R6584

Sub. Code
511201

M.Sc. DEGREE EXAMINATION, APRIL – 2022.

Second Semester

Mathematics

LINEAR ALGEBRA

(CBCS – 2019 onwards)

Time : 3 Hours　　　　　　　　　　Maximum : 75 Marks

Part A　　　　　　$(10 \times 2 = 20)$

Answer **all** questions.

Each question carries 2 marks.

1. If V is a finite-dimensional vector space, prove that any two bases of V have the same number of elements.

2. Show that the vectors $\alpha_1 = \{1, 0, -1\}$, $\alpha_2 = \{1, 2, 1\}$, $\alpha_3 = \{0, -3, 2\}$ form a basis for R^3.

3. Find the range, rank, null space, and nullity for the zero transformation on a finite-dimensional space V.

4. Define linear functional on a vector space V with an example.

5. Suppose f, g, and h are polynomials over the field F such that $f \neq 0$ and $fg = fh$. Prove that $g = h$.

6. Define algebraically closed field.

7. Define diagonalizable of a matrix.

8. If A is an invertible $n \times n$ matrix over a field, show that det $A \neq 0$.

9. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & -8 \\ 0 & -1 \end{bmatrix}$ find an invertible, real matrix P such that $P^{-1}AP$ and $P^{-1}BP$ are both diagonal.

10. If E_1 and E_2 are projections onto independent subspaces, prove that $E_1 + E_2$ is a projection or not?

Part B $(5 \times 5 = 25)$

Answer **all** questions by choosing either (a) or (b).

11. (a) Let V be a vector space over the field F. Prove that the intersection of any collection of subspaces of V is a subspace of V.

Or

(b) If R be a non-zero row-reduced echelon matrix. Show that the non-zero row vectors of R form a basis for the row space of R.

12. (a) If A is an $m \times n$ matrix with entries in the field F, prove that row rank (A) = *column rank* (A).

Or

(b) Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n.

13. (a) Suppose that F be a field, Show that the intersection of any number of ideals in $F[x]$ is an ideal.

Or

(b) Suppose f and d are non-zero polynomials over a field F such that deg $d \leq deg\, f$. Prove that there exists a polynomial g in $F[x]$ such that either $f - dg = 0$ or $deg\ (f - dg) < deg\, f$.

14. (a) Prove that a linear combination of n-linear functions is n-linear.

Or

(b) If A be a 2×2 matrix over a field F, and suppose that $A^2 = 0$, show that det $(cI - A) = c^2$ for each scalar c.

15. (a) If A is a complex 5×5 matrix with characteristic polynomial $f = (x-2)^3(x+7)^2$ and minimal polynomial $p = (x-2)^2(x+7)$, what is the Jordan form for A?

Or

(b) If A is the companion matrix of a monk polynomial p, prove that p is both the minimal and the characteristic polynomial of A.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

Each question carries 10 marks.

16. Let V be a vector space which is spanned by a finite set of vectors $\beta_1, \ldots, \beta_m$. Prove that any independent set of vectors in V is finite and contains no more than m elements.

17. If V be a finite-dimensional vector space over the field F, and let W be a subspace of V. Prove that $dim\ W + dim\ W^\circ = dim\ V$.

18. State and prove Taylor's Formula.

19. If V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

20. State and prove Generalized Cayley – Hamilton Theorem.

———————

R6585

Sub. Code
511202

M.Sc. DEGREE EXAMINATION, APRIL – 2022

Second Semester

Mathematics

REAL ANALYSIS – II

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

Each question carries 2 marks.

1. Define Riemann-Stieltjes integral.
2. Define Holder's inequality.
3. Find limit for the sequence of functions

$$f_n(x) = \frac{x^2}{(1+x^2)^n}, \ (x \text{ real} ; n = 0, 1, 2, \ldots)$$

4. Define uniform convergent sequence.
5. Define Pointwise boundedness of a sequence of functions.
6. If $f \in \mathcal{B}$ and $g \in \mathcal{B}$ then prove that max $(f, g) \in \mathcal{B}$ and min $(f, g) \in \mathcal{B}$, where $\mathcal{B}$ is uniform closure of algebra $\mathcal{A}$.
7. Define analytic function and give an example.
8. Find the radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$.

9. Suppose $0 < \delta < \pi$, $f(x) = 1$ if $|x| \leq \delta$, $f(x) = 0$ if $\delta \leq |x| \leq \pi$ and $f(x + 2\pi) = f(x)$ for all x. Compute Fourier coefficients of f.

10. Define Beta function.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

Or

(b) If $f \in \mathcal{R}(\alpha)$ on $[a, b]$ and $|f(x)| \leq M$ on $p[a, b]$ prove that

$$\left|\int_a^b f d\alpha\right| \leq M[\alpha(b) - \alpha(b)].$$

12. (a) State and prove the Cauchy criteria for uniform convergence.

Or

(b) If $\{f_n\}$ is a sequence of continuous function on E and if $f_n \to f$ uniformly on E, prove that f is continuous on E.

13. (a) If K is a compact metric space, if $f_n \in \mathcal{C}(K)$ for $n = 1, 2, \ldots$, and $\{f_n\}$ converges uniformly on K, prove that $\{f_n\}$ is equicontinuous on K.

Or

(b) Prove that for every interval $[-a, a]$ there is a sequence of real polynomial P_n such that $P_n(0) = 0$ and such that

$$\lim_{n \to \infty} P_n(x) = |x|$$

uniformly on $[-a, a]$.

14. (a) Suppose $\sum c_n$ converges, put $f(x) = \sum_{n=0}^{\infty} c_n x^n (-1 < x < 1)$, prove that

$$\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$$

Or

(b) Define :

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$$

Prove that f has derivatives of all orders at $x = 0$, and that $f^n(0) = 0$ for $n = 1, 2, \ldots$

15. (a) Let the sequence of complex functions $\{\phi_n\}$ be an orthogonal on $[a, b]$. Let

$$s_n(x) = \sum_{m=1}^{n} c_m \phi_m(x)$$

be the n^{th} partial sum of the Fourier series of f and suppose

$$t_n(x) = \sum_{m=1}^{n} c_m \gamma_m \phi_m(x).$$

Prove that

$$\int_a^b |f - s_n|^2 dx \leq \int_a^b |f - t_n|^2 dx$$

Or

(b) If for some x, there are constants $\delta > 0$ and $M < \infty$ such that

$$|f(x+t) - f(x)| \leq M|t|$$

for all $t \in (-\delta, \delta)$, show that

$$\lim_{N \to \infty} s_N(f : x) = f(x).$$

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

Each question carries 10 marks.

16. Suppose $f \in \mathcal{R}(\alpha)$ on $[a, b]$ $m \leq f \leq M$, ϕ is continuous on $[m, M]$ and $h(x) = \phi f(x))$ on $[a, b]$ prove that $h \in \mathcal{R}(\alpha)$ on $[a, b]$.

17. Suppose $\{f_n\}$ is a sequence of differentiable function on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, prove that $\{f_n\}$ converges uniformly on $[a, b]$ to a function f and

$$f'(x) = \lim_{n \to \infty} f_n'(x) \ (a \leq x \leq b).$$

18. State and prove Stone-Weierstrass theorem.

19. Derive Taylor's series.

20. State and prove the Parseval's theorem.

R6586

Sub. Code
511203

M.Sc. DEGREE EXAMINATION, APRIL – 2022

Second Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

Each question carries 2 marks.

1. Define conjugate harmonic function and give example.

2. Prove that $f(z)$ is analytic if and only if $\overline{f(\bar{z})}$ is analytic.

3. Define index of a point with respect to a closed curve.

4. Prove that $n(-\gamma, a) = -n(\gamma, a)$, where $n(\gamma, a)$ is the index of the point a with respect to a closed curve γ.

5. Expand Taylor's series about 0 for $f(z) = e^z$.

6. Find the removable singularities for $f(z) = \dfrac{\sin^2 z}{z}$.

7. Show that the mean-value formula remains valid for $u = \log|1+z|, z = 0, r = 1$.

8. How many residues are there for $f(z)=\frac{1}{(z^2-1)^2}$.

9. Define Bernoulli number.

10. Define Schwarzian derivative of function.

Part B (5 × 5 = 25)

Answer **all** questions, by choosing either (a) or (b).

11. (a) If all zeros of a polynomial $P(z)$ lie in a half-plane, prove that the all zeros of the derivative $P'(z)$ lie in the same half-plane.

Or

(b) Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$.

12. (a) Derive the Cauchy's integral formula.

Or

(b) Compute

(i) $\int_{|z|=1} e^z z^{-n}\, dz$ and (ii) $\int_{|z|=2} z^n (1-z)^n dz$.

13. (a) Prove that the non-constant analytic function maps open sets onto open sets.

Or

(b) State and prove the Schwarz's lemma.

14. (a) State and prove maximum principle for harmonic function.

Or

(b) Prove that $\int_{-\infty}^{\infty} \frac{e^{ix}}{x} dx = \pi i$.

15. (a) The Fibonacci numbers are defined by $c_0 = 0$, $c_1 = 1$, $c_n = c_{n-1} + c_{n-2}$. Show that the c_n are Taylor coefficient of a rational function.

Or

(b) If $f(z)$ is analytic in the region Ω containing z_0, prove that the representation

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \ldots + \frac{f^n(z_0)}{n!}(z - z_0)^n + \ldots$$

is valid in the largest disk of center z_0 contained in Ω.

Part C (3 × 10 = 30)

Answer any **three** questions.

Each question carries 10 marks.

16. (a) If z_1, z_2, z_3, z_4 are distinct points in the extended plane and T any linear transformation, prove that $(T_{z1}, T_{z2}, T_{z3}, T_{z4}) = (z_1, z_2, z_3, z_4)$.

(b) Prove that the cross ratio z_1, z_2, z_3, z_4 is real if and only if the four points lie on a circle or on a straight line.

17. State and prove the Cauchy's theorem for rectangle.

18. Prove that a region Ω is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω and all points a which do not belong to Ω.

19. (a) State and prove the Rouche's theorem.

(b) State and prove residue theorem.

20. Derive the Laurent series.

R6587

Sub. Code
511501

M.Sc. DEGREE EXAMINATION, APRIL – 2022.

Second Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

Each question carries 2 marks.

1. Define C^m–function.

2. If γ be a curve $r = r(\mathrm{u})$, and let P, Q be two neighbouring points on the curve have parameters u_0, u. Write the unit tangent vector to γ at P.

3. Define center of spherical curvature, and give its position vector.

4. Define circular helix.

5. Give an example for two systems of parametric curves are orthogonal.

6. Write the tangential part and tangential components of a vector.

7. Define double family of curves.

8. Write the characteristic property of a geodesic.

9. Find the geodesic curvature of the parametric curve $v = c$.

10. Write the Liouville's formula for K_g.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the equations of the circular helix $r = (\alpha cos u, \alpha sin u, bu), \quad -\infty < u < \infty$, where $a > 0$ and show that the length of one complete turn of the helix is $2\pi c$, where $c = \sqrt{(a^2 + b^2)}$.

Or

(b) If a curve is given in terms of a general parameter u show that the equation of the osculating plane corresponding to $[R - r(0), r'(0), r''(0)] = 0$, is $[R - r, \dot{r}, \ddot{r}] = 0$.

12. (a) If a curve lies on a sphere, show that ρ and σ are related by $\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$

Or

(b) Show that the involutes of a circular helix are plane curves.

13. (a) Derive the formula for the magnitude of a tangential vector in terms of its components.

Or

(b) Show that the curves bisecting the angles between the parametric curves are given by $Edu^2 - Gdv^2 = 0$.

14. (a) Find the orthogonal trajectories of the sections on the paraboloid by the planes z = constant.

Or

(b) A particle is constrained to move on a smooth surface under no force except the normal reaction. Prove that its path is a geodesic.

15. (a) Prove that if the orthogonal trajectories of the curves v = constant are geodesics, then H^2/E is independent of u.

Or

(b) Prove that the Gaussian curvature of the surface given (in Monge form) by $z = f(x, y)$ is $(rt - s^2)(1 + p^2 + q^2)^{-2}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

Each question carries 10 marks.

16. Find the curvature and torsion of the curve of intersection of the two quadratic surfaces $ax^2 + by^2 + cz^2 = 1,\ a'x^2 + b'y^2 + c'z^2 = 1$.

17. Explain contact between curves and surfaces.

18. Find the coefficients of the direction which makes a D angle $\frac{1}{2}\pi$ with the direction whose coefficients are (l, m).

19. Prove that the curves of the family $v^2/u^2 =$ constant are geodesics on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2 \quad (u > 0, v > 0)$.

20. State and prove Gauss Bonnet theorem.

———————

R6588

Sub. Code
511401

M.Sc. DEGREE EXAMINATION, APRIL – 2022

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define normed linear space.

2. Define dual space.

3. Define orthogonal.

4. Write the statement of the completion theorem.

5. Define Hilbert adjoint operator.

6. Prove that $(\alpha T)^* = \overline{\alpha} T^*$, where α is any scalar and T is a bounded linear operator from H_1 to H_2.

7. Show that a norm on a vector space X is a sublinear functional on X.

8. State Hahn-Banach theorem.

9. Prove that every subsequence of (x_n) converges weakly to x.

10. Define closed linear operator.

Part B $(5 \times 5 = 25)$

Answer **all** the questions, choosing either (a) or (b).

11\. (a) Let $T : D(T) \to Y$ be a linear operator where $D(T) C X$ and X, Y are normed space then prove that,

(i) T is continuous if and only if T is bounded.

(ii) If T is continuous at a single point, it is continuous.

Or

(b) State and prove the Riesz's lemma.

12\. (a) State and prove null space lemma.

Or

(b) Prove that an orthogonal set is linearly independent.

13\. (a) State and prove the properties of Hilbert-adjoint operators.

Or

(b) Let the operators $U : H \to H$ and $V : H \to H$ be unitary (Here, H is a Hilbert space), Prove that

(i) $\|U\| = 1,$ provided $H \neq \{0\}$

(ii) U is normal.

14\. (a) Show that the normed space X_n itself is separable if the dual space X' of X is separable.

Or

(b) Prove that every Hilbert space is reflexive.

15. (a) State and prove closed graph theorem.

Or

(b) Prove that strong convergence implies weak convergence with the same limit.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that the dual space of l^p is l^q where $1 < p < +\infty$ and q is the conjugate of p.

17. State and prove Schwarz inequality and triangle inequality.

18. State and prove the existence of Hilbert adjoint operator T^* of T with the norm $\|T^*\| = \|T\|$.

19. State and prove uniform boundedness theorem.

20. State and prove open mapping theorem.

R6589

Sub. Code
511402

M.Sc. DEGREE EXAMINATION, APRIL – 2022

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. State the properties of conditional probability.

2. Define continuous random variable.

3. Define joint cumulative distribution function.

4. Define moment generating function of a random vector.

5. Define Binomial distribution.

6. Write the pdf and mgf of χ^2 distribution.

7. Define t-distribution.

8. Define sample range and sample median.

9. State weak law of large numbers.

10. Define convergence in distribution.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Urn one contains 2 black and 3 red balls; urn two contains 3 black and 2 red halls. We toss an unbiased coin to decide on the urn to draw from but we do not know which is which. Suppose the first ball drawn is black and it is put back. What is the probability that time second ball drawn from the same urn is also black?

Or

(b) Let X have the pdf $f(x) = \begin{cases} 2(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$. Find the value of $E[6X + 3X^2]$.

12. (a) Let X_1 and X_2 have the pdf $f(x_1, x_2) = \begin{cases} 8x_1x_2 & \text{if } 0 < x_1 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$ Calculate the value of $E[7X_1X_2^2 + 5X_2]$.

Or

(b) Let the joint pdf of X_1 and X_2 be $f(x_1, x_2) = \begin{cases} x_1 + x_2 & \text{if } 0 < x_1 < 1.0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$. Show that X_1 and X_2 are dependent.

13. (a) Prove that $\mu = \sigma^2 = \lambda$ for a Poisson distribution with parameter λ.

Or

(b) Derive the formula for mgf of $\Gamma(\alpha, \beta)$ distribution.

14. (a) Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample of size 4 from a distribution having pdf $f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$. Calculate the value of $P(1/2 < Y_3)$.

Or

(b) Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample of size 3 from a distribution with pdf $f(x) = 1.0 < x < 1$, zero elsewhere. Let $Z = (Y_1 + Y_3)/2$ be the midrange of the sample. Find time pdf of Z.

15. (a) Prove that if X_n converges to X in probability, then X_n, converges to X is distribution.

Or

(b) Let $\overline{X}$ denote the mean of a random sample of size 100 from a distribution that is χ^2 (50). Compute an approximate value of $P(49 < \overline{X} < 51)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X be a random variable such that $P(X \le 0) = 0$ and let $\mu = E[X]$ exist. Show that $P(X \ge 2\mu) \le \frac{1}{2}$.

17. Let (X_1, X_2) be a random vector such that the variance of X_2 finite. Prove that

(a) $E[E(X_2|X_1)] = E(X_2)$.

(b) $Var[E(X_2|X_1)] \le Var(X_2)$.

18. Suppose (X,Y) has a $N_2(\mu,\Sigma)$ distribution where $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$. Prove that $E(Y|x) = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1)$ where ρ is the correlation coefficient.

19. State and prove Student's Theorem.

20. State and prove Central Limit Theorem.

R6590

Sub. Code
511403

M.Sc. DEGREE EXAMINATION, APRIL – 2022

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS – 2019 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Subgraph of a graph with an example.

2. Define a clique of a graph G.

3. Define: A k-connected graph G.

4. Define block of a graph G.

5. Define maximum matching of a graph G.

6. What is Ramsey number of a graph G?

7. Define chromatic number of a graph G.

8. What is edge chromatic number of a graph G?

9. Define faces of a graph G.

10. Write down the Euler's formula with clear explanation.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) For a simple graph G, Prove that atleast one of the graphs G and G is connected.

Or

(b) Prove that for a simple graph G with n vertices and k components then, $|E_G| \le \frac{(n-k)(n-k+1)}{2}$.

12. (a) Let e be a cut-edge connected graph G. Then prove that $G-e$ has atleast two components.

Or

(b) Show that, for any odd prime n, the edges of K_n can be partitioned into $\frac{n-1}{2}$ edge - disjoint hamiltonian cycles.

13. (a) Show that every nonempty regular bipartite graph has a perfect matching.

Or

(b) Show that, for all k and $l, r(k,l) = r(l,k)$.

14. (a) For a bipartite graph G, show that $\chi'(G) = \delta_{\max}(G)$.

Or

(b) Show that for a chromatically k-critical graph G, then no vertex of G has degree less than $k-1$.

15. (a) (i) Show that the dual of any planar graph is connected.

(ii) If G is a plane graph, then show that

$$\sum_{f \in F} d(f) = 2m$$

Or

(b) For a simple planar graph C on atleast three vertices, then show that $m \le 3n - 6$, Furthermore, $m = 3n - 6$ if and only if every planar embedding of G is a triangulation.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. For a connected graph G, show that the following are equivalent:

(a) G is Eulerian.

(b) The edge of every vertex in G is even.

(c) E_G is the union of the edge-sets of a set of edge-disjoint cycles of G.

17. For a simple n vertex graph G, where ≥ 3, such that $\deg(x) + \deg(y) \ge n$ for each pair of non-adjacent vertices x and y, then show that G is Hamiltonian.

18. Prove that, for any two integers $k \ge 2$ and $l \ge 2$,

$r(k,l) \le r(k,l-1) + r(k-1,l)$

Furthermore, if $r(k,l-1)$ and $r(k-1,l)$ are both even, and the inequality holds strictly.

19. For a simple graph G, show that there exists a proper edge-colouring of G, that uses atmost $\delta_{\max}(G)+1$ colors.

20. State and Prove the Five Color theorem.

———————