$. In other words, the number of elements conjugate to a in G if the index of the normalizer of a in G.

Or

(b) (i) Define commutator subgroup and give example.

(ii) Prove: $N(a)$ is a subgroup of G.

13. (a) State and prove second part of p-sylow's theorem.

Or

(b) If G and G' are isomorphic abelian groups then prove that, for every integer s, $G(s)$ and $G'(s)$ are isomorphic.

14. (a) If $a+bi$ is not a unit of $J[i]$ then prove that, $a^2+b^2>1$.

Or

(b) If φ is a homomorphism of R into R', then prove the following:

(i) $\varphi(0)=0$.

(ii) $\varphi(-a)=-\varphi(a)$ for every $a \in R$.

15. (a) State and prove Gauss Lemma.

Or

(b) If p is a prime number of the form $4n+1$, then prove that, the congruence $x^2 \equiv -1$ mode p.

Part C (3 × 10 = 30)

Answer any **three** of the following.

16. Prove: A subgroup N of G is a normal subgroup of G if and only if the product of two right cosets of N in G is again a right coset of N in G.

17. Solve the following:

(a) Conjugancy is an equivalence relation on G.

(b) If G is an abelian of order $o(G)$ and $p^n | o(G), p^{n+1} | o(G)$, there is a Unique subgroup of G of order p^n.

18. State and prove third part of p-sylow's theorem.

19. Prove that, If R is a commutative ring with identity and M is an ideal of R, then M is a maximal of R iff $R|M$ is a field.

20. Prove that, If R is a unique factorization domain, then so is $R[x]$.

R5795

Sub. Code
511102

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2021

First Semester

Mathematics

REAL ANALYSIS – I

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define compact spaces with an example.

2. Define perfect sets with an example.

3. Define subsequences with an example.

4. Prove the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is convergent.

5. Define root test.

6. Give an example of a series which is not convergence but absolutely convergence.

7. Give an example of a function which is continuous but not differentiable.

8. Define monotonic functions.

9. Define derivatives of higher order.

10. Define L'Hospital rule.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that set $\mathbb{N} \times \mathbb{N}$ is countable.

Or

(b) Prove that $\mathbb{R}$ is not compact.

12. (a) Prove that every convergent sequence is bounded.

Or

(b) State and prove that necessary condition for convergence of series.

13. (a) Show that the series $1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots$ converges.

Or

(b) Prove that every absolutely convergent series is convergent.

14. (a) If f is continuous on $[a,b]$, then prove that it is bounded on $[a,b]$.

Or

(b) Explain the types of discontinuities with an example.

15. (a) Let $f(x) = |x| - 1$, for all $x \in \mathbb{R}$. Then show that f is derivable at all points except $x = 0$.

Or

(b) State and prove mean value theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that every infinite subset of a countable set A is countable.

17. Suppose s_n, t_n, are sequences and $\lim_{n\to\infty} s_n = s$, $\lim_{n\to\infty} t_n = t$. Then prove that

    (a) $\lim_{n\to\infty}(s_n + t_n) = s + t$

    (b) $\lim_{n\to\infty}(s_n t_n) = st$

    (c) $\lim_{n\to\infty}(cs_n) = cs$ for any number c.

18. (a) Test the convergence of the series $\sum u_n$, where $\sum u_n = \sqrt{n^4 + 1} - n^2$.

    (b) Show that the series $1 + x + \frac{x^2}{2!} + \cdots$ converges absolutely for all values of x.

19. (a) Let $A \subset \mathbb{R}$, let f and g be functions on A to $\mathbb{R}$ and let $b \in \mathbb{R}$. Suppose that $c \in A$ and that f and g are continuous at c. Then prove that

    (i) $f + g, f - g, fg, bf$ are continuous at c.

    (ii) f/g is continuous at c, when $h(x) \neq 0$.

    (b) If f is continuous on $[a,b]$, then prove that it is bounded on $[a,b]$.

20. State and prove Cauchy's mean value theorem.

R5796

Sub. Code
511103

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2021

First Semester

Mathematics

DIFFERENTIAL EQUATIONS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. When do we say two functions are linearly independent?

2. Define non-homogeneous equation of order two.

3. Write standard form of Chebyshev equation.

4. Show that the coefficient of x^n in $P_n(x)$ is $\frac{(2n)!}{2^n (n!)^2}$.

5. Define a partial equation of second order with two variables.

6. Write a short note on Jacobi's method.

7. Define one-dimensional wave equation.

8. Is the one dimensional diffusion equation parabolic? Justify your answer.

9. Define Laplace equation.

10. State interior Dirichlet problem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove the necessary and sufficient condition for the Wronskian of two functions is non-zero.

Or

(b) Find all solutions of the following equations:

(i) $y'' + 4y = \cos x$

(ii) $y'' + y = 2 \sin x \sin 2x$.

12. (a) Let φ_1 be a solution of $L(y) = 0$ on an interval I. and suppose $\varphi_1(x) \neq 0$ on I. If $v_2, v_3, \ldots, v_n$ is any basis on I for the solutions of the linear equation $\varphi_1 v^{(n-1)} + \ldots + \left[n\varphi_1^{(n-1)} + a_1(n-1)\varphi_1^{(n-2)} + \ldots + a_{n-1}\varphi_1\right]v = 0$ of order $n-1$, and if $v_k = u'_{k'}(k = 2, \ldots, n)$, then show that $\varphi_1, u_2\varphi_1, \ldots, u_n\varphi_1$ is a basis for the solutions of $L(y) = 0$ on I.

Or

(b) Consider the following equations $y'' = \frac{2}{x^2} y = 0 (0 < x < \infty)$. Find the basis of the solutions of the given equation.

13. (a) Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$.

Or

(b) Find the integral surface of the linear partial differential equation $x\left(y^2/z\right)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0, z=1$.

14. (a) Derive telegraphy equation.

Or

(b) Prove that a linear combination of solutions of the homogeneous linear partial differential equation is also a solution.

15. (a) Show the existence of the solution of the interior Neumann problem.

Or

(b) State exterior Dirichlet problem and solve it.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let b be continuous on an interval I, and let $\varphi_1, \varphi_2, \ldots, \varphi_n$ be n linearly independent solutions of $L(y) = 0$ on I. Then show that every solution ψ of $L(y) = b(x)$ can be written as linear combination of these n solutions together with the particular solution ψ_p.

17. Find a basis for the solutions of Legendre equation.

18. Explain Charpit's method in detail and find a complete integral of the equation $p^2 + x + q^2 y = z$.

19. Solve the equation

(a) $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 y^2}$.

(b) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{xy^2} = x - y$.

20. Explain the concept of elementary solutions of Laplace's equation.

R5797

Sub. Code
511104

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2021

First Semester

Mathematics

ANALYTIC NUMBER THEORY

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. If $(a,b) = (a,c) = 1$ prove that $(a, bc) = 1$.

2. Define möbious function.

3. Define mutual visibility of two lattice points.

4. Write the Legendre's identity.

5. Write the Chebyshev's function.

6. Define "little oh" notation

7. Define reduced residue system.

8. State the fundamental theorem of algebra.

9. Define quadratic residue.

10. Define Diophantine equations.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that given any two integers a and b, there is a common divisor d or a and b of the form $d = ax + by$, where x and y are integers. Prove also that every common divisor of a and b divides d.

Or

(b) Suppose $n \geq 1$, prove that $\log n = \sum_{d|n} \wedge(d)$.

12. (a) Use Euler's summation formula to deduce the following for $x \geq 2$:

$$\sum_{2 \leq n \leq x} \frac{1}{n \log n} = \log(\log x) + A + O\left(\frac{1}{x \log}\right),$$

where A is constant.

Or

(b) If $x \geq 2$ prove that

$$\sum_{n \leq x} \frac{d(n)}{n} = \frac{1}{2}\log^2(x) + 2C \log x + O(1),$$

where C is Euler's constant.

13. (a) Derive the upper and lower bounds on the size of the with prime.

Or

(b) Prove that there is a constant A such that

$$\sum_{p \leq x} \frac{1}{p} = \log \log x + A + O\left(\frac{1}{\log x}\right), \forall x \geq 2 .$$

14. (a) Assume $(a, m) = d$ and suppose that $d|b$. Prove that the linear congruence $ax \equiv b(\text{mod } m)$ has exactly d solutions modulo m and list all the solutions.

Or

(b) Find all n for which $\varphi(n) \equiv 2 \pmod 4$.

15. (a) State and prove Euler's criterion.

Or

(b) Suppose P is an odd positive integer, prove that

$$(-1|P) = (-1)^{\left(\frac{P-1}{2}\right)}$$

and $$(2|P) = (-1)^{\left(\frac{P^2-1}{8}\right)}.$$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the fundamental theorem of arithmetic.

17. Prove that

(a) two lattice points (a, b) and (m, n) are mutually visible if and only if $(a-m)$ and $(b-n)$ are relatively prime.

(b) the set of lattice points visible from the origin has density $6/\pi^2$.

18. Let p_n denote the n^th^ prime. Prove that the following asymptotic relations are logically equivalent:

$$\lim_{x\to\infty} \frac{\pi(x)\log x}{x} = 1$$

$$\lim_{x\to\infty} \frac{\pi(x)\log \pi(x)}{x} = 1$$

$$\lim_{x\to\infty} \frac{p_n}{n\log n} = 1$$

19. (a) Prove that for any prime p all the coefficients of the polynomial, prove that

$$f(x) = (x-1)(x-2)\ldots(x-p+1) - x^{(p-1)} + 1$$

are divisible by p.

(b) A yardstick divided into inches is again divided into 70 equal parts. Prove that among the four shortest divisions two have left endpoints corresponding to 1 and 19 inches. What are the right endpoints of the other two?

20. (a) If p and q are distinct odd primes and if χ is the quadratic character mod p. Prove that

$$G(1,\chi)^{(q-1)} = (q|p) \sum_{\substack{r_1 \bmod p \\ r_1+\ldots+r_q \equiv q (\bmod p)}} \cdots \sum_{r_q \bmod p} (r_1 \ldots r_q | p).$$

(b) If P and Q are positive odd integers with $(P, Q) = 1$, then prove that

$$(P|Q)(Q|P) = (-1)^{(P-1)(Q-1)/4}.$$

R5798

Sub. Code
511512

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2021

First Semester

Mathematics

OBJECT ORIENTED PROGRAMMING AND C++

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. What is C++?

2. The operator ———— is called insertion operator and the operator ———— is called extraction operator.

3. Write the form of function prototyping.

4. Describe: 'return by reference' in a function.

5. Write the general form of class declaration.

6. State the general format for calling a member function

7. Task of constructor is ————

8. Define 'operator function'.

9. Inheritance is ———— feature of OOP.

10. Write the general form of defining derived class.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Explain the general structure of C++ program.

Or

(b) Describe any six keywords used in C++ program.

12. (a) Illustrate scope resolution operator with a suitable C++ program.

Or

(b) Apply the concept of call by reference to swap two values in a C++ program.

13. (a) Describe two different ways of defining member functions in a class.

Or

(b) Interpret member function of one class can be friend function of another class in a C++ program.

14. (a) Explain the concept of copy constructor with an example C++ program.

Or

(b) Illustrate overloading unary minus operator with a C++ program.

15. (a) Interpret how to make a private member inheritable.

Or

(b) Write a program to implement the concept of virtual base class.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Explain the following:

    (a) Basic concepts of OOPS

    (b) Benefits of OOPs

    (c) Applications OOPs

17. Apply the concept of recursive function to find factorial value and hence examine sin x in series for given degree value x in a C++ program.

18. Assess dynamic initialization of objects to find compound interest value in a C++ program.

19. Diagnose the overloading binary operator to find sum of complex numbers in a C++ program.

20. Compare different forms of inheritances available in C++.

———————

R5799

Sub. Code
511301

M.Sc. Mathematics DEGREE EXAMINATION, NOVEMBER – 2021

Third Semester

MECHANICS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Classify the forces that act on a body based on their mode of application.

2. Define non holonomic constraints.

3. Interpret the statement: Generalized momentum is linear in $\dot{q}$

4. Show that the generalized momentum corresponding to each ignorable coordinates is a constant.

5. State the necessary and sufficient condition for a stationary value of a definite integral $I = \int_{x}^{x} f[y(x),\ y'(x), x]\, dx$.

6. Describe Jacobi's form of the principle of least action.

7. What do you mean by Hamilton's principal function?

8. State two Stackel conditions for a separable system.

9. Define point transformation.

10. What do you mean by Lagrange bracket expression for two variables?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the principle of virtual work.

Or

(b) A particle of mass in suspended by a massless wire of length $r = a + b \cos \omega t$, $a > b > 0$ to form a spherical pendulum. Find the equations of motion.

12. (a) Express the kinetic energy of a system in terms of the generalized coordinates.

Or

(b) Derive the Jacobi integral for a conservative system described by standard nonholonomic form of Lagrange's equations.

13. (a) Obtain the Hamilton's canonical equations of motion for a holonomic system.

Or

(b) Derive the modified Hamilton's principle for a holonomic system.

14. (a) State and prove Jacobi's theorem,

Or

(b) Prove that Liouville conditions are sufficient for the separability of an orthogonal system.

15. (a) Show that the rheonomic transformation $Q = \sqrt{2q}\ e^t \cos\rho,\ P = \sqrt{2q}\ e^t \sin\ \rho$ is canonical.

Or

(b) State and prove the Poisson's theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Compute the generalized forces for a system with an applied force and moment.

17. Derive the standard form of a Lagrange's equation for a holonomic system.

18. Determine Euler-Lagrange equation.

19. Use the Jacobi form of the principle of least action to obtain the orbit for Kepler problem.

20. Evaluate the relationship between Lagrange and Poisson brackets and show that $L = P^{-1}$.

R5800

Sub. Code
511302

M.Sc. DEGREE EXAMINATION, NOVEMBER 2021.

Third Semester

Mathematics

TOPOLOGY

(CBCS – 2019 onwards)

Time : 3 Hours　　　　　　　　　　Maximum : 75 Marks

Part A　　　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Define box topology.

2. Let $X = \{1,2,3.4\}$ and $Y = \{a,b,c,d\}$,
$\tau_1 = \{\phi, X, \{1\}, \{1,2\}, \{1, 2, 3\}\}$ and
$\tau_2 = \{\phi, Y, \{a\}, \{b\}, \{a,b\}, \{b,c,d\}\}$.
Define $f : X \to Y$ as $f(1) = b, f(2) = c, f(3) = d,$
$f(4) = c$. Prove or disprove that f is continuous.

3. Define punctured Euclidean space.

4. Define uniform topology.

5. Define Lebesgue number.

6. Define countably compact.

7. Define sorgenfrey plane.

8. Define perfectly normal space.

9. Define locally compact space.

10. Write the countable intersection property.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11\. (a) Let X be a topological space. Suppose that $\mathscr{C}$ is a collection of open sets of X such that for each $x \in X$ and each open set U of X, there is an element C of $\mathscr{C}$ such that $x \in C \subset U$. Prove that $\mathscr{C}$ is a basis for the topology of X.

Or

(b) Let $\mathscr{C}$ be a collection of subsets of the set X, Suppose that ϕ and X are in $\mathscr{C}$ and that finite union and arbitrary intersection of elements of $\mathscr{C}$ are in $\mathscr{C}$. Show that the collection $\mathscr{T} = \{X - C \mid C \in \mathscr{C}\}$ is a topology on X.

12\. (a) State and prove the intermediate value theorem.

Or

(b) Prove that a space X is locally path connected if and only if for every open set U of X, each path component of U is open in X.

13\. (a) Prove that every compact subset of a Hausdorff space is closed.

Or

(b) Prove that a metric space X is complete if every Cauchy sequence in X has a convergent subsequence.

14\. (a) Prove that the subspace of a Hausdorff space is Hausdorff and the subspace of regular space is regular.

Or

(b) Prove that a regular space with a countable basis is normal.

15. (a) Let X be a Hausdorff space. Then X is locally compact at x if and only if for every neighbourhood U of x, there is a neighbourhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$.

Or

(b) Prove that every locally compact Hausdorff space is completely regular.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If each space X_α is a Hausdorff space, then prove that ΠX_α is a Hausdorff space in both the box and product topologies.

17. If L is a linear continuum in the order topology, then prove that L is connected and so is every interval and ray in L.

18. Let X be a simply ordered set having the least upper bound property. In the order topology, prove that each closed interval in X is compact.

19. State and prove Tietze extension theorem.

20. State and prove Tychonoff theorem.

R5801

Sub. Code
511303

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2021

Third Semester

Mathematics

OPTIMIZATION TECHNIQUES

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** the questions.

1. Define spanning tree.

2. Define cut and cut capacity.

3. Define a basic solution of $AX = b$.

4. Show that the following set is convex:
$C = \{(x_1, x_2) \mid x_1 + x_2 \leq 1, x_1 \geq 0, x_2 \geq 0\}$

5. Define value of the game.

6. Define saddle point of a game.

7. Define absolute and relative minimum.

8. Define control matrix of the constrained derivative method.

9. Define separable function.

10. Define a quadratic programming model.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Write Dijkstra's algorithm.

Or

(b) Using Floyd's algorithm to determine the shortest route between any two nodes the network given in figure 1.

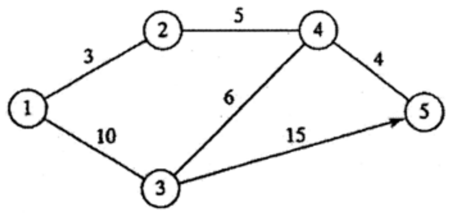

Figure 1

12. (a) Determine and classify (as feasible and infeasible) all the basic solution of the following system of equations

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}.$$

Or

(b) Write the revised simplex algorithm.

13. (a) The following game give A's payoff. Determine the values of p and q that will make the entry (2, 2) of the game a saddle point:

	B_1	B_2	B_3
A_1	1	q	6
A_2	p	5	10
A_3	6	2	3

Or

(b) Consider the following game: The pay off is for player A.

	B_1	B_2	B_3
A_1	2	1	3
A_2	2	4	1
A_3	5	6	2

Write the player B's linear program.

14. (a) Derive the necessary and sufficient condition for X_0 to be an extreme point of $f(X)$.

Or

(b) Write the KKT condition for the following problem:

Min $f(X) = x_1^4 + x_2^2 + 5x_1x_2x_3$

Subject to

$x_1^2 - x_2^2 + x_3^3 \leq 10$

$x_1^3 + x_2^2 + 4x_3^2 \geq 20$

15. (a) Write the algorithm for Dichotomous method.

Or

(b) Write the algorithm for Golden section method.

Part C (3 × 10 = 30)

Answer any **three** questions.

16.

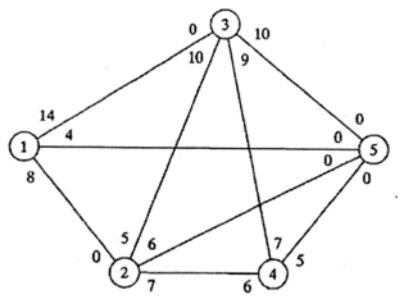

Figure 2

Determine the maximal flow and the optimum flow in each arc for the above network.

R5801

17. Solve the following linear programming model using upper-bounding algorithm

max $z = 2x_1 + x_2$

subject to $x_1 + x_2 \le 3$

$$0 \le x_1 \le 2\,,\ 0 \le x_2 \le 2.$$

18. Solve the following game graphically. The payoff is for player A.

	B_1	B_2	B_3	B_4
A_1	2	1	3	–1
A_2	4	3	2	6

19. Solve the linear programming problem using Lagrangean method:

Max $x = 2x_1 + 3x_2$

Subject to

$$x_1 - x_2 + x_3 = 5$$
$$x_1 - x_2 + x_4 = 3$$
$$x_1, x_2 x_3, x_4 \ge 0.$$

20. Solve the problem:

Max $z = x_1 + x_2^4$

Subject to

$$3x_1 + 2x_2 \le 9$$
$$x_1, x_2 \ge 0$$

R5802

Sub. Code
511503

M.Sc. DEGREE EXAMINATION, NOVEMBER – 2021

Third Semester

Mathematics

MULTIVARIATE CALCULUS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. When f is said to be differentiable at x?

2. Define contraction mapping.

3. Prove that a linear operator A on R^n is invertible if $\det[A] \neq 0$

4. Define Jacobian's operator.

5. Define flip with example.

6. Define a differential form of order $k \geq 1$ in E.

7. Define affine mapping.

8. Give an example for boundary of the oriented affine k – simplex.

9. Define area integral of f over Φ.

10. State Stokes theorem.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Suppose E is an open set in R^n, f maps E into R^m, f is differentiable at $x_0 \in E, g$ maps an open set containing $f(E)$ into R^k and g is differentiable at $f(x_0)$. Then prove that the mapping F of E into R^k by $F(x) = g(f(x))$ is differentiable at x_0 and $F'(x_0) = g'(f(x_0))f'(x_0)$.

Or

(b) If X is a complete metric space and if φ is a contraction of X into X, then prove that there exists one and only one $x \in X$ such that $\varphi(x) = x$.

12. (a) If $[A]$ and $[B]$ are n by n matrices, then prove that $\det([B][A]) = \det[B]\det[A]$.

Or

(b) Define projection operator and explain its properties.

13. (a) Suppose $\omega = \Sigma_I b_I(\mathbf{x})dx_I$ is the standard presentation of a k-form w in an open set $E \subset R^n$. If $\omega = 0$ in E, then prove that $b_I(\mathrm{x}) = 0$ for every increasing k –index I and for every $x \in E$.

Or

(b) Explain change of variables on a multiple integrals.

14. (a) Suppose T is a C' mapping of an open set $E \subset R^n$ into an open set $V \subset R^m$, S is a C' mapping of V into an open set $W \subset R^p$ ω and a is a k – form in W, ω_s is a k – form in V and both $(\omega_s)_T$ and ω_{ST} are k – forms in E, where ST is defined by $(ST)(x) = S(T(x))$. Then prove that $(\omega_S)_T = \omega_{ST}$.

Or

(b) Discuss about the positively oriented boundaries.

15. (a) Suppose E is a convex open set in R^n, $f \in C'(E)$, p is an integer, $1 \le p \le n$ and $(D_j f)(x) = 0, (p < j \le n, x \in E)$. Then prove that there exists an $F \in C'(E)$ such that $(D_p F)(x) = f(x), (D_j F)(x) = 0, (p < j \le n, x \in E)$.

Or

(b) State and prove Green's theorem.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove inverse function theorem.

17. Suppose $m.n, r$ are nonnegative integers, $m \ge r.n \ge r, F$ is a C' mapping of an open set $E \subset R^n$ into R^m and $F'(x)$ has rank r for every $x \in E$. Fix $a \in E$, put $A = F'(a)$, let Y_1 be the range of A and let P be a projection in R^m whose range is Y_1. Let Y_2 be the null space of P. Then prove that there are open sets U and V in R^n with $a \in U, U \subset E$ and there is a $1-1C'$ mapping H of V on to U such that $F(H(x)) = Ax + \varphi(Ax), (x \in V)$ where φ is a C' mapping of the open set $A(V) \subset Y_1$ into Y_2.

18. Suppose F is a C' mapping of an open set $E \subset R^n$ into R^n, $0 \in E, F(0) = 0$ and $F'(0)$ is invertible. Then prove that there is a neighborhood of 0 in R^n in which a representation $F(x) = B_1 ... B_{n-1} G_n \circ ... \circ G_1(x)$ is valid. Also, prove that each G_i is a primitive C' mapping in some neighborhood of $0, G_i(0), G_i'(0)$ is invertible and each B_i is either a flip or the identify operator.

19. Prove that

(a) If ω and λ are k– and $m-$ forms, respectively, of class C' in E, then $d(\omega \wedge \lambda) = (d\omega) \wedge \lambda + (-1)^k \omega \wedge d\lambda$.

(b) If ω is the class C'' in E, then $d^2\omega = 0$.

20. If ψ is a k- chain of class C'' in an open set $V \subset R^m$ and if ω is a $(k-1)$ form of class C' in V, then prove $\int_{\psi} d\omega = \int_{\partial\omega} \omega$.
